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Quantum Computing: The Alternative Way[1]

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ABSTRACT

An alternative way of quantum computing is presented. This approach is more general than the one based on the notion of the quantum bit. Mathematically it is based on the quantum set theory of Gaisi Takeuti. A possible implementation of this quantum computer belonging to this quantum computing could be to build up by centering it on the quantum liquids He^3 and He^4 at low temperature (below 3K) and by applying the quasi-particle approach of L.D. Landau.

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Introduction

Ladies and gentlemans I am Miklós Banai, a retired theoretical physicist from Hungary, from Budapest. I deeply acknowledge the invitation and the help to the organizers of this conference. My talk consists of free main blocks. The first block deals with the formulation of the problem and the theoretical way I hope to lead to its solution. The second block is devoted to some extent to the technical details necessary to understand the essence, the point of this approach. In the third block the presented apparatus is applied to computing to arrive at a mathematically well-defined and theoretically implementable quantum computing and quantum computer which is more general than the approach based on the notion of the quantum bit.

Let us start with the first block, with our famous compatriot, John von Neumann.

Digitalization by John von NEUMANN
Budapest, 1925-26, Hungary
The Mathematical Foundation Of Quantum Mechanics (QM) was formulated by John von Neumann. He observed the basic significance of the (0, 1), i. e. digital valued functions, projections in this mathematical framework.

He generalized this idea and proposed: Let we describe the objects of the world around us with their digital reprezentations and let we mechanize the reprezentation, i. e. build up a **computer** which implements the assignment and creates the digital representation.

The engine/motor of the computer is the device implementing the (0,1) valued operations, i.e **the processor**.

Coding, Decoding
Between Human and **Mashine**: the solusion is the digitalization (of John von Neumann); the base of it:
In cases of finitely many degrees of freedom we have: Every observables can be derived from (0, 1)-valued elementary observables! The instruments of them:

- programing language,
- interpreter (translating the program to binary form),
- computer (memory + processor).

Question
Is the digitalization the sloly solution or does there existe a more general approach of the problem?
The answer: There does existe a more general approach!
For, in case of systems with infinitely many degrees of freedom (e.g. EM field), the observables are not built up from (0,1)-valued elementary observables but (0, e(x), 1) (trialy) valued ones, where e(x) can have infinitely many „true-false" values (see [1]).
Such a non trivial three valued logic is **the quantum logic of von Neumann**. **Gaisi Takeuti** showed, that one can derive a mathematics, based on this logic, similar to the one based on the two valued (true, false) logic but which is more „gigantic" than the old one. This is the mathematics based on **the quantum set theory of Takeuti**.
In this framework we can built up a mathematicaly well-defined quantum computing and quantum computer which is a real alternative of the approach based on the notion of the quantum bit (see ref.s [2-4]).

Geometrical Illustration
We have a problem to be solved. We have to study 100 diferent alternatives to obtain the solution.
Our computers in these days are able to compute the 100 alternatives consecutively. The professionals expect from the „quantum computer" that it will be able to sudy the 100 alternatives in parallel, simultaneously. In this way it reduces the time of the solution radically.

How?
We can illustrate the geometrical picture of the caracteristic mathematical objects in the quantum set theory of Gaisi Takeuti by the next figure (Figure 1.). One can see in the figure that the elementary systems constituting the whole system are side by side in space. The collective states of the system are given by

[1]ICAICC-25, Orlando, Florida, 09/05/2025, keynote talk

the sections of the compact cylinder which are evolving thogether (parallel) in time. The appropriate mathematical object, the local state space of the system describes the system **coherently (not in a loosely way)**. (Γ represents the event space of the underlying spacetime, $H{\times}\Gamma$ over it is representing the local state space H_A of the system with infinitely many degrees of freedom. At the event p there is a state Hilbert space H_p describing the states of the elementary system of finitely many degrees of freedom at the event/point of the underlying spacetime. Thus one can think of the local state space as a kind of a Hilbert bundle. The section Ψ represents a local state of the whole system.)

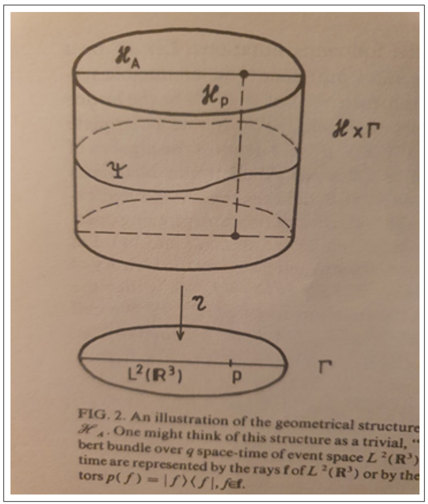

Figure 1

Briefly the main points

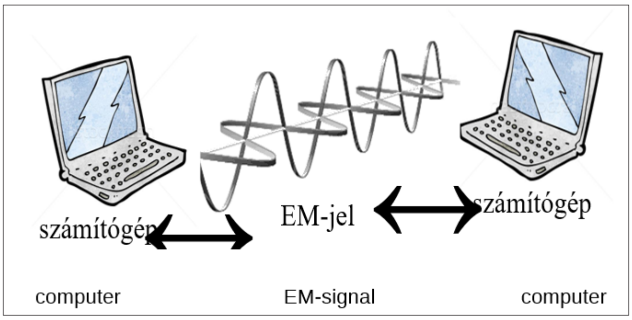

Figure 2

Quantum theory of point-like objects →digitalization
The logical values: P := {0, 1} (binary valued classical logic)

Quantum theory of extended objects/systems →"trialization"
The logical values: P := {0, e_x, 1} (three valued quantum logic)

The coresponding Mathematics 1

It is the mathematics corresponding to the quantum theory of point-like systems (of finitely many degrees of freedom), i.e. the matematics in the (classical) univers $V^{(0,1)}$ of set theory.
The method of computing is:

- digitalization,
- with digital computer
- and digital processor.

The coresponding Mathematics 2

It is the mathematics corresponding to the quantum theory of extended systems (of infinitely many degrees of freedom), i. e. the mathematics in the quantum univers $V^{(L)}$ of the quantum set theory of Gaisi Takeuti.
The method of computing should be:

- "trialization",
- with "trial" computer,
- with "trial" processor,

i. e. with a quantum computer basen on the qunatum logic of von Neumann.

Some Technical Details

Let we consider the illustrative case of N real classically relativistic scalar fields of Lagrangian density

$$L(t,\boldsymbol{x}) = [1/2\Sigma_{\alpha=1}{}^{N}(\partial_\mu\phi^\alpha\partial_\mu\phi^\alpha - m_\alpha{}^2\phi_\alpha{}^2) - V(\phi_1, \ldots, \phi_N)] \quad (t,\boldsymbol{x}),\ (t,\boldsymbol{x})\in \boldsymbol{M}^4 \quad (4.1)$$

This system consists of an infinite collection of identical classical anharmonic oscillators of N degrees of freedom connected in space. Then the corresponding quantum field theory (QFT) should consist of an infinite collection of identical quantum anharmonic oscillators of N degrees of freedom connected in space.

Really, the alternative quantization method substitutes the individual members of the system by their quantum mechanical counterparts [5]. The *local state pace* H_A is an A-valued Hilbert space (Hilbert A-module) of the form $L^2(\boldsymbol{R}^N)\otimes A$ [the tensor product of the complex separable Hilbert space $L^2(\boldsymbol{R}^N)$ and the C*-algebra A of bounded operators of $L^2(\boldsymbol{R}^3)$]. In this approach the quantized system is described **coherently** because the algebra of bounded operators $B(H_A) = B(H)\otimes A$ of the local state space H_A is a **factor** [6] (which means that we can not divide it into two factors).

Von Neumann's basic theorem of QM, namely that the canonical commutation relations (CCR's) have a unique solution up to unitary equivalence, has an extended form in this framework:
a B-irreducible[2] set of unitary operators in the A-valued Hilbert space H_A satisfying the CCR's is uniquely determined up to A-unitary equivalence [5].

In this way this extension of von Neumann's theorem offers the possibility that one formulates QFT in terms of the A-valued Hilbert spaces in the same unique way, up to A-unitary equivalence as QM is formulated in terms of complex Hilbert spaces up to unitary equivalence [5].
The dynamics of the system is described by the unitary map

$$t \rightarrow exp\ (-iHt)$$

of H_A onto itself, where H is the **local Hamiltonian** of the system obtained by replacing the *Hamiltonian density* of the classical system with its operator counterpart one gets by the quantization algorithm.

$$H = H(\phi, \pi, \partial\phi) = $$
$$= 1/2\Sigma_{\alpha=1}{}^{N}\,[\pi_\alpha{}^2 + (\partial\phi_\alpha)^2 + m_\alpha{}^2\phi_\alpha{}^2] + V(\phi_1, \ldots, \phi_N),$$

where the fields ϕ_α and their canonical momentum densities π_α as operators in H_A satisfy the CCR's [5]. The classical equations of motion become well-defined operator equations in H_A and the local states (the ray's $\boldsymbol{\Phi}$ of H_A, i.e. for all $\Phi\in\boldsymbol{\Phi}$ we have $\langle\Phi|\Phi\rangle_A = 1$, where $\langle|\rangle_A$ denotes the A-valued inner product in H_A and 1 is the unity operator of A) are governed by the local Schrödinger equation [5]:

$$i\hbar\partial\Phi(t)/\partial t = 1/2\Sigma_{\alpha=1}{}^{N}[\pi_\alpha{}^2 + (\partial\phi_\alpha)^2 + m_\alpha{}^2\phi_\alpha{}^2]\Phi(t) + V(\phi_1, \ldots, \phi_N)\ \Phi(t),\ \Phi\in H_A \quad (4.1)$$

[2]A system of bounded operators in an A-valued Hilbert space is B-irreducible if the set of bounded operators commuting with all the members of the system is equal to the Abelian von Neumann algebra B.

Then we arrived at the main result:
Proposition: Different alternatives [for the individual members of the infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space] given by an initial value of the evolution equation described by a section of the „non commutative" Hilbert bundle can be computed in **parallel**.
One can apply the **extension** of the perturbation theory of QM to solve this equation by using the interaction picture [5]. The local Hamiltonian of the free fields is

$$H_0 = 1/2\Sigma_{\alpha=1}^N[\pi_\alpha^2 + (\partial\phi_\alpha)^2 + m_\alpha^2\phi_\alpha^2] = \Sigma_{\alpha=1}^N (N_\alpha + 1/2)p_0^\alpha,$$

where $p_0^\alpha = (\mathbf{p}^2 + m_\alpha^2)^{1/2}$, $\mathbf{p}^2 = (-i\hbar\partial)^2 = -\hbar^2\Delta$, Δ is the Laplace operator, and $N_\alpha = a_\alpha^+ a_\alpha$, a_α^+ is the creation while a_α is the annihilation operator in the local Fock space F_A of the free fields [5].

p_0^α is the energy operator of a Klein-Gordon-like free particle of mass m_α, i.e. its Hamiltonian operator. In this framework the Haag-theorem does not block to solve the local Schrödinger equation for non-trivial interactions in the local Fock space F_A of the free fields [5].

We apply this formalism to a system of Lagrangian density (4.1) considered localized in space to a cube with side-edges *a*. In that case the basic Hilbert space in the Takeuti's approach reduces to the Hilbert space $L^2([0,a]^3)$ of the square integrable functions over the domain of the cube.

We diagonalize p_0^α. It means the solutions of the eigenvalue equations

$$p_0^\alpha\phi_n = e^\alpha{}_n\phi_n, \quad \phi_n\in L^2([0,a]^3)$$

The wave functions of norm 1 have the form

$$\phi_{n1,n2,n3}(x,y,z) = (2/a)^3 1/2 \sin (n_1\pi/a)x \sin (n_2\, \pi/a)y \sin (n_3\, \pi/a)z \qquad (4.2)$$

and

$$e^\alpha{}_{n1,n2,n3} = [\pi^2\hbar^2 a^{-2}(n_1^2+ n_2^2+ n_3^{\ 2}) + m_\alpha^2]^{1/2},\ n_1, n_2, n_3 = 1, 2, 3, ...$$

The quantized system localised it in a cube in space has a discrete energy spectra in the free field approximation.

Application in Computing
The basic Hilbert space is $L^2([0,a]^3)$ spanned by the orthonormal functions of the relation (4.2) which set of functions constitutes a basis for this Hilbert space. *L* is the lattice of all closed linear subspaces of $L^2([0,a]^3)$ (the quantum logic of von Neumann). Then the totality of all *L*-valued functions provides the universe $V^{(L)}$ of Takeuti.

In the "quantum mathematics" based on $V^{(L)}$, the real numbers defined by Dedekind's cuts are self-adjoint operators of the Hilbert space $L^2([0,a]^3)$ as it was shown by Takeuti. Therefore the "quantum real numbers" are self-adjoint operators and the algebra of them is the algebra of these operators. The binary numbers are replaced by the "quantum binary numbers", in symbols (0, 1) → (0, *p*(*X*), *1*) [$p^2(X) = p(X)$, the orthogonal projector of the closed linear subspace *X* of $L^2([0,a]^3)$, i.e. *X* is an element of *L*]. In this way we have in symbols:
the machine-made code of a classical program has the form of (1, 0, 0, 1, 1, ….),
then
the machine-made code of a "quantum program" should have the form of (*p(X), 1, 0, p(Y), p(Z), …, 0, …*).

The unity operator *1* of $L^2([0,a]^3]$) belongs to the **true** logical value, the zero operator *0* belongs to the **false** logical value, while the projection operators *p*(*X*), *p*(*Y*), *p*(*Z*), … to the **true-false** values, e.g. *p*(*X*) is **true** on the subspace *X* while it is false outside *X* (on the difference subspace *L**X*). Clearly the number of the true-false values is **infinite**.

The local state space $H_A= L^2(R^N)\otimes A$ is isomorphic to the countably infinite direct sum $H_A = \Sigma_1{}^\infty\oplus A$ of the Hilbert *A*-module *A*. This means that we can represent H_A with infinite column vectors with operator entries from *A*. The local states are represented by the rays of norm *1* (the unity operator of *A*) in H_A. The expectation value of a local bounded observable *F* in the local state Φ in H_A is given by the formula

$$Exp\ F = <\Phi|F|\Phi>_A \in A$$

using the *A*-valued inner product of H_A.
The local Hamiltonian *H* is a real number valued function in $V^{(L)}$:

$$H = \Sigma_{[n]}\, E_n\, P(\phi_n)$$

where $P(\phi_n)$ is the orthogonal projector of the one dimensional subspace of $L^2([0,a]^3)$ spanned by the ray belonging to the eigenstate ϕ_n, while E_n is also a hermitian element of *A* from the spectrum of the local Hamiltonian (which is a hermitian operator in the *A*-valued Hilbert space H_A). Therefore:

$$E_n = \Sigma_{[m]}\, e_m P(\phi_m)$$

where the ordinary non-negative real number e_m is from the spectrum of E_n (which of course may has not only discrete but continuous spectrum, too).

Thus the local Hamiltonian of the quantized system is a hermitian valued function in $V^{(L)}$ with values of form

$$H(\Phi) = Exp\ H = \Sigma_{[n]}\, e_n P(\phi_n) \in A$$

Then one can express the expectation value of the local Hamiltonian *H* as a linear combination of binary number valued functions in $V^{(L)}$ having the form

$$b = \Sigma_{[n]}\, b(n)P(\phi_n\,),\ b(n) = 0 \text{ or } 1$$

where $P(\phi_n)$ is the orthogonal projector in $L^2([0,a]^3)$ belonging to the eigenstate ϕ_n. The set of these binary numbers is a subset of the set of all binary numbers in $V^{(L)}$.

Thus one can evaluate, in finite linear combinations, the evolution of the quantized system, localised in a cube, in the local Fock space F_A of the free fields by applying the eigenstates of the energy operators of the Klein-Gordon-like particles of mass m_α, and thus in a finite steps of recursions. Therefore one can approach (or at least estimate) the real numbers in the universe $V^{(L)}$ by linear combinations of "quantum binary numbers" in this Takeuti's universe.

Conclusion
A physical system having eigenstates of form in the relation (4.2) can help us to solve the system's evolution equation by exciting it and measuring its eigenstates and the corresponding eigenvalues while inserting the results in the appropriate mathematical relations.

Therefore the cube of a "well and appropriately tuned up" rigid or condensed body may be an essential part of the physical implementation of the processor for a "quantum computer" of this type [4].

As a closing note we remember that L. D. Landau described the quantum liquids He^3 and He^4 at low temperature (below 3K) by applying the quasi particle approach outlined in [3,4]. He called the quasi particles (elementary excitations) as "rotons". Thus these physical materials and rotons may be the candidates for building the processor of **a "quantum computer" of this type based on quantum set theory offering a more general framework than the one based on the notion of the quantum bit**.

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