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Magnon-Phonon Localization and Quasielastic Spectral Response in Quasi-One-Dimensional Heisenberg Ferromagnets

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ABSTRACT

Localized soliton excitations in a quasi-one-dimensional isotropic Heisenberg ferromagnet are investigated by explicitly accounting for coupling between the magnetic subsystem and longitudinal lattice vibrations. In the low-temperature regime, spin operators are bosonized using the Holstein-Primakoff transformation. In the continuum limit, the coupled dynamics of the magnetic excitation and lattice deformation reduces to a nonlinear Schrödinger equation. A complete set of working expressions is given for the spin-phonon Hamiltonian, phonon energy, continuum equation, one-soliton solution, localization width, energy, momentum, effective mass, and dynamic structure factor. The analysis shows that magnon-phonon coupling produces self-consistent spatial localization of the spin excitation over a finite length scale. As the coupling tends to zero, the localization width diverges and the excitation continuously approaches an extended spin wave. In the low-temperature, low-velocity regime, a dilute soliton subsystem generates a quasielastic central component in the scattering spectrum. Its width and integrated intensity depend on temperature, wave vector, localization length, and effective mass. This central component provides an experimentally accessible signature of nonlinear localized modes in quasi-one-dimensional magnetic systems and can be probed by neutron scattering and, where applicable, infrared spectroscopy.

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Introduction

Low-dimensional magnetic systems are of particular interest in nonlinear physics because spatial confinement enhances collective effects and facilitates the formation of stable localized states. In quasi-one-dimensional ferromagnets, conventional magnons and phonons may coexist with particle-like nonlinear excitations that preserve their localization during propagation. Direct observation of an individual soliton is difficult; therefore, neutron- and electromagnetic-scattering spectra provide essential information, because the soliton subsystem can manifest itself through characteristic features of the dynamic structure factor [1-5]. Recent studies have further established magnon-phonon coupling as a central mechanism governing hybrid spin-lattice excitations in magnetic materials [6]. Inelastic neutron-scattering measurements have directly identified magnon-polaron modes arising from magnon-phonon hybridization, while contemporary theory has clarified how symmetry and phonon angular momentum regulate the coupling strength [7,8]. Temperature-dependent ferromagnetic-resonance experiments have also demonstrated coherent magnon-phonon interaction in ferromagnetic thin-film hybrid systems [9].

The localization mechanism is based on the mutual feedback between a magnetic excitation and lattice deformation. A change in the intersite distance modifies the exchange integral, while the magnetic excitation itself generates a local deformation. As a result, a self-consistent moving structure is formed in which the magnetic and elastic subsystems propagate together. At low temperatures, the spin problem can be represented in bosonic variables, and the continuum limit then leads to a nonlinear Schrödinger equation. This physical picture is consistent with the broader modern understanding that coupling between spin excitations and lattice vibrations can strongly reshape both magnonic and phononic dynamics [6,8].

The aim of this work is to construct a self-contained theoretical description of magnon-phonon localization, provide the complete set of expressions required to characterize the soliton mode, and identify a measurable spectral signature suitable for experimental detection.

Materials and Methods

Theoretical Model

Consider a quasi-one-dimensional chain of magnetic sites with Heisenberg exchange and longitudinal lattice vibrations. The Hamiltonian of the system is written as

$$\hat{H} = T + U - \sum_j J_{j,j+1} \left(\hat{\mathbf{S}}_j \cdot \hat{\mathbf{S}}_{j+1} + \delta \hat{S}_j^z \hat{S}_{j+1}^z \right) \quad \frac{dx_j}{dt} = \frac{P_j}{m}, \quad \frac{dP_j}{dt} = -\frac{\partial H}{\partial x_j}, \quad H = \langle \Psi | \hat{H} | \Psi \rangle. \quad (7)$$

where $J_{j,j+1}$ is the exchange integral between neighboring sites, δ is the exchange-anisotropy parameter, $\mathbf{S}$ is the spin operator at site j , and T and U are the kinetic and potential energies of lattice vibrations. Introducing the standard raising and lowering operators,

$$\hat{S}_j^\pm = \hat{S}_j^x \pm i\hat{S}_j^y.$$

the Hamiltonian can be represented as

$$\hat{H} = T + U - \sum_j J_{j,j+1} \left[\frac{1}{2} \left(\hat{S}_j^+ \hat{S}_{j+1}^- + \hat{S}_j^- \hat{S}_{j+1}^+ \right) + (1 + \delta) \hat{S}_j^z \hat{S}_{j+1}^z \right].$$

The isotropic case $\delta = 1$ is considered below. The kinetic and potential energies of longitudinal chain vibrations are

$$T = \frac{m}{2} \sum_j \left(\frac{dx_j}{dt} \right)^2, \quad U = \frac{mv_0^2}{2a} \sum_j (x_{j+1} - x_j - a)^2. \quad (3)$$

where v_0 is the sound velocity in the crystal, a is the lattice constant, and m is the mass associated with a lattice site.

In the low-temperature regime, spin operators are transformed to bosonic creation and annihilation operators by the Holstein-Primakoff transformation:

$$\begin{aligned} \hat{S}_j^+ &= \sqrt{2S} a_j \sqrt{1 - \frac{a_j^\dagger a_j}{2S}}, \\ \hat{S}_j^- &= \sqrt{2S} a_j^\dagger \sqrt{1 - \frac{a_j^\dagger a_j}{2S}}, \\ \hat{S}_j^z &= S - a_j^\dagger a_j. \end{aligned} \quad (4)$$

At low density of spin deviations, this transformation reduces to

$$\langle a_j^\dagger a_j \rangle \ll 2S: \quad \hat{S}_j^+ \simeq \sqrt{2S} a_j, \quad \hat{S}_j^- \simeq \sqrt{2S} a_j^\dagger, \quad \hat{S}_j^z = S - a_j^\dagger a_j.$$

A low-lying one-particle excitation in the Schrödinger representation is described by the wave function

$$|\Psi(t)\rangle = \sum_j \chi_j(t) a_j^\dagger |0\rangle, \quad a_j |0\rangle = 0. \quad (5)$$

The time evolution of the state is governed by the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H} |\Psi\rangle. \quad (6)$$

The coordinate x_j of the center of the localized deformation is simultaneously treated as the classical coordinate of a particle of mass m :

Thus, the quantum equation for the magnetic excitation and the classical equations of motion for the lattice form a closed self-consistent system.

Results and Theoretical Analysis

Continuum Limit and Nonlinear Schrödinger Equation

The exchange integral depends on the intersite separation. In the vicinity of the equilibrium configuration, its expansion is written as

$$J_{j,j+1} = J + a \frac{\partial J}{\partial x_j} + \frac{a^2}{2} \frac{\partial^2 J}{\partial x_j^2} + \dots$$

Assume that the magnetic excitation and the lattice deformation propagate self-consistently with velocity v . Passing to the continuous coordinate ξ and the field $\chi(\xi, t)$, the coupled system reduces to the nonlinear equation

$$\begin{aligned} i\hbar \frac{\partial \chi(\xi, t)}{\partial t} &= \left[T + U - J_0 S^2 N + \frac{4J_1^2 S}{mv_0^2} \right] \chi(\xi, t) \\ &\quad - J_0 S \frac{\partial^2 \chi(\xi, t)}{\partial \xi^2} - \frac{4J_1^2 S^2}{mv_0^2} |\chi(\xi, t)|^2 \chi(\xi, t). \end{aligned} \quad (8)$$

$$J_0 = J(a = 1), \quad J_1 = -\frac{\partial J}{\partial Y} > 0.$$

The cubic nonlinear term is proportional to J_1^2 , where J_1 characterizes the variation of the exchange integral with deformation. Therefore, spatial localization of the magnetic wave packet is a direct consequence of magnon-phonon coupling. The sensitivity of magnetic dynamics to spin-lattice coupling is also supported by recent theoretical and experimental studies of hybrid magnon-phonon systems [6,8,9].

Soliton Solution and Localization Parameters

For a quasi-one-dimensional isotropic ferromagnet with magnon-phonon interaction, and within the low-velocity approximation, the localized solution of the nonlinear Schrödinger equation can be written as

$$\chi(\xi, t) = (2\mu)^{-1/2} \frac{\exp[i(k\xi - \omega t + \Phi_0)]}{\cosh[(\xi - vt + \xi_0)/\mu]}. \quad (9)$$

The parameters of the solution are determined by

$$\begin{aligned} k &= \frac{\hbar v}{2J_0 S}, \quad \mu = \frac{J_0 m v_0^2}{J_1^2 S}, \quad \hbar\omega = \varepsilon + \frac{\hbar^2 v^2}{4J_0 S} + \varepsilon_0, \\ \varepsilon &= -\frac{(J_1 S)^4}{J_0 S (m v_0^2)^2}, \quad \varepsilon_0 = J_0 S^2 N. \end{aligned} \quad (10)$$

The parameter μ defines the characteristic localization width. Equation (10) gives $\mu \propto J_1^{-2}$: stronger magnon-phonon coupling narrows the localized region, whereas in the limit $J_1 \rightarrow 0$ one obtains $\mu \rightarrow \infty$ and the excitation approaches an ordinary extended spin wave.

For the mean spin projection in the continuum approximation, one obtains

$$\langle S_j^z \rangle = S - |\chi(\xi, t)|^2 = S - \frac{1}{2\mu} \operatorname{sech}^2\left(\frac{\xi - vt + \xi_0}{\mu}\right) \quad (11)$$

Figure 1 shows the normalized spatial profile of the one-soliton excitation. The dimensionless comoving coordinate is $\zeta = (\xi - vt + \xi_0)/\mu$, while $2\mu|\chi(\xi, t)|^2 = \operatorname{sech}^2\zeta$. The maximum at $\zeta = 0$ identifies the soliton center. For $|\zeta| \gg 1$, the profile decreases exponentially, demonstrating strong spatial localization. The scale μ controls the width of the central region: decreasing μ produces a narrower, more strongly localized excitation, whereas increasing μ gives a more extended state.

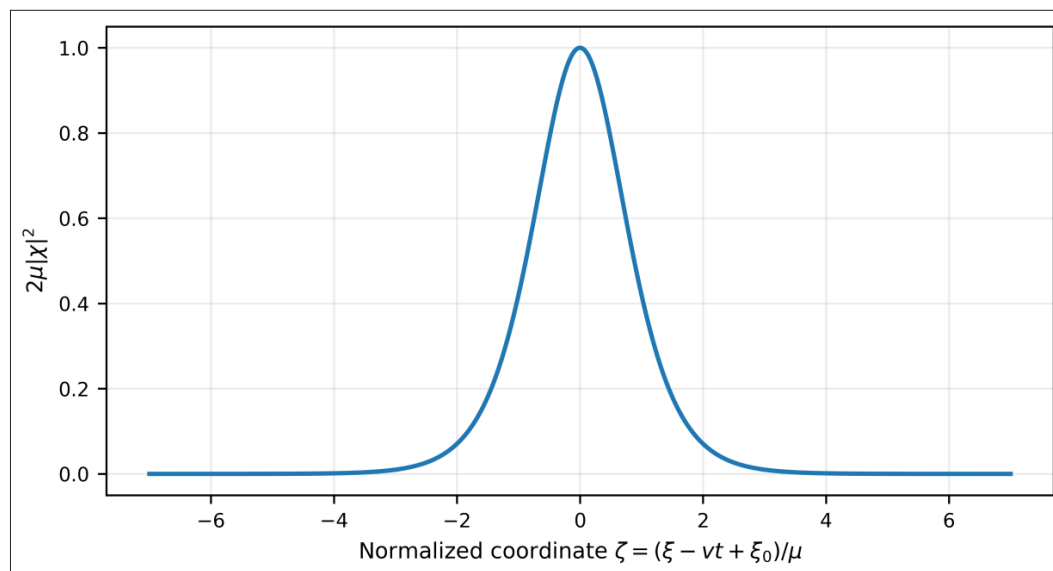


Figure 1: Normalized spatial profile of the localized one-soliton excitation, $2\mu|\chi(\xi, t)|^2 = \operatorname{sech}^2\zeta$, where $\zeta = (\xi - vt + \xi_0)/\mu$. The maximum at $\zeta = 0$ corresponds to the soliton center, and μ defines the characteristic localization width.

The analytical dependence of the localization width on the coupling parameter is summarized in Figure 2. Because $\mu \propto J_1^{-2}$, even a moderate increase in magnon-phonon coupling produces a substantial decrease in localization width. Physically, the magnetic excitation generates a local deformation, which modifies the exchange interaction and creates an effective self-consistent trapping potential for the same excitation. As the coupling is weakened, this feedback becomes ineffective and the localized state continuously evolves into an extended spin wave.

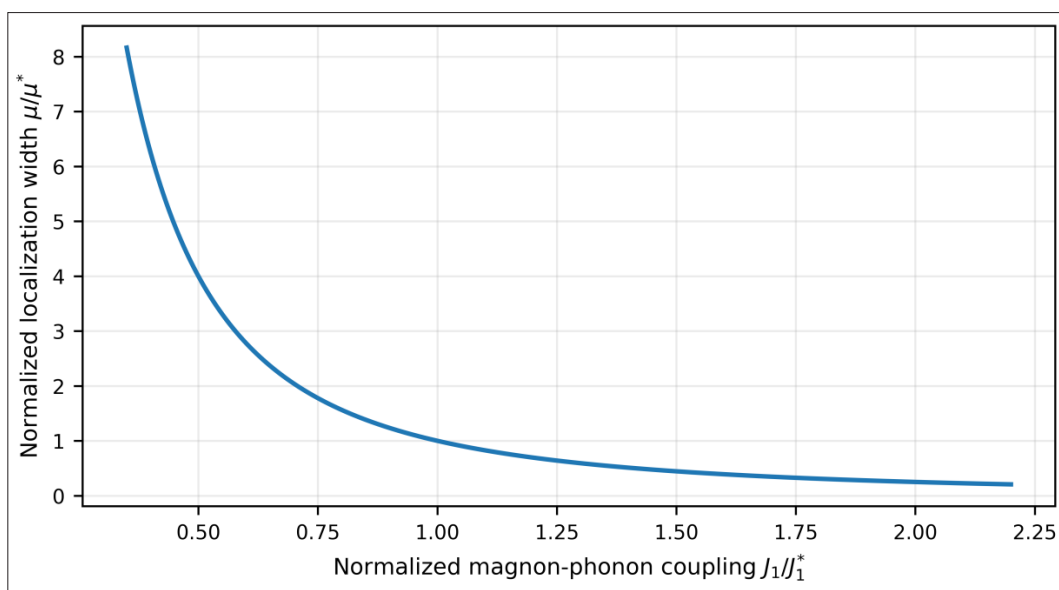


Figure 2: Normalized dependence of the localization width μ/μ^* on the magnon-phonon coupling strength J_1/J_1^* . The curve illustrates the scaling law $\mu \propto J_1^{-2}$: increasing the coupling produces rapid spatial compression of the soliton state.

Energy, Momentum, and Effective Mass

In the low-velocity regime $v \ll v_0$, the soliton admits a quasiparticle interpretation. Its momentum, energy, and effective mass are given by

$$P = m_s v, \quad E = \frac{m_s v^2}{2} - \frac{(J_1 S)^4}{J_0 S (m v_0)^2},$$

$$m_s = m_0 + \frac{4(J_1 S)^4}{J_0 S m^2 v_0^6}, \quad m_0 = \frac{\hbar^2}{2J_0 S a^2}. \quad (12)$$

The quantity m_0 is the mass of a free magnon in the continuum approximation. The additional contribution to m_s originates from the joint motion of the magnetic excitation and the lattice deformation. Hence, the soliton differs from a free magnon not only by spatial localization but also by its inertial characteristics.

Dynamic Structure Factor

For a dilute equilibrium gas of solitons, the dynamic structure factor per unit system length can be expressed through the mean soliton density and the form factor of an individual excitation:

$$S_s(q, \omega) = \bar{n}_s S_1(q, \omega), \quad \bar{n}_s = \frac{N_s}{l}. \quad (13)$$

Here N_s is the number of solitons on a length l , and $S_1(q, \omega)$ is the dynamic form factor of a single soliton, obtained from the Fourier transform of the spin correlation function after averaging over phase space.

After integrating over the soliton-center coordinate and the velocity distribution, the soliton contribution to the dynamic structure factor takes the form

$$S_s(q, \omega) = \bar{n}_s \frac{m_s}{2\pi|q|} \left[\frac{\pi q \mu / 2}{\sinh(\pi q \mu / 2)} \right]^2$$

$$\times \frac{\exp \left[-\frac{m_s}{2\Theta} \left(\frac{\omega}{q} \right)^2 \right]}{(2am_s\Theta)^{1/2}}. \quad (14)$$

Equation (14) connects the spectral response with wave vector q , localization width μ , effective mass m_s , and the thermal parameter Θ . The soliton mode redistributes spectral weight toward the quasielastic region and produces a central component near $\omega \approx 0$.

Figure 3 illustrates the expected temperature broadening of this quasielastic central component. At lower temperature, the thermal distribution of soliton velocities is narrower and the spectral line is concentrated near zero frequency. Increasing temperature broadens the velocity distribution and therefore the central spectral component. In experiment, both the width and integrated intensity of this feature are informative and should be analyzed as functions of temperature and wave vector q .

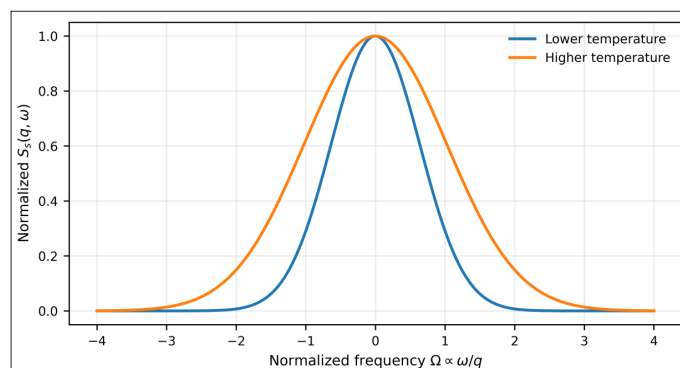


Figure 3: Normalized schematic quasielastic central component of the soliton contribution $S_s(q, \omega)$ for two temperature regimes. Increasing temperature broadens the central peak near $\omega \approx 0$. The curves are shown in normalized units and do not represent experimental data.

Range of Validity and Experimental Interpretation

The present approximation applies to the low-temperature and low-velocity regime. The quasielastic component is analyzed under the conditions

$$E_0 \gg \Theta, \quad \frac{\omega}{q} \ll v_0.$$

Under these conditions, the width and integrated intensity of the central peak depend on temperature and wave vector. Slow neutrons are particularly suitable for probing low-frequency spin dynamics; in quasi-one-dimensional magnetic systems, the quasielastic central component can therefore serve as a signature of the soliton mode [1,3,4]. More generally, inelastic neutron-scattering experiments have shown that magnon-phonon hybridization can generate distinct collective modes in measured excitation spectra [7], supporting the use of scattering observables as diagnostics of coupled spin-lattice dynamics.

A similar correlation-function approach may be applied to infrared scattering in systems that support localized nonlinear excitations. Quantitative comparison with a particular experiment requires numerical values of J_0 , J_1 , m , v_0 , a , S , temperature, and the equilibrium soliton density. Without these material-specific parameters, the derived expressions determine the functional form of the response but not its absolute intensity.

Discussion

A key feature of the model is the self-consistent character of localization. The magnetic excitation changes the deformation of the chain, the deformation modifies the exchange integral, and the resulting feedback produces an effective nonlinearity for the magnetic wave packet. The localized state therefore forms dynamically rather than being imposed by an external potential.

The soliton solution is characterized by a propagation velocity, phase, and finite localization width. Switching off the magnon-phonon coupling causes the width to increase without bound and the excitation to delocalize. The energy relation is supplemented by an effective mass, which makes a quasiparticle description natural for the statistical treatment of a dilute soliton gas.

The dynamic structure factor provides the direct link between the spatial soliton profile and an experimentally measurable spectral response. A combined analysis of the central quasielastic

component, its linewidth, integrated intensity, and q dependence offers a practical route for testing the soliton interpretation of low-frequency excitations in quasi-one-dimensional magnetic materials. The broader experimental literature on coherent and hybridized magnon-phonon modes reinforces the importance of spectroscopic linewidths, dispersions, and temperature dependences as probes of spin-lattice coupling [7,9].

Conclusion

A complete theoretical description of magnon-phonon localization in a quasi-one-dimensional isotropic Heisenberg ferromagnet has been presented. The treatment includes the spin-phonon Hamiltonian, the Holstein-Primakoff transformation, self-consistent equations of motion, the continuum nonlinear Schrödinger equation, the localized soliton solution, its energy-momentum characteristics, and the dynamic structure factor.

Magnon-phonon coupling sets the finite localization width and produces an additional contribution to the effective mass. As the coupling is reduced, the localized soliton continuously transforms into an ordinary spin wave. In the scattering spectrum, the soliton subsystem is predicted to generate a quasielastic central component, providing a basis for experimental diagnostics of nonlinear localized modes by neutron scattering and complementary optical techniques.

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Not applicable.

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