

International Conference on Artificial Intelligence and Cybersecurity (ICAIC 2025)

Conference Proceedings

November 27-28, 2025 (Virtual)

Boolean-Algebraic Framework for Maximal-Degree U-k-Seminets: Theoretical Foundations and Computational Advances

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Abstract

A Boolean-algebraic framework for maximal-degree U-k-seminets is presented, unifying combinatorial and algebraic properties. This work extends Aczel's quasigroup theory and Belousov's k-net constructions by introducing a computational framework for U-k-seminets of maximal degree μ . Key results include: (1) explicit bounds for μ in terms of set cardinality t and t -order d ($\mu = t-d+2$), (2) existence conditions for non-equipotent sets, and (3) inequalities governing μ and t ($(t+2)/2 < \mu \leq t$). Theorems are validated via tabulated solutions for $m = t - d$, demonstrating scalable applications in finite geometry and network design. The framework bridges partial quasigroups and block designs, offering algorithmic tools for seminets with maximal degree constraints.

Keywords: U-k-Seminets, Maximal Degree, T-Order, Boolean Algebra, Partial Quasigroups, Combinatorial Designs, Finite Geometry, Network Coding

References:

1. Stošović, D., Katić, A., & Galić, D. (2023). k-regular decomposable incidence structure of maximum degree. *Mathematica Moravica*, 27(2), 127–136.
2. Galić, R. (1997). The maximal number of U-k-seminets of the maximal degree. *Mathematical Communications*, 2(1), 21–25.
3. Galić, R., Ćajić, E., Stojanović, Z., & Galić, D. (2023). Stochastic methods in artificial intelligence. *Research Square*. <https://doi.org/10.21203/rs.3.rs.3597781/v1>
4. Ušan, J., & Galić, R. (2000). (m,n)-rings as algebras with only one operation. *Mathematical Communications*, 5(2), 133–141.
5. [5] Galić, R., Ćajić, E., Shabani, E., & Ramaj, V. (2024). Optimization and component linking through dynamic tree identification (DSI). *Journal of Mathematical Techniques and Computational Mathematics*, 3(1), 1–9. 25
6. Aigner, M. (1979). *Combinatorial theory*. Springer-Verlag.
7. Barbut, M., & Monjardet, B. (1970). *Ordre et classification*. Librairie Hachette.
8. Birkhoff, G. (1984). *Lattice theory* (3rd ed.). American Mathematical Society.
9. Blyth, T. S. (2005). *Lattices and ordered algebraic structures*. Springer.
10. Crawley, P., & Dilworth, R. P. (1973). *Algebraic theory of lattices*. Prentice Hall.
11. Davey, B. A., & Priestley, H. A. (1992). *Introduction to lattices and order*. Cambridge University Press.
12. Devid'c, V. (1979). *Zadaci iz apstraktne algebre*. Naučna knjiga.
13. Devid'c, V. (1972). *Matematička logika, Prvi dio: Klasična logika sudova*. Matematički institut Beograd.
14. Grätzer, G. (2003). *General lattice theory*. Birkhäuser Verlag.
15. Grimaldi, R. P. (2004). *Discrete and combinatorial mathematics* (5th ed.). Pearson Education.
16. Halmos, P. R. (1974). *Lectures on Boolean algebras*. Springer-Verlag.
17. Lidl, R., & Pilz, G. (1984). *Applied abstract algebra*. Springer-Verlag.
18. Galić, D., Katić, A., Galić, R., & Ćajić, E. (2025). An algorithmic-modeling approach to the classification of network structures using Boolean algebra. *Preprints*. <https://doi.org/10.20944/preprints202505.1203.v1>
19. Mendelson, E. (1970). *Boolean algebra and switching circuits*. McGraw-Hill