

## Computing in Terms of Takeuti's Quantum Set Theory

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### ABSTRACT

In this letter we discuss computing in terms of Gaisi Takeuti's quantum set theory using an explicit example of a rigid body of cuboid form. The universe  $V^{(L)}$  of Takeuti will be determined. A set of real numbers in this Universe are also explicitly described including a set of binary numbers. Thus we arrive at the foundations of von Neumann' theory of computing in terms of ordinary binary numbers. Then we see that this extension of computing to the Universe  $V^{(L)}$  provides a sound, mathematically well defined theory of quantum computing.

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### Introduction

Let us consider the well-known example of a free (quasi) particle of mass  $m$  moving in a box of cuboid form [1]. The time evolution is determined by the Schrödinger equation

$$i\hbar\partial\psi/\partial t = H\psi, \quad \psi \in L^2([0,a],[0,b],[0,c]) \quad (1)$$

where  $L^2([0,a],[0,b],[0,c])$  is the Hilbert space of the square integrable functions over the domain of the cuboid form with side-edges  $a, b, c$ , and

$$H = -(\hbar^2/2m)\Delta + V(x,y,z)$$

is the Hamiltonian of the system, where  $\Delta$  is the Laplace operator and the potential function is of the form

$$V(x,y,z) = V(x) + V(y) + V(z), \\ V(x) = V(y) = V(z) = 0, \text{ if } 0 < x < a, 0 < y < b, 0 < z < c; \infty \text{ otherwise.}$$

Then the Schrödinger equation (1) becomes three one dimensional equations having solutions of the form

$$\begin{aligned} \psi_1(x) &= A \sin k_1 x + B \cos k_1 x, & 0 < x < a, \\ \psi_2(y) &= C \sin k_2 y + D \cos k_2 y, & 0 < y < b, \\ \psi_3(z) &= F \sin k_3 z + G \cos k_3 z, & 0 < z < c, \end{aligned}$$

where

$$(k_1)^2 + (k_2)^2 + (k_3)^2 = (2m/\hbar^2)E$$

and

$$\psi_1(x)\psi_2(y)\psi_3(z) = \psi(x,y,z)$$

Since on the wall of the cuboid  $\psi=0$ , the wave functions of norm 1 have the form

$$\psi_{n_1,n_2,n_3}(x,y,z) = (8/abc)^{1/2} \sin(n_1\pi/a)x \sin(n_2\pi/b)y \sin(n_3\pi/c)z \quad (2)$$

The eigenvalues of the quasi particle's energy are discrete in the form

$$E_{n_1,n_2,n_3} = (\pi^2\hbar^2/2m)(n_1^2/a^2 + n_2^2/b^2 + n_3^2/c^2), \quad n_1, n_2, n_3 = 1, 2, 3, \dots \quad (3)$$

Then, in the special case when  $a = b = c$ , i.e. the cuboid form is a cube, we have

$$n_1^2 + n_2^2 + n_3^2 = (2ma^2/\pi^2\hbar^2) E_{n_1,n_2,n_3}, \quad n_1, n_2, n_3 = 1, 2, 3, \dots \quad (4)$$

Thus with these relations, formulas and mathematical objects we can determine the components of Takeuti's approach. The basic Hilbert space is the state space  $L^2([0,a],[0,b],[0,c])$  spanned by the orthonormal functions in (2) which set of functions constitutes a basis for this Hilbert space.  $L$  is the lattice of all closed linear subspaces of  $L^2([0,a],[0,b],[0,c])$ . Then the totality of all  $L$ -valued functions provides the Universe  $V^{(L)}$  for us [remember: the totality, the set of all (0,1)-valued functions (the characteristic functions of the sets in classical set theory) gives the Universe  $V$  in classical set theory] [2].

In the "quantum mathematics" based on  $V^{(L)}$ , the real numbers defined by Dedekind's cuts are self-adjoint operators of the basic Hilbert space  $L^2([0,a],[0,b],[0,c])$  as it was shown by Takeuti in [2]. Therefore the "quantum real numbers" are self-adjoint operators and the algebra of them is the algebra of these operators. The binary numbers are replaced by the "quantum binary numbers", namely in symbols  $(0, 1) \rightarrow (0, p(X), 1) [p^2(X) = p(X)]$ , the orthogonal projector of the closed linear subspace  $X$  of  $L^2([0,a],[0,b],[0,c])$ , i.e.  $X$  is an element of  $L$ . In this way we have in symbols:

the machine-made code of a classical program has the form of  $(1, 0, 0, 1, 1, \dots)$ , then

the machine-made code of a “quantum program” should have the form of  $(p(X), 1, 0, p(Y), p(Z), \dots, 0, \dots)$ .

Therefore the Hamiltonian of the system is a real number valued function in this Takeuti's Universe and we can write it in the form

$$H = \sum_{[n]} E_n P(\psi_n) \quad (5)$$

where  $P(\psi_n)$  is the orthogonal projector of the one dimensional subspace of  $L^2([0,a],[0,b],[0,c])$  spanned by the ray belonging to the eigenstate  $\psi_n$ . Then one can express the Hamiltonian  $H$  as a linear combination of binary number valued functions in  $V^{(L)}$  having the form

$$b = \sum_{[n]} e(n)P(\psi_n), \quad e(n) = 0 \text{ or } 1, \quad (6)$$

where  $P(\psi_n)$  is the orthogonal projector in  $L^2([0,a],[0,b],[0,c])$  according to the relation of (5). The set of these binary numbers is a subset of the set of all binary numbers in  $V^{(L)}$  (the elements of this set of self-adjoint operators in  $L^2([0,a],[0,b],[0,c])$  do all commute with the Hamiltonian  $H$ ).

Therefore one can approach the real numbers in the Universe  $V^{(L)}$  by linear combinations of “quantum binary numbers” in this Takeuti's Universe. In this way one can extend the methods of von Neumann's computing in a straightforward way to the Takeuti's Universe  $V^{(L)}$  arriving then at a mathematically well defined and implementable extension of computing, at a quantum computing [3]. The cuboid form of rigid body in our example might be an essential part of the physical implementation of the processor for a quantum computer of this type.

We note that, as it is well known, L. D. Landau described the quantum liquids  $\text{He}^3$  and  $\text{He}^4$  at low temperature (below 3 K) by applying the quasi particle approach, outlined above, called the quasi particles (elementary excitations) as rotons”. Thus these physical materials might be the candidates for building the processor of a quantum computer of this type.

## References

1. Constantinescu F, Magyari E (1968) Quantum Mechanics, Problems. Technica Publishing House, Bucharest.
2. Takeuti G (1981) Quantum Set Theory, in Current Issues of Quantum Logic. Plenum Press, New York and London 303-322.
3. Banai M (2023) Computing based on Quantum Set Theory. Int Journal of Computer Science and Information Technologies 14: 35-36.

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