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A New Approach to Optimizing Aggregates in Asphalt - Evaluated Using Physical Properties Measured in the Goode and Lufsey Study

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ABSTRACT

In a previously published article, it was found that maximizing the Percent Packing Efficiency was effective in optimizing aggregates in asphalt to achieve an improved property balance. This study then found that the Percent Packing Efficiency can be further increased for an optimum aggregate particle size distribution by increasing the ratio, Z, between particles. In addition, the Percent Packing Efficiency has been found to approach an upper limit as the number of particles are increased in an optimum particle size distribution with a constant value of Z. A property analysis of the Goode and Lufsey study data for aggregates in asphalt also found that the minimums for the Mineral Voids and the Air Voids corresponded with maximums of Bulk Specific Density when the Percent Packing Efficiency approaches its maximum value. These results confirmed that the Percent Packing Efficiency is a significant measure of the particle packing efficiency for aggregate blends in asphalt.

Since Gradation 3 generated in the Goode and Lufsey study had the best balance of properties, this gradation was identified as the standard for their recommended maximum density line. As anticipated, the Goode and Lufsey maximum density standard gradation did yield the maximum Percent Packing Efficiency followed in order by other standard gradations including a Stone Matrix Gradation, a Superpave Gradation and finally a Bailey Gradation. In general, the analysis of the 24 gradations in the Goode and Lufsey study found that the Percent Packing Efficiency appears to be a very useful tool in predicting the gradation performance.

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Introduction

Several attempts have been made over the years to elucidate the best theoretical and/or practical approaches to characterize the blending of different pseudo-spherical particle sizes to maximize the packing fraction [1-22]. However, optimizing aggregates in asphalt generally has remained primarily a trial-and-error process. Consequently, a more theoretical blending approach to optimize the packing of pigments in coatings would be expected to have the potential to significantly improve the packing of the asphalt-aggregate blends.

One of the first attempts to define a maximum density line for aggregate gradations was proposed by Fuller and Thompson using the following formula [1]:

$$P = 100(d/D)^n \qquad (1)$$

Where

P = Percent passing a sieve with an opening size d
D = Maximum particle diameter
n = Parameter based on the gradation fineness

A later study by Nijboer found that an exponent of n ≈ 0.45 was effective in characterizing a maximum density line[2]. One of the first attempts to describe mathematically the optimization of the maximum packing fraction for aggregates was proposed by Furnas [3]. Furnas proposed a model for pseudo spherical aggregates where smaller particles would fit into the voids available from larger particles. An example of such a distribution involving three different interpenetrating particle sizes is shown in Figure 1.

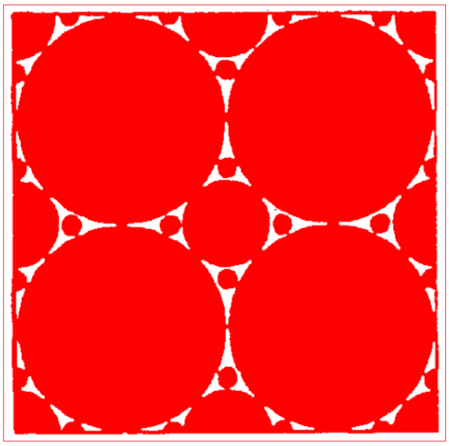


Figure 1: Schematic of Maximum Packing for a Blend of Spherical Particles

Furnas also found that it was easier to generate a straight line for maximum packing as described by Nijboer and Fuller if the ratio between sieve particle sizes was approximately 2 for the aggregate blend [3]. The article by Furnas [3] also recommended that the ratio between particles be approximately the square root of 2 or $2^{1/2}$ which would require a slope of $n \approx ½$ or 0.5 for equation (1) instead of a value of $n \approx 0.45$ as proposed by Nijboer [2].

Goode and Lufsey extended the work of Furnas and Nijboer but again indicated that a slope of $n = 0.45$ gave the best fit of the experimental data [5]. In addition, the maximum density straight line described by Goode and Lufsey also passed through the origin requiring an intercept of zero to be consistent with equation (1). While the work of Goode and Lufsey was extensive experimentally it did not address an extension of the mathematical analysis of Furnas. The classic asphalt aggregate gradation study of Goode and Lufsey was successful in extending the early work of Thompson, Nijboer, Furnas, Anderegg to achieve a straight line maximum density standard gradation that was found to be a very useful standard that continues to be used today [1-5]. The classic Goode and Lufsey study was generated using 24 asphalt aggregate gradations that used the same 10 particle sizes but with very different particle size distributions. In addition, the technique used by Goode and Lufsey relied heavily on using approximately the same matrix asphalt to insure that the change in the measured properties was dependent primarily on the particle size distribution of the aggregates. Consequently, a maximum packing fraction for each gradation should only be dependent on the establishment of an optimum particle size distribution with these same 10 different particle sizes. The Goode and Lufsey study relied heavily on using an extensive trial an error approach to optimize the aggregate particle size distributions.

Conversely, a separate successful study [21] evaluated several different asphalt matrix materials while attempting to keep the aggregate particle size distribution approximately constant. Since both the asphalt matrix and the distribution of the aggregates within the blend are important a model that can address both of these components in the final product would be preferred.

To characterize the blending of different pseudo-spherical particle sizes to maximize the packing fraction Kaeuffer found experimentally that an optimum particle size distribution for pigments in a paint could be achieved when the accumulated volume fraction for the distribution was directly proportional to the square root of the particle diameter [6]. In other words, according to Kaeuffer, the accumulated volume fraction from the smallest particle size up to the largest particle size considered, D_β, was assumed to be dependent on the square root of the particle diameter as:

$$\sum_{i=1}^{i=\beta} f_i = a\sqrt{D_\beta} \tag{2}$$

Where
f_i = volume fraction of particles of the i^{th} type

Note that equation (2) is very similar to Fuller and Thompson's equation (1); however, the exponent is $n = 0.5$ instead of $n = 0.45$ [1]. The volume fraction addressed in equation (3) would be identical to the mass fraction if all particles had approximately the same specific gravity; otherwise, the volume fraction for all particles would need to be corrected for specific gravity.

Sudduth then extended the experimental observations by Kaeuffer and was able to show from theoretical considerations that to maximize both the average particle size ratio of $\overline{D}_5 / \overline{D}_1$ and to maximize the packing fraction that it was desirable to have an "optimum particle size distribution" characterized by the following set of equations [11-18, 22].

$$f_i = \frac{V_i}{V_T} = \frac{N_i D_i^3}{\sum_{i=1}^{n} N_i D_i^3} \tag{3}$$

$$f_{imax} = \frac{\sqrt{D_i}}{\sum_{i=1}^{n} \sqrt{D_i}} \tag{4}$$

$$\varphi_{nult} = 1 - (1 - \varphi_m)^n \tag{5}$$

$$\varphi_n = \varphi_{nult} - (\varphi_{nult} - \varphi_m)\, e^{\alpha(1-(\overline{D}_5/\overline{D}_1))} \tag{6}$$

$$\overline{D}_5 = \frac{\sum_{i=1}^{n} N_i D_i^5}{\sum_{i=1}^{n} N_i D_i^4} \tag{7}$$

$$\overline{D}_1 = \frac{\sum_{i=1}^{n} N_i D_i}{\sum_{i=1}^{n} N_i} \tag{8}$$

Where
V_i = Volume of the ith particles
V_T = Total volume of all particles
f_i = Volume fraction of particles of the ith type
f_{imax} = Maximum volume fraction of particles of the ith type
$\overline{D}_1$ = Number average particle size or the 1st moment average particle size

$\overline{D}_5$ = 5th moment average particle size
N_i = Number of particles of the ith particle size
D_i = Diameter of the ith particle size.
n = Number of different particle sizes in mixture
α = Particle distribution constant = 0.268
φ_n = Maximum packing fraction
φ_{nult} = Ultimate packing fraction
φ_m = Monodisperse Packing fraction

At this point a fundamental observation reported by Lee is important to review [9]. Lee found that monodisperse spherical particles always yield essentially the same random packing fraction independent of particle size. Lee found that the monodisperse loose random packing fraction is $\varphi_m = 0.589$ and that the dense random packing fraction is 0.639.

Figure 1 was the type of model used by Sudduth to theoretically develop Equation (5) [12]. The ultimate packing fraction described by equation (5) can then be calculated as a function of the number of particle sizes in the blend using the data from Lee as indicated in Table 1 [9].

Table 1: Theoretical Ultimate Packing Fraction, φ_{nult}, Based on the Number of Particles for Both Loose and Dense Monodisperse Packing Fractions

$$\varphi_{nult} = 1 - (1 - \varphi_m)^n$$

Number of Particles, n	φ_m	φ_{nult}	Number of Particles, n	φ_m	φ_{nult}
1	0.589	0.5890	1	0.639	0.6390
2	0.589	0.8311	2	0.639	0.8697
3	0.589	0.9306	3	0.639	0.9530
4	0.589	0.9715	4	0.639	0.9830
5	0.589	0.9883	5	0.639	0.9939
6	0.589	0.9952	6	0.639	0.9978
7	0.589	0.9980	7	0.639	0.9992
8	0.589	0.9992	8	0.639	0.9997
9	0.589	0.9997	9	0.639	0.9999
10	0.589	0.9999	10	0.639	1.0000
11	0.589	0.9999	11	0.639	1.0000
12	0.589	1.0000	12	0.639	1.0000

Note in Table 1 that only 10 particles are theoretically needed to maximize the packing using either the loose monodisperse packing fraction or the dense monodisperse packing fraction. Since Goode and Lufsey used only 10 particles in their study, they had the potential to achieve close to the maximum packing fraction.

Calculation of $\bar{D}_5/\bar{D}_1$ for an Optimum Particle Size Distribution and for the Particle Size Distributions in the Goode and Lufsey Study

Note that equation (4) can be used directly to maximize both the average particle size ratio of $\bar{D}_5/\bar{D}_1$ and to optimize the volume fraction, f_{imax}, for each particle in an optimum particle size distribution directly by knowing each diameter in the particle size distribution. However, in the Goode and Lufsey study the volume fractions, fi, for each particle in each particle size distribution have previously been summarized in their study. The volume fractions for each particle, fi , can then be calculated as:

$$f_i = \frac{V_i}{V_T} = \frac{N_i D_i^3}{\sum_{i=1}^{n} N_i D_i^3} \tag{9}$$

Taking the ratio of two volume fractions yields

$$\frac{f_i}{f_k} = \frac{V_i}{V_k} = \frac{N_i D_i^3}{N_k D_k^3} \tag{10}$$

Rearranging gives

$$N_i = N_k \left(\frac{D_k^3}{D_i^3}\right)\left(\frac{f_i}{f_k}\right) \tag{11}$$

Assuming that $N_k = 1$ for largest particle diameter, then equations 9, 10 and 11 can then be used to calculate number of particles for each particle size, N_i. Consequently, then the ratio $\bar{D}_5/\bar{D}_1$ can be calculated using equation (12)

$$\bar{D}_x = \frac{\sum_{i=1}^{n} N_i D_i^x}{\sum_{i=1}^{n} N_i D_i^{x-1}} \tag{12}$$

where

V_i = Volume of the ith particles
V_k = Volume of the kth particles
V_T = Total volume of all particles
f_i = Volume fraction of particles of the ith type
f_k = Volume fraction of particles of the kth type
$\bar{D}_1$ = Number average particle size or the 1st moment average particle size
$\bar{D}_5$ = 5th moment average particle size
$\bar{D}_x$ = Xth moment average particle size
N_i = Number of particles of the ith particle size
D_i = Diameter of the ith particle size.
n = Number of different particle sizes in mixture

In a previous publication the importance of developing an aggregate formulation based on an optimum particle size distribution described by equation (4) was found to be very important to achieve the full benefit of the Percent Packing Efficiency [22]. In general, an optimum particle size distribution is characterized with volume fractions being generated as the square root of a particle diameter divided by the sum of the square roots of all the particle diameters in the distribution.

An optimum distribution was also found to achieve an excellent duplication of the volume fractions that McGeary achieved when he maximized the packing fractions for the binary, tertiary and quaternary particle size distributions for his nuclear reactor application [10, 14, 22]. In addition, McGeary's experimental maximum packing fractions were also very close to the ultimate packing fractions (equation 5) that could be achieved for binary, tertiary and quaternary particle size distributions.

It has also been found that the particles in the formulation of Goode and Lufsey's maximum density line were also found to be well characterized using close to an optimum particle size distribution. In addition, it has also been found that when an optimum particle size distribution is generated with a constant ratio, Z, between particle sizes that the accumulated volume fraction vs the square root of the particle diameters must necessarily yield a straight line.

Characterization of an Optimum Particle Size Distribution with a Constant Ratio Z between Particles

Sudduth has shown previously that if an optimum particle size distribution does have a constant ratio between particles, Z, then the accumulated volume fractions vs the square root of the diameters will necessarily form a straight line with the following applicable equations [12-18]:

$$D_i = D_1 Z^{i-1} \tag{13}$$

$$D_{Max} = Z^{n-1} D_{Min} = Z^{n-1} D_1 \tag{14}$$

$$f_{imax} = \frac{\sqrt{D_i}}{\sum_{i=1}^{n} \sqrt{D_i}} \tag{15}$$

$$\sum_{i=1}^{i=\beta} f_i = a\sqrt{D_\beta} + b \tag{16}$$

$$a = \left(\frac{1}{\sqrt{D_1}}\right)\left(\frac{\sqrt{Z}}{\sqrt{Z^n} - 1}\right) \tag{17}$$

$$b = \left(\frac{1}{1 - \sqrt{Z^n}}\right) \tag{18}$$

$$\sum_{i=1}^{i=\beta} f_i = 0 \quad \text{then} \qquad (\sqrt{D_\beta})_{y=0} = -b/a = (D_1/Z)^{1/2} \tag{19}$$

$$\overline{D}_x = \frac{\sum_{i=1}^{n} N_i D_i^x}{\sum_{i=1}^{n} N_i D_i^{x-1}} = \left(\frac{Z^{(x-3.5)} - 1}{Z^{(x-2.5)} - 1}\right)\left(\frac{Z^{(x-2.5)n} - 1}{Z^{(x-3.5)n} - 1}\right) D_1 \tag{20}$$

$$\overline{D}_5 / \overline{D}_1 = \frac{1}{Z^{(n-1)}}\left(\frac{Z^{1.5} - 1}{Z^{2.5} - 1}\right)^2 \left(\frac{Z^{2.5n} - 1}{Z^{1.5n} - 1}\right)^2 \tag{21}$$

$$R = D_{Max} / D_{Min} = Z^{n-1} \tag{22}$$

Where

V_i = Volume of the ith particles

V_T = Total volume of all particles

a = Slope of straight line

b = Intercept of straight line

f_i = Volume fraction of particles of the ith type

f_{imax} = Maximum volume fraction of particles of the ith type

D_i = Diameter of the ith particle size.

D_{Min} = D1 = Diameter of the smallest particle size in the particle distribution

D_{Max} = Diameter of the maximum particle size in the particle distribution

n = Number of different particle sizes in mixture

However, if there is not a constant ratio between particles then many times an optimum particle size distribution will form an approximate straight line where:

$$a = \frac{1 - f_{min}}{\sqrt{D_{max}} - \sqrt{D_{min}}} \tag{23}$$

$$b = \frac{f_{min}\sqrt{D_{max}} - \sqrt{D_{min}}}{\sqrt{D_{max}} - \sqrt{D_{min}}} \tag{24}$$

Note that if the ratio between particles, Z, is a constant such that $Z > 1$, then the value of the intercept, b , must necessarily be negative as indicated by equation 18. However, when the ratio between particles is not a constant, then the calculated intercept can be positive using equation 24 when

$$f_{min}\sqrt{D_{max}} > \sqrt{D_{min}} \tag{25}$$

Development of the Percent Packing Efficiency

The ratio of the maximum particle diameter, D_{max}, to the minimum particle diameter, D_{min}, in any particle distribution can be characterized with the symbol R as

$$R = D_{max} / D_{min} \tag{26}$$

Since $\overline{D}_5$ can be no greater than the maximum particle D_{max} and the value $\overline{D}_1$ can be no smaller than of the minimum particle diameter, D_{min}, then in general:

$$\overline{D}_5 / \overline{D}_1 \le R \qquad \text{and} \qquad (\overline{D}_5 / \overline{D}_1)/R \le 1.0 \tag{27}$$

However, there is one complication. For a monodisperse particle size distribution then

$$\overline{D}_5 / \overline{D}_1 = 1.0 \text{ and } R = 1.0 \text{ yielding } (\overline{D}_5 / \overline{D}_1)/R = 1 \tag{28}$$

To get around the monodisperse case, the efficiency of particle packing for any particle size distribution then needs to be modified to yield the following equation.

$$\text{Percent Packing Efficiency} = 100\,((\overline{D}_5 / \overline{D}_1) - 1)/R) \tag{29}$$

The Percent Packing Efficiency introduced in a previous publication as defined by equation 29 would then have the following range [22].

$0 \le$ Percent Packing Efficiency ≤ 100

For an optimum particle size distribution with a constant ratio , Z, between particles then the Percent Packing Efficiency as a function of the number of particles in the distribution can be calculated as using equations 30 and 31 as

$$\overline{D}_5 / \overline{D}_1 = \frac{1}{Z^{(n-1)}}\left(\frac{Z^{1.5} - 1}{Z^{2.5} - 1}\right)^2 \left(\frac{Z^{2.5n} - 1}{Z^{1.5n} - 1}\right)^2 \tag{30}$$

$$R = D_{Max} / D_{Min} = Z^{n-1} \tag{31}$$

A plot of Percent Packing Efficiency vs Ratio Z between particle and the number of particles in the distribution is shown in Figure 2

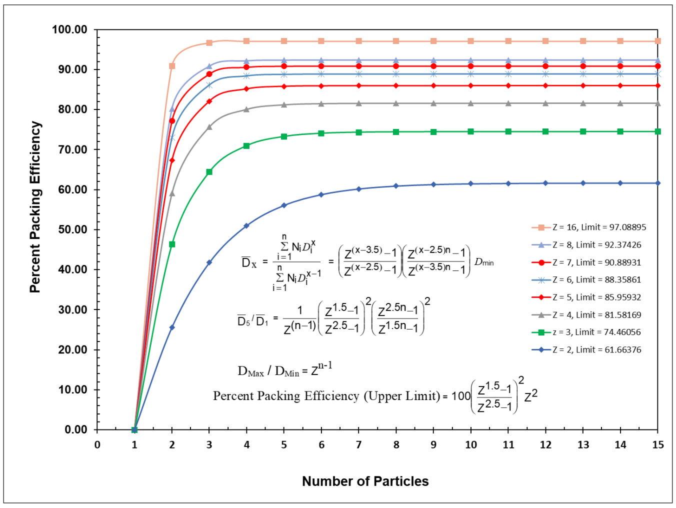


Figure 2: Percent Packing Efficiency vs Ratio Between Particles, Z, and Number of Particle in Distribution

The upper limit of Percent Packing Efficiency as n ⟶ ∞ then yields

$$\text{Percent Packing Efficiency (Upper Limit)} = 100\left(\frac{Z^{1.5}-1}{Z^{2.5}-1}\right)^2 Z^2$$

The Upper Limits of the Percent Packing Efficiency for each value of the ratio between particles, Z, are summarized in Table 2. Table 2 also includes the lower limit for the number of particles needed in the distribution of to reach the upper limits for the Percent Packing Efficiency. The ultimate packing fractions are also included in Table 2 for the number of particles needed in the distribution of to reach the upper limits for the Percent Packing Efficiency.

In addition, there is both a theoretical and a practical limit for the value of Z. The theoretical limit of Z has been shown in previous publications (18 &22) to be between 5.92 and 7.67. McGeary also found experimentally that the preferred upper limit of Z to be approximately Z = 7. Therefore, as indicated in both Figure 2 and Table 2 the values of Z for Z = 5 and Z =7 are marked in red since this is the approximate range for an optimum value for the ratio Z between particle sizes.

However, the practical limit of Z depends first on the largest particle in the distribution which for aggregates in asphalt is typically a particle size of approximately 1 inch. Therefore, the value of Z would then be dependent primarily on the smallest particle size available. In addition, if the number of particles in an optimum particle size distribution is small then the ultimate packing fractions would be expected to be significantly reduced. In general, a value Z of 2 or 4 would appear to be the most practical for most aggregate asphalt applications at this time.

Table 2: Summary of the Upper Limits for the Percent Packing Efficiency vs the Ratio Between Particle and Lower Limit of Number of Particles in Distribution

Z	*Maximum Percent Packing Efficiency*	*Number of Particles to Limit*	*Ultimate Packing Fraction* $\varphi_{ult} = 1-(1-\varphi_m)^n$
1.000	*0.000*		*0.5890*
1.095	*39.312*	*140*	*1.0000*
2.000	*61.664*	*20*	*1.0000*
3.000	*74.461*	*12*	*1.0000*
4.000	*81.582*	*11*	*0.9999*
5.000	*85.959*	*9*	*0.9997*
6.000	*88.859*	*9*	*0.9997*
7.000	*90.889*	*8*	*0.9992*
8.000	*92.374*	*7*	*0.9980*
16.000	*97.089*	*6*	*0.9952*
32.000	*98.932*	*5*	*0.9883*
64.000	*99.616*	*4*	*0.9715*
128.000	*99.863*	*3*	*0.9306*

Note that a maximum Percent Packing Efficiency of 39.312 can be quite low for particulate fillers in a coating application where the ratio between particle is typically in the range of Z= 1.095. However, the packing fraction can still continue to increase as long as the ratio of $\overline{D}_5/\overline{D}_1$ continues to increase as indicated in equation (6) independent of the value of the Percent Packing Efficiency and as long as Z >1.

Characterization of the Maximum Density Line using Gradation 3 from the Goode and Lufsey Study

While the work of Goode and Lufsey was extensive experimentally it did not address an extension of the mathematical analysis of Furnas. The maximum density straight line found by trial and error in the Goode and Lufsey study has been designated as Gradation 3 in Table 3. In general, the experimental gradations of Goode and Lufsey did maintain the predominant ratio of approximately 2 between sieve sizes as recommended by Furnas.

While the work of Goode and Lufsey was successful experimentally it did not yield an improved understanding of the theoretical description of the original maximum density line proposed by Fuller and Thompson. Two recent articles did address additional theoretical considerations regarding the blending of aggregates; however, the resulting models did not yield the straight line described by Goode and Lufsey [23,24].

The accumulation of volume fractions for Gradation 3 from the Goode and Lufsey Study have been plotted in Figure 3 as a function of the square root of the aggregate diameter. Note in Figure 3 that Gradation 3 from the Goode and Lufsey study does plot as a straight line. The formulation for Gradation 3 from the Goode and Lufsey Study has been summarized in Table 3. However, note that the volume fractions for Gradation 3 plotted in Figure 4 only plot as a straight line between the first particle and the last two particles and as long as the ratio between particles was again approximately constant (Z=2).

Table 3: Goode and Lufsey's Maximum Density Aggregate Gradation 3

Gradation = 3 - G&L
Max D5/D1= 175.0667
Max Particle Size= 13.13
Min. Particle Size= 0.075

Percent Packing Efficiency = 63.8624
Actual D5/D1= 112.8017
Actual D5 = 9.126201
Actual D1 = 0.080905

Ratio Dia	Particle Number	Sieve Number	Diameter, Millimeters	SQRT(Dia)	Volume Fraction	Accu Volume Fraction
	1	200	0.075	0.2739	0.0970	0.0970
2.00	2	100	0.15	0.3873	0.0350	0.1320
2.00	3	50	0.3	0.5477	0.0480	0.1800
2.00	4	30	0.6	0.7746	0.0660	0.2460
1.97	5	16	1.18	1.0863	0.0910	0.3370
2.00	6	8	2.36	1.5362	0.1230	0.4600
2.01	7	4	4.75	2.1794	0.1700	0.6300
2.00	8	3/8 in.	9.5	3.0822	0.2300	0.8600
1.32	9	1/2 in.	12.5	3.5355	0.1200	0.9800
1.05	10	0.525 in	13.13	3.6235	0.0200	1.0000
			SUM =	17.0267	1	

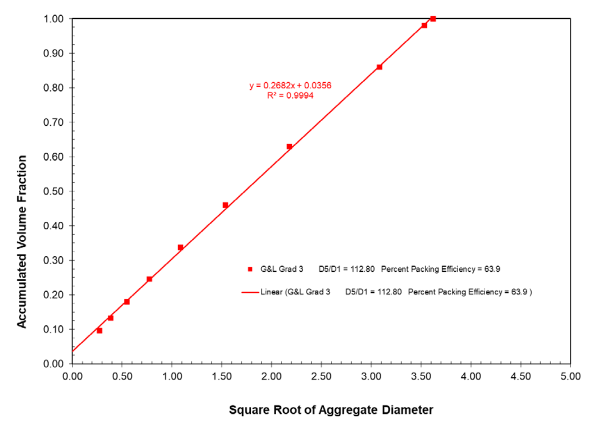


Figure 3: Accumulated Volume Fraction vs Square Root of Particle Diameter for G& L Gradation 3

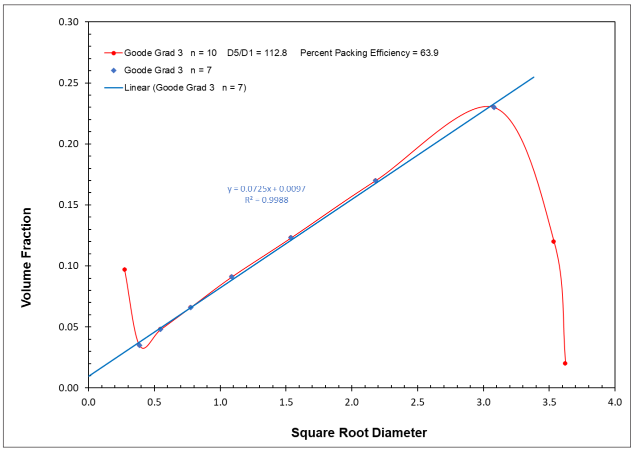


Figure 4: Volume Fraction vs Square Root of Particle Diameter for G& L Gradation 3

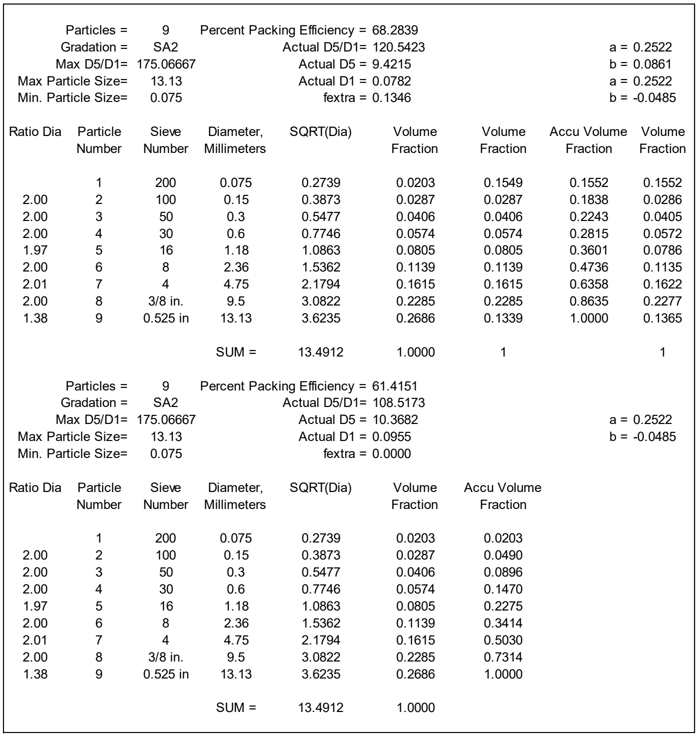
Note that the results indicated in Figure 4 are clearly inconsistent with the calculated results evaluated for an optimum particle size distribution. In particular, the results in Figure 4 indicate that the smallest particle for Gradation 3 had a significantly larger volume fraction in proportion to its smaller size and the largest two particles have significantly smaller volume fractions relative to their larger size.

At this point it has been found useful to review the procedure used to optimize the particle size distributions for coating applications to develop a duplicate of Goode and Lufsey's Gradation 3 using more fundamental considerations instead of the trial-and-error approach used by Goode and Lufsey.

Development of a Duplicate of Goode and Lufsey's Gradation 3 Straight line from Fundamental Considerations Using Generalized Viscosity Model Technology

Since Gradation 3 from the Goode and Lufsey study appears to be one of the standards being used by the asphalt industry, it appeared to be prudent to develop a duplicate model in this study to compare directly with Gradation 3. Of particular interest in this instance was which combination of particles would be most appropriate to try to match Gradation 3. Notice in Table 3 that the particle size distribution for Gradation 3 used by Goode and Lufsey had a constant ratio distribution of Z = 2 up to the particle that was 3/8 inch in diameter. The last two particles were the ½ in particle and the 0.525-inch particle that had ratios between particle sizes of 1.332 and 1.050 respectively. Consequently, it was found useful in this study to eliminate the ½ inch particle but keeping the 0.525 in particle size as the final maximum particle to maximize the particle ratio from 3/8-inch particle to 1.38 as indicated in Table 4. Note in Table 4 and in Figure 5 that GradationSA2 was initially calculated with only 9 particles by eliminating the ½ in particle. The volume fractions for Gradation SA2 before shifting were evaluated as an optimum particle size distribution using equation 4.

Table 4: Gradation SA2 (with 9 Particles) Before and After Shifting (from this study)



Particles = 9 Percent Packing Efficiency = 68.2839
Gradation = SA2 Actual D5/D1= 120.5423 a = 0.2522
Max D5/D1= 175.06667 Actual D5 = 9.4215 b = 0.0861
Max Particle Size= 13.13 Actual D1 = 0.0782 a = 0.2522
Min. Particle Size= 0.075 fextra = 0.1346 b = -0.0485

Ratio Dia	Particle Number	Sieve Number	Diameter, Millimeters	SQRT(Dia)	Volume Fraction	Volume Fraction	Accu Volume Fraction	Volume Fraction
	1	200	0.075	0.2739	0.0203	0.1549	0.1552	0.1552
2.00	2	100	0.15	0.3873	0.0287	0.0287	0.1838	0.0286
2.00	3	50	0.3	0.5477	0.0406	0.0406	0.2243	0.0405
2.00	4	30	0.6	0.7746	0.0574	0.0574	0.2815	0.0572
1.97	5	16	1.18	1.0863	0.0805	0.0805	0.3601	0.0786
2.00	6	8	2.36	1.5362	0.1139	0.1139	0.4736	0.1135
2.01	7	4	4.75	2.1794	0.1615	0.1615	0.6358	0.1622
2.00	8	3/8 in.	9.5	3.0822	0.2285	0.2285	0.8635	0.2277
1.38	9	0.525 in	13.13	3.6235	0.2686	0.1339	1.0000	0.1365
			SUM =	13.4912	1.0000	1		1

Particles = 9 Percent Packing Efficiency = 61.4151
Gradation = SA2 Actual D5/D1= 108.5173
Max D5/D1= 175.06667 Actual D5 = 10.3682 a = 0.2522
Max Particle Size= 13.13 Actual D1 = 0.0955 b = -0.0485
Min. Particle Size= 0.075 fextra = 0.0000

Ratio Dia	Particle Number	Sieve Number	Diameter, Millimeters	SQRT(Dia)	Volume Fraction	Accu Volume Fraction
	1	200	0.075	0.2739	0.0203	0.0203
2.00	2	100	0.15	0.3873	0.0287	0.0490
2.00	3	50	0.3	0.5477	0.0406	0.0896
2.00	4	30	0.6	0.7746	0.0574	0.1470
1.97	5	16	1.18	1.0863	0.0805	0.2275
2.00	6	8	2.36	1.5362	0.1139	0.3414
2.01	7	4	4.75	2.1794	0.1615	0.5030
2.00	8	3/8 in.	9.5	3.0822	0.2285	0.7314
1.38	9	0.525 in	13.13	3.6235	0.2686	1.0000
			SUM =	13.4912	1.0000	

When the ratio between particles is not the same (Z=2), then a straight line would not necessarily to be expected for the accumulated volume fraction vs the square root of the particle diameters. However, shifting the curve to achieve a straight line can be achieved by adding an extra volume fraction, f_{Extra}, to the volume fraction of the smallest particle and subtracting the extra volume fraction for the largest particle volume fraction. Note in Figure 5 that the first 8 particles did form a straight line with a slope of a = 0.2522.

A new straight line was formed by shifting the lower curve to make one straight line that ended at $\sum_{i=1}^{i=\beta} f_i = 1$. The shift value of f_{Extra} = 0.0861- (-0.0485) = 0.1346 was then the difference between these straight-line equations which actually was simply the difference between these intercept values. This new straight line then yielded the higher Percent Packing Efficiency of 68.3 compared with the unshifted Gradation SA2 value for the Percent Packing Efficiency of 61.4.

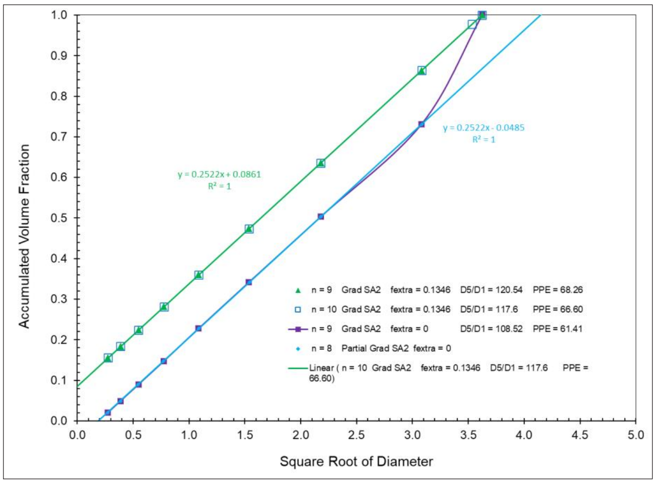


Figure 5: Gradation SA2 with 9 & 10 Particles Before and After Shifting vs Square Root of Particle Diameter

After the new straight line was developed for the accumulated volume fraction vs the square root of the particle diameter, this straight line equation was used to calculate the accumulated volume fraction values for both the ½ inch particle and the 0.525 inch particle. Keep in mind that the volume fractions for each of these particle sizes were determined from the difference between the calculated values from the accumulated volume fractions forming the new straight line as indicated in Figure 5. Note particularly that the calculated values for Gradation SA2 shifted with 9 particles yielded the higher Percent Packing Efficiency of 68.3 compared with the unshifted Gradation SA2 Percent Packing Efficiency of 61.4.

However, Gradation SA2 shifted with 10 particles yielded a Percent Packing Efficiency of 66.6. Therefore, in this case adding the extra particle did not yield an increase in the value of the Percent Packing Efficiency. However, as indicated in Table 5 Gradation SA2 with 10 particles did still have a Percent Packing Efficiency = 66.6 while Gradation 3 from the Goode and Lufsey study had a Percent Packing Efficiency = 63.8. Therefore, it does appear that Gradation SA2 with 10 particles did yield a better packing efficiency than Gradation 3 from the Goode and Lufsey study.

Figure 6 shows a direct comparison between the straight lines for Gradation 3 from the Goode and Lufsey study and Gradation SA2 with 10 particles from this study. Both of these straight lines in Figure 6 are quite close even though Gradation SA2 with 10 particles was calculated from theoretical considerations while Gradation 3 from the Goode and Lufsey study was obtained by trial-and-error processing.

Table 5: Goode and Lufsey Gradation 3 and Gradation SA2 (Both with 10 Particles)

Particles = 10
Gradation = 3 - G&L Percent Packing Efficiency = 63.8624
Max D5/D1= 175.0667 Actual D5/D1= 112.8017 a = 0.2661
Max Particle Size= 13.13 Actual D5 = 9.126201 b = 0.0375
Min. Particle Size= 0.075 Actual D1 = 0.080905

Ratio Dia	Particle Number	Sieve Number	Diameter, Millimeters	SQRT(Dia)	Volume Fraction	Accu Volume Fraction
	1	200	0.075	0.2739	0.0970	0.0970
2.00	2	100	0.15	0.3873	0.0350	0.1320
2.00	3	50	0.3	0.5477	0.0480	0.1800
2.00	4	30	0.6	0.7746	0.0660	0.2460
1.97	5	16	1.18	1.0863	0.0910	0.3370
2.00	6	8	2.36	1.5362	0.1230	0.4600
2.01	7	4	4.75	2.1794	0.1700	0.6300
2.00	8	3/8 in.	9.5	3.0822	0.2300	0.8600
1.32	9	1/2 in.	12.5	3.5355	0.1200	0.9800
1.05	10	0.525 in	13.13	3.6235	0.0200	1.0000
			SUM =	17.0267	1	

Particles = 10 Percent Packing Efficiency = 66.6032
Gradation = SA2 Actual D5/D1= 117.6000 a = 0.2522
Max D5/D1= 175.0667 Actual D5 = 9.1916 b = 0.0861
Max Particle Size= 13.13 Actual D1 = 0.0782 a = 0.2522
Min. Particle Size= 0.075 fextra = 0.1346 b = -0.0485

Ratio Dia	Particle Number	Sieve Number	Diameter, Millimeters	SQRT(Dia)	Extra Volume Fraction	Extra Volume Fraction	Accu Volume Fraction	Volume Fraction
	1	200	0.075	0.2739	0.0161	0.1507	0.1552	0.1552
2.00	2	100	0.15	0.3873	0.0227	0.0227	0.1838	0.0286
2.00	3	50	0.3	0.5477	0.0322	0.0322	0.2243	0.0405
2.00	4	30	0.6	0.7746	0.0455	0.0455	0.2815	0.0572
1.97	5	16	1.18	1.0863	0.0638	0.0638	0.3601	0.0786
2.00	6	8	2.36	1.5362	0.0902	0.0902	0.4736	0.1135
2.01	7	4	4.75	2.1794	0.1280	0.1280	0.6358	0.1622
2.00	8	3/8 in.	9.5	3.0822	0.1810	0.1810	0.8635	0.2277
1.32	9	1/2 in.	12.5	3.5355	0.2076	0.2076	0.9778	0.1143
1.05	10	0.525 in	13.13	3.6235	0.2128	0.0782	1.0000	0.0222
			SUM =	17.0267	1	1		1

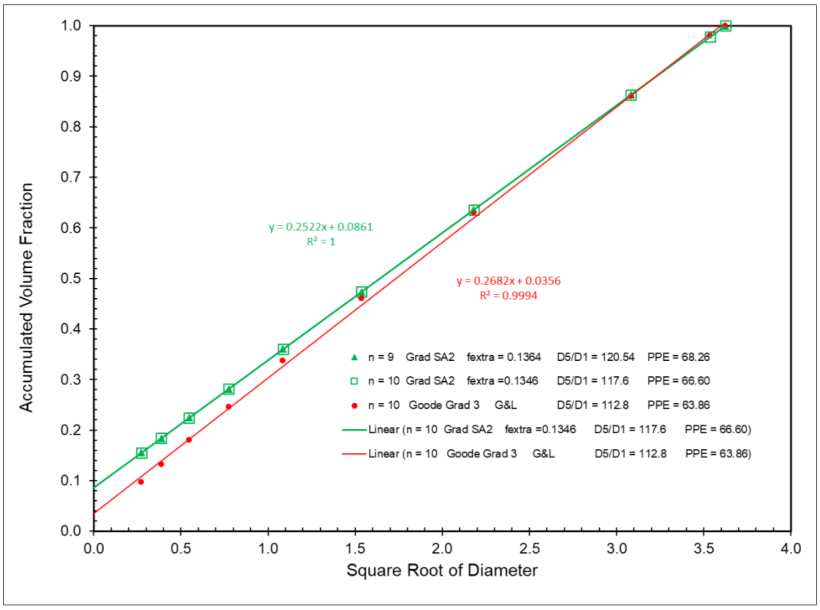


Figure 6: Accumulated Volume Fraction vs Square Root of Particle Diameter for Gradations SA2 (n=10) & Gradation 3

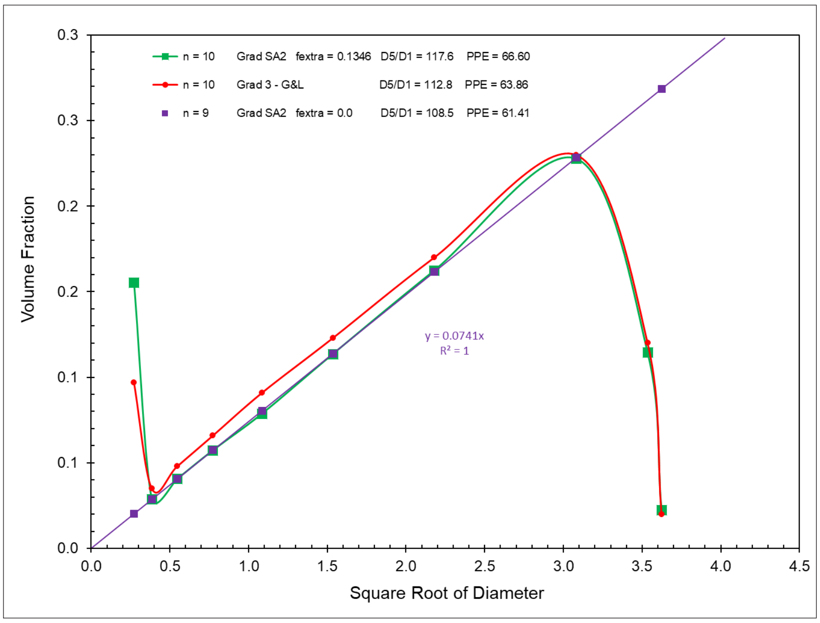


Figure 7: Volume Fraction vs Square Root of Particle Diameter for Gradations 3 & SA2 (n=10)

At this point it was found useful to look at the distribution of volume fractions generated between Gradation SA2 with 10 particles and the volume fractions for Gradation 3 generated by Goode and Lufsey as shown in Figure 7. Note in Figure 7 that all of the volume fractions calculated for Gradation SA2 with 9 particles before shifting fell in straight line that went through the origin. Amazingly, after shifting Gradation SA2 with 10 particles appears to match Gradation 3 distribution in Figure 7 almost exactly. It has also been found that the center particles in the formulation of Goode and Lufsey's maximum density line have also been found to be well characterized using an optimum particle size distribution that was developed for Gradation SA2. This major observation means the calculated volume fractions generated using the new approach identified in this study have been able to match the trial-and-error approach that Goode and Lufsey used to generate the Gradation 3 distribution.

Unfortunately, Figure 7 also indicates that the volume fractions for Gradation 3 distribution generated by Goode and Lufsey showed a significant amount of extra volume fraction added to the smallest particles compared with Gradation SA2 with 9 particles without shifting. Conversely Figure 7 also indicates that the Gradation 3 distribution also removed a significant volume fraction from the largest particle size compared with the Gradation SA2 with 9 particles before shifting. The shifted Gradation SA2 with 10 particles did match the Gradation 3 distribution from the Goode and Lufsey study. However, Gradation SA2 with 10 particles did have a higher value of Percent packing Efficiency = 66.6 while Gradation 3 from the Goode and Lufsey study had a lower value of Percent Packing Efficiency = 63.8.

Evaluation of a Model for the Packing of Spherical Particle as Originally Proposed by Furnas

One of the first to attempt to describe mathematically the optimization of the maximum packing fraction for aggregates was proposed by Furnas [4]. Furnas proposed a model for pseudo spherical aggregates where smaller particles would fit into the voids available from larger particles. An example of such a distribution involving three different interpenetrating particle sizes is shown in Figure 8.

In Table 6 the first two estimates of the volume fraction of the three particle sizes in Figure 8 are based on the volume and the surface area for each of the three particle sizes. The Case III is the optimum particle size distribution discussed earlier where the volume fraction for each particle that was calculated as the square root of each particle diameter divided by the sum of the square roots of all the particle diameters in the blend. The numbers of each particle size for Case III were evaluated using Equations (9) to (12) as discussed earlier [13-18]. The accumulated volume fractions vs the square root of the diameters for these three cases are plotted in Figure 9. The volume fractions per particle for these three cases plotted in Figure 10 clearly indicate that the volume fraction for each particle was roughly proportional to its particle diameter with the smaller particle having a smaller volume fraction and the larger particles having a larger volume fraction.

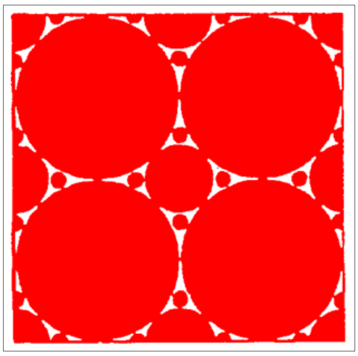
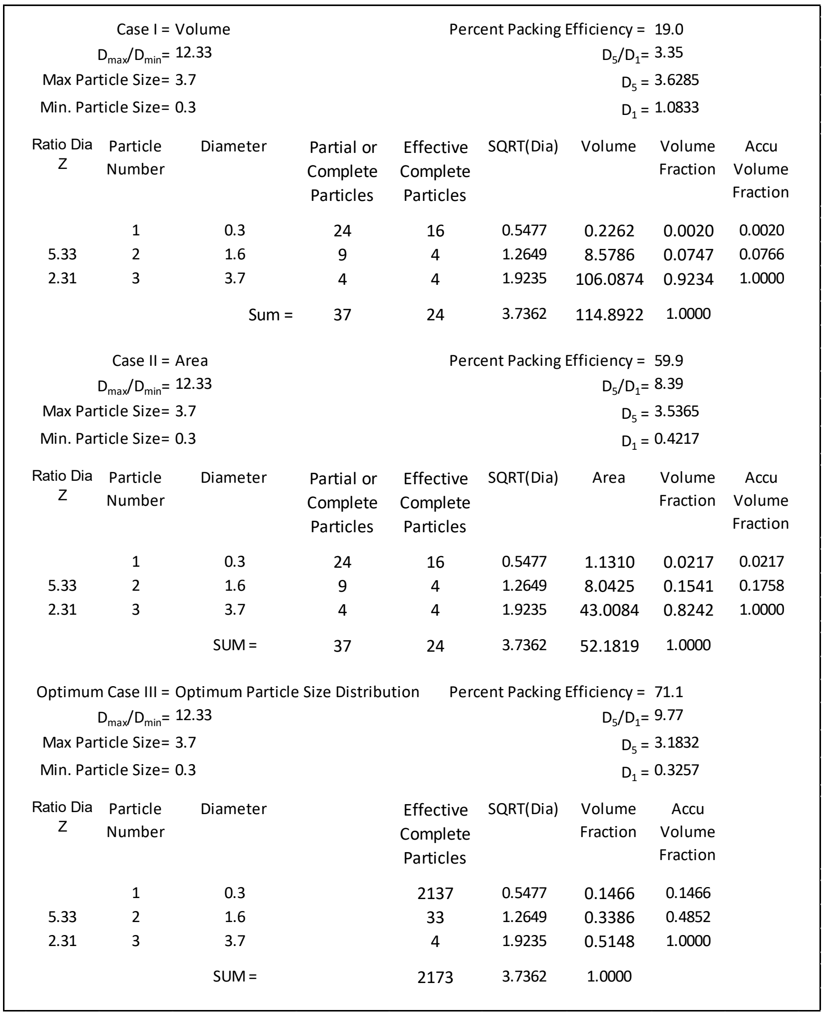


Figure 8: Schematic of Maximum Packing for a Blend of Three Different Sized Spherical Particles

Table 6: Volume Fraction Estimates per Particle Size for Three Particle Blend in Figure 8



Case I = Volume
D_{max}/D_{min}= 12.33
Max Particle Size= 3.7
Min. Particle Size= 0.3

Percent Packing Efficiency = 19.0
D_5/D_1= 3.35
D_5 = 3.6285
D_1 = 1.0833

Ratio Dia Z	Particle Number	Diameter	Partial or Complete Particles	Effective Complete Particles	SQRT(Dia)	Volume	Volume Fraction	Accu Volume Fraction
	1	0.3	24	16	0.5477	0.2262	0.0020	0.0020
5.33	2	1.6	9	4	1.2649	8.5786	0.0747	0.0766
2.31	3	3.7	4	4	1.9235	106.0874	0.9234	1.0000
		Sum =	37	24	3.7362	114.8922	1.0000	

Case II = Area
D_{max}/D_{min}= 12.33
Max Particle Size= 3.7
Min. Particle Size= 0.3

Percent Packing Efficiency = 59.9
D_5/D_1= 8.39
D_5 = 3.5365
D_1 = 0.4217

Ratio Dia Z	Particle Number	Diameter	Partial or Complete Particles	Effective Complete Particles	SQRT(Dia)	Area	Volume Fraction	Accu Volume Fraction
	1	0.3	24	16	0.5477	1.1310	0.0217	0.0217
5.33	2	1.6	9	4	1.2649	8.0425	0.1541	0.1758
2.31	3	3.7	4	4	1.9235	43.0084	0.8242	1.0000
		SUM =	37	24	3.7362	52.1819	1.0000	

Optimum Case III = Optimum Particle Size Distribution
D_{max}/D_{min}= 12.33
Max Particle Size= 3.7
Min. Particle Size= 0.3

Percent Packing Efficiency = 71.1
D_5/D_1= 9.77
D_5 = 3.1832
D_1 = 0.3257

Ratio Dia Z	Particle Number	Diameter	Effective Complete Particles	SQRT(Dia)	Volume Fraction	Accu Volume Fraction
	1	0.3	2137	0.5477	0.1466	0.1466
5.33	2	1.6	33	1.2649	0.3386	0.4852
2.31	3	3.7	4	1.9235	0.5148	1.0000
		SUM =	2173	3.7362	1.0000	

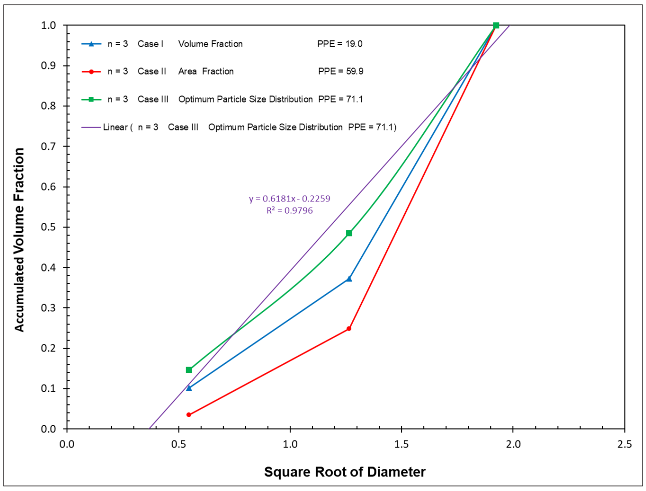


Figure 9: Accumulated Volume Fractions vs Square Root of Particle Diameter for Cases I – III

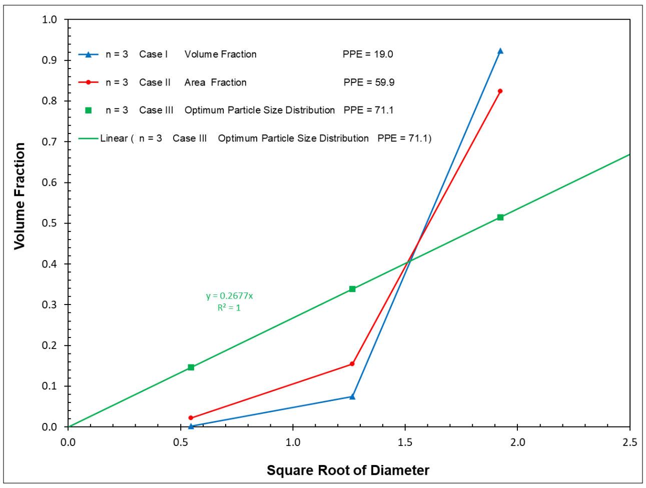


Figure 10: Volume Fractions per Particle vs Square Root of Particle Diameter for Cases I – III

Note in Figure 9 that for Case III the accumulated volume fractions vs the square root of the particle diameter did plot as an approximate straight line. As indicated in Table 6 the average ratio between the particles was approximately 3.8 that was not a constant ratio between particles. However, since the average ratio between particles was greater that Z = 2 then based partially on the results in Table 2 the Percent Packing Efficiency was greater for case III than the Percent Packing Efficiency for gradation 3.

Comparison of the Calculated Percent Packing Efficiency for All 24 Gradations in the Goode and Lufsey Study with Their Measured Properties

The calculations of the Percent Packing Efficiencies for all 24 of the Goode and Lufsey gradations are summarized in Table 7. The calculated values of Percent packing Efficiency along with the properties measured for each gradation are also shown graphically in Figure 11. The properties measured for each of these gradations are then summarized by gradation number in Figures 12-15.

Citation: Richard D Sudduth (2026) A New Approach to Optimizing Aggregates in Asphalt - Evaluated Using Physical Properties Measured in the Goode and Lufsey Study. Journal of Civil Engineering Research & Technology. SRC/JCERT-226. DOI: doi.org/10.47363/JCERT/2026(8)200

Table 7: Measured Property Results from Goode and Lufsey Study Plus Calculated Values for D5/D1 and Percent Packing Efficiency

	Goode and Lufsey Gradation	Bulk Specific Gravity	Mineral Voids Per Cent	Air Voids Per Cent	Marshall Stability, Pounds	Marshall Flow	Calculated D_5/D_1	$R=D_{max}/D_{min}$	Percent Packing Efficiency
	1	2.344	16.2	4.2	1620	9	111.26	175.067	62.98
	2	2.384	14.7	2.5	1930	10	112.18	175.067	63.51
Max Blend	3	2.392	14.4	2.1	1650	10	112.80	175.067	63.86
	4	2.373	15.1	2.9	1280	9	112.48	175.067	63.68
	5	2.334	16.4	4.4	1000	9	112.60	175.067	63.75
	6	2.29	17.9	6.2	770	9	111.71	175.067	63.24
	7	2.286	18.7	7	810	7	96.99	175.067	54.83
	8	2.304	17.9	6.1	860	8	98.21	175.067	55.53
	9	2.32	17.3	5.4	990	8	99.71	175.067	56.38
	10	2.331	16.8	4.8	1100	9	101.40	175.067	57.35
	11	2.318	17.2	5.3	1030	8	102.81	175.067	58.16
	12	2.33	16.7	4.7	1070	9	104.95	175.067	59.38
	13	2.24	20.5	9	570	7	92.76	175.067	52.41
	14	2.265	19.5	7.9	710	7	93.77	175.067	52.99
	15	2.277	18.9	7.2	800	7	94.86	175.067	53.61
	16	2.274	19	7.3	780	8	96.22	175.067	54.39
	17	2.282	18.6	6.9	860	8	97.77	175.067	55.27
	18	2.326	17.1	5.2	980	7	101.03	175.067	57.14
	19	2.35	16.1	4.1	1220	8	102.65	175.067	58.06
	20	2.355	15.8	3.8	1230	9	104.38	175.067	59.05
	21	2.346	16.1	4.1	1080	8	106.50	175.067	60.27
	22	2.306	17.7	5.9	960	8	106.55	175.067	60.29
	23	2.343	16.3	4.3	1080	8	111.24	175.067	62.97
	24	2.374	15.1	2.9	1260	8	115.54	175.067	65.43

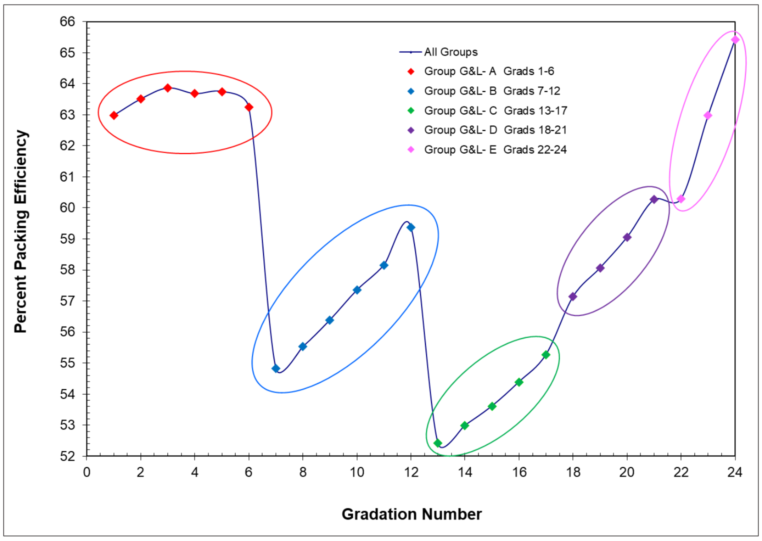


Figure 11: Calculated Percent Packing Efficiency vs Gradation Number for All 24 Gradation in the Goode and Lufsey Study

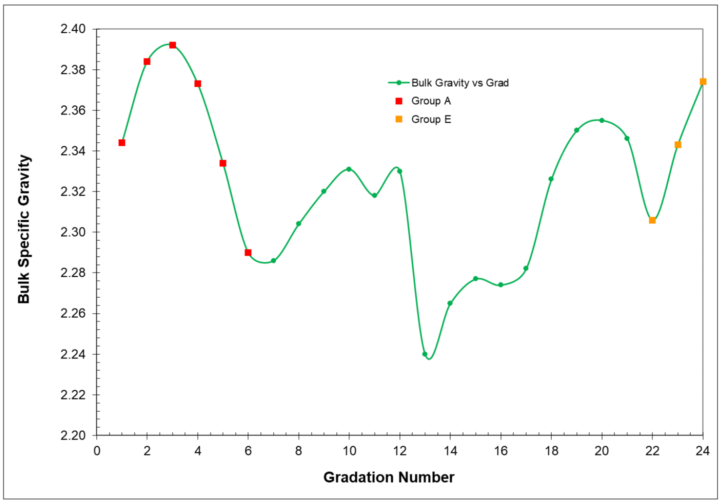


Figure 12: Bulk Specific Gravity vs Gradation Number for All 24 Gradation in the Goode and Lufsey Study

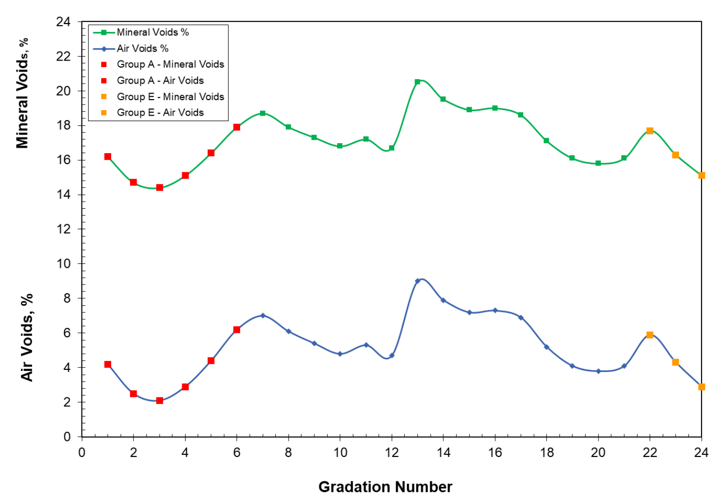


Figure 13: Mineral and Air Voids vs Gradation Number for All 24 Gradation in the Goode and Lufsey Study

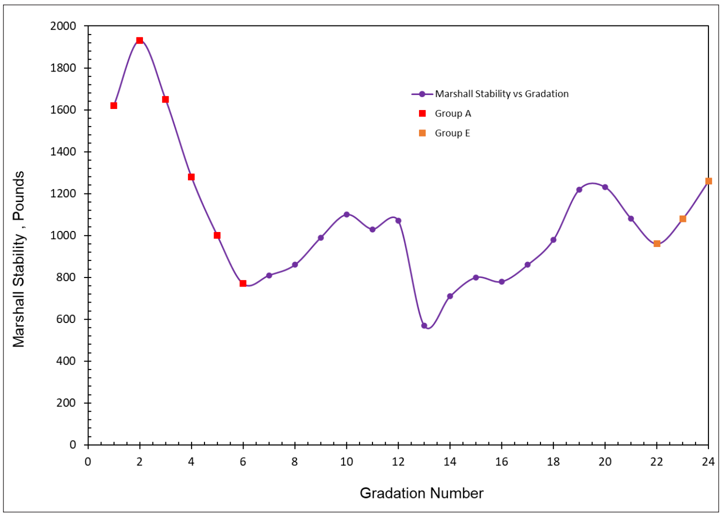


Figure 14: Marshall Stability vs Gradation Number for All 24 Gradation in the Goode and Lufsey Study

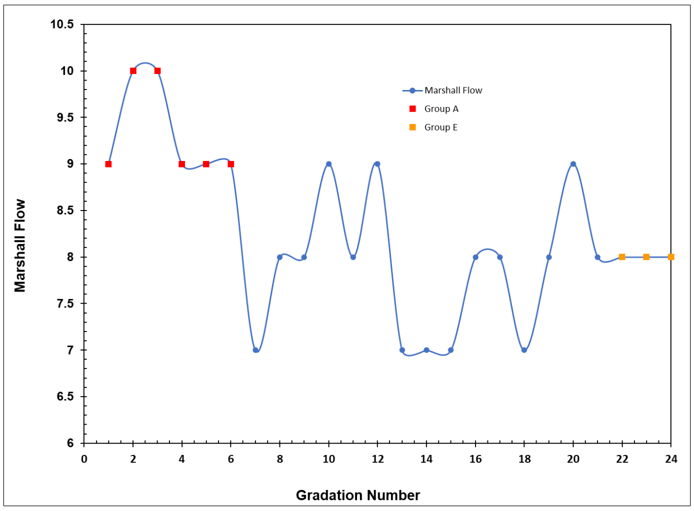


Figure 15: Marshall Flow vs Gradation Number for All 24 Gradation in the Goode and Lufsey Study

It is interesting to note in Figure 11 that the Percent Packing Efficiency did successfully separate all 24 different sets of gradation mixtures generated by Goode and Lufsey into five clearly defined separate groups. It was also surprising to see that each of these 5 groups appeared to form essentially a straight line between the Percent Packing Efficiency and the Goode and Lufsey gradation number. This result indicated that the volume fractions indicated in Figures 11 were generated by Goode and Lufsey for each group in such a consistent way that each gradation was not only consistent within each group but that each group also was different enough to clearly be recognized as a separate group. This can only be described as an amazing achievement.

Of the five separate groups separated by Percent Packing Efficiency, the group A consisting of Gradations 1-6 appeared to offer the best optimization of the 5 different properties characterized in the Goode and Lufsey Study. Within group A, Gradation 3 was found to yield the maximum value of Percent Packing Efficiency and this was the same Gradation identified by Goode and Lufsey as their preferred choice to use as a standard for their recommended maximum density line.

Comparison of Figures 12 and 13 clearly indicates that the minimums for the % Mineral Void and the % Air Voids as well as the maximums for the Bulk Specific Gravity appear to correspond with the maximums for the Percent Packing Efficiency. These results strongly suggest that the Percent Packing Efficiency appears to be as significant indicator of the packing efficiency of the aggregate blends. The Percent Packing Efficiency appears to indicate when a particle size distribution is packed efficiently as well as when a particle distribution is not packed well. This was most clearly indicated in the Goode and Lufsey gradation 13 that had the poorest balance of properties as a result of having the lowest Percent Packing Efficiency and consequently the largest mineral and air voids.

The fact that Percent Packing Efficiency correlates so well with the void content and bulk specific gravity and also to some extent the Marshal Stability does suggest that it may be possible to define a lower limit of Percent Packing Efficiency to be able to avoid unstable asphalt/aggregate applications.
The two groups most effected by both Marshall Stability and Marshall Flow appear to be Group A and Group C in Figure s 14 & 15. In general, Group A had the highest values of Percent Packing Efficiency and Group C had the lowest values of Percent Packing Efficiency.

The best relationship between the physical properties measured by Goode and Lufsey and the Percent Packing Efficiency was found for Gradations 7-21 as indicated in Figures 16-20. Since the shapes of the volume fraction curves for Groups A and E are obviously different from those of Groups B, C and D relative to the Percent Packing Efficiency. Groups A and E appear to indicate that they could require additional analysis that will not be addressed at this time.

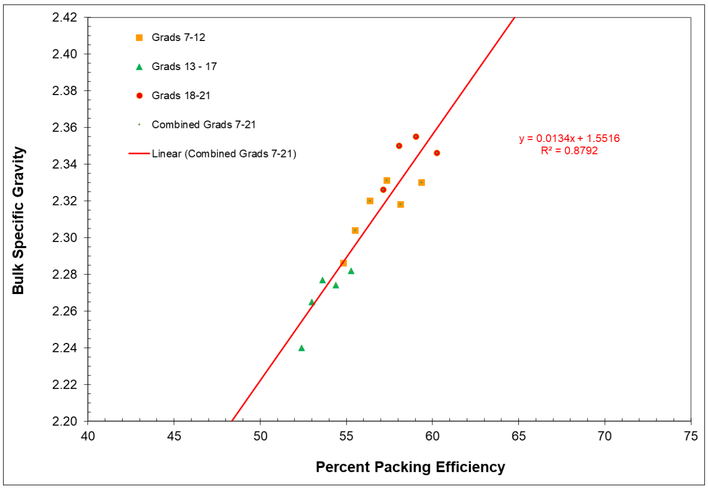


Figure 16: Bulk Specific Gravity vs Percent Packing Efficiency for G&L Gradations 7-21

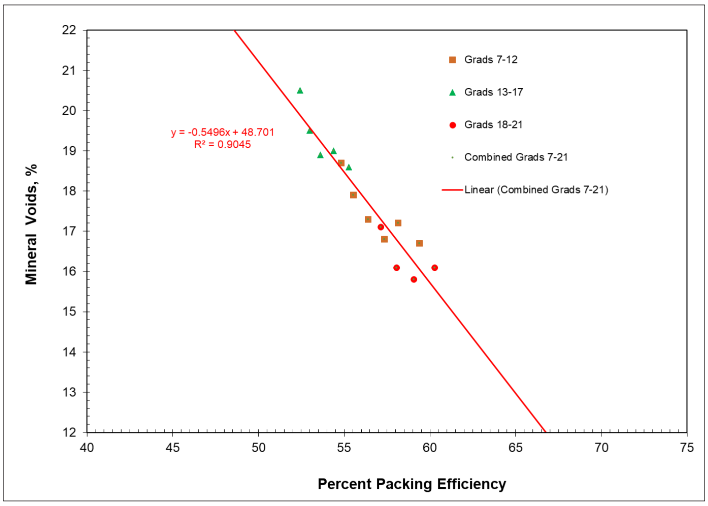


Figure 17: % Mineral Voids vs Percent Packing Efficiency for G&L Gradations 7-21

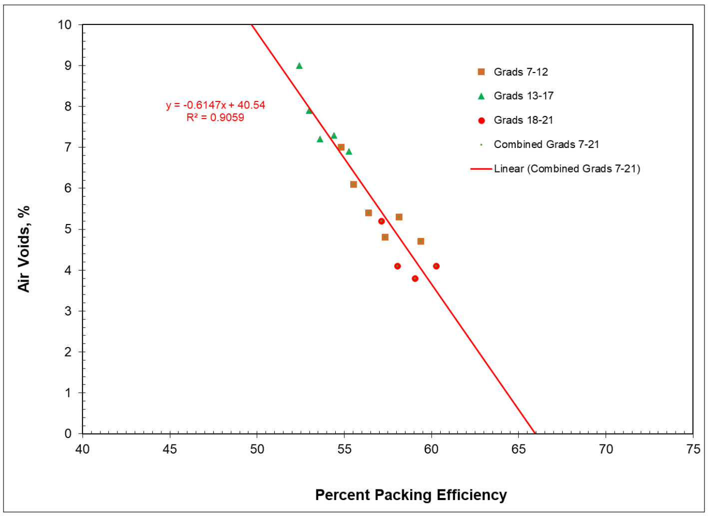


Figure 18: % Air Voids vs Percent Packing Efficiency for G&L Gradations 7-21

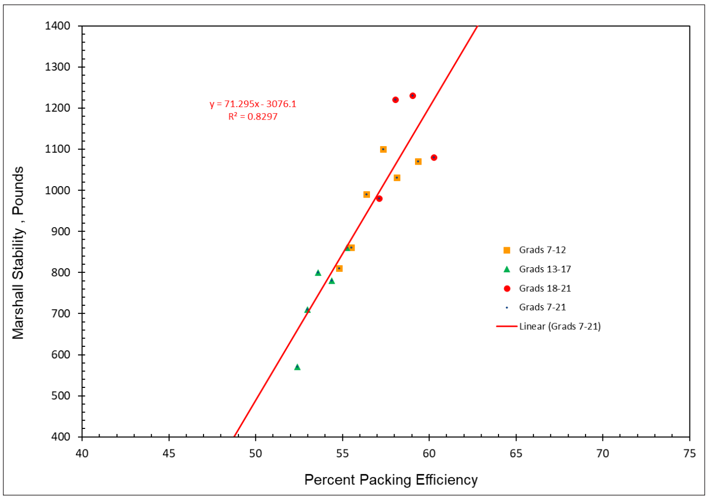


Figure 19: Marshall Stability vs Percent Packing Efficiency for G&L Gradations 7-21

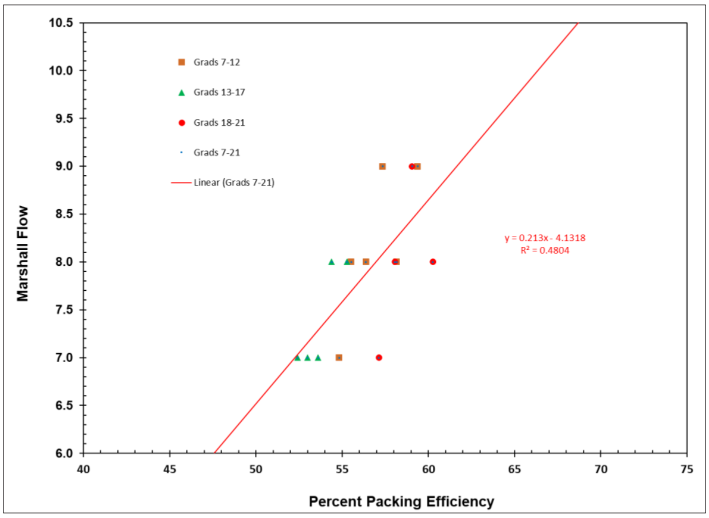


Figure 20: Marshall Flow vs Percent Packing Efficiency for G&L Gradations 7-21

In general, most of the properties measured by Goode and Lufsey gave reasonably good correlations with the Percent Packing Efficiency. Since the measurements of the Marshall Flow had so few data points it was not easy to get a good correlation with Percent Packing Efficiency. Nevertheless, the results in Figures 16-19 show that in general the Percent Packing Efficiency gave a favorable influence on most of the properties measured by Goode and Lufsey for the Gradations generated in their study.

Within the Goode and Lufsey study the following four properties appeared to form effective linear relationship with each other including:

- Mineral Voids % vs Bulk Specific Gravity
- Air Voids % vs Bulk Specific Gravity
- Marshall Stability vs Bulk Specific Gravity
- Mineral Voids % vs Air Voids %
- In general , all of the properties measured by Goode and Lufsey (particularly for Gradation 7-21) except for Marshall Flow did appear to form a generally good direct relationship with the values of Percent Packing efficiency

Comparison of the Particle Volume Fractions for All 24 Gradations in the Goode and Lufsey Study

The volume fractions of all 24 of the Goode and Lufsey Gradations that were used to make the calculations for Percent Packing Efficiencies in Table 7 are indicated in Figures 21-25. At this point it is useful to note that the volume fraction for all 24 of the Goode and Lufsey particle size distributions appeared to be similar to Gradation 3 in that they all had a significant drop in the volume fractions for the largest particles and many had added volume fractions to the smallest particle sizes. While the justification of this result is not apparent, Goode and Lufsey may have modeled the other gradations in their study from the successful development of Gradation 3.

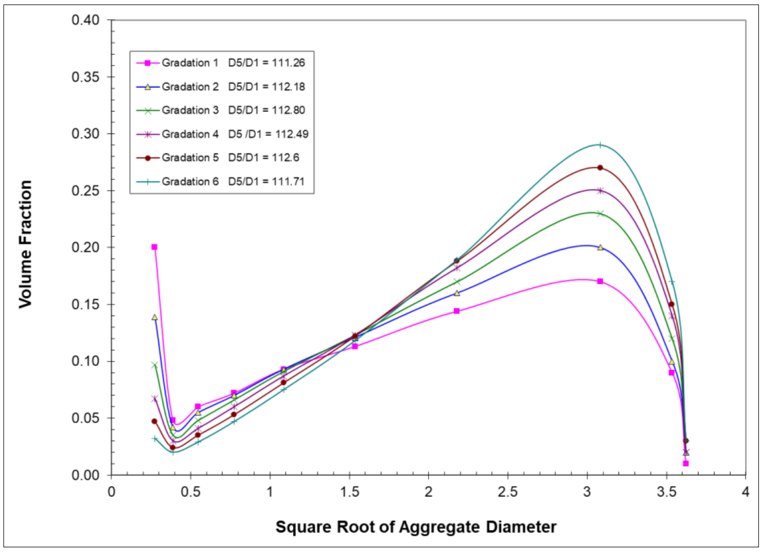


Figure 21: Volume fraction vs Square Root of Aggregate Diameter (Group A – G&L Gradations 1-6)

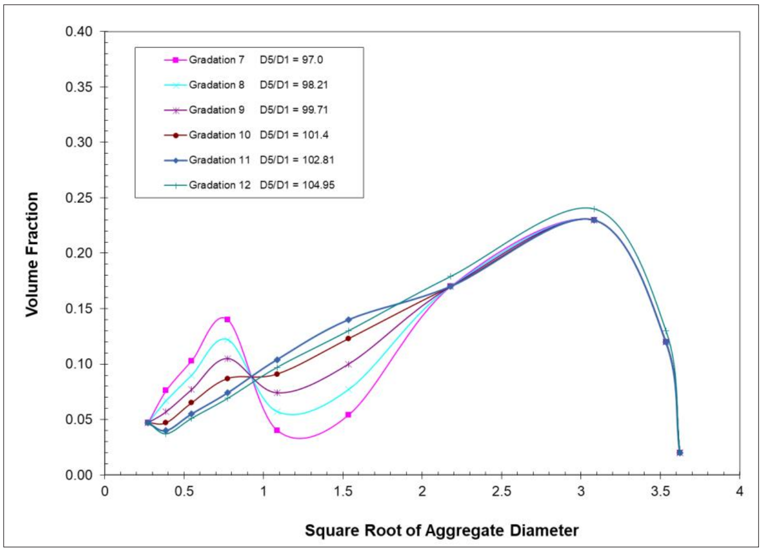


Figure 22: Volume fraction vs Square Root of Aggregate Diameter (Group B – G&L Gradations 7-12)

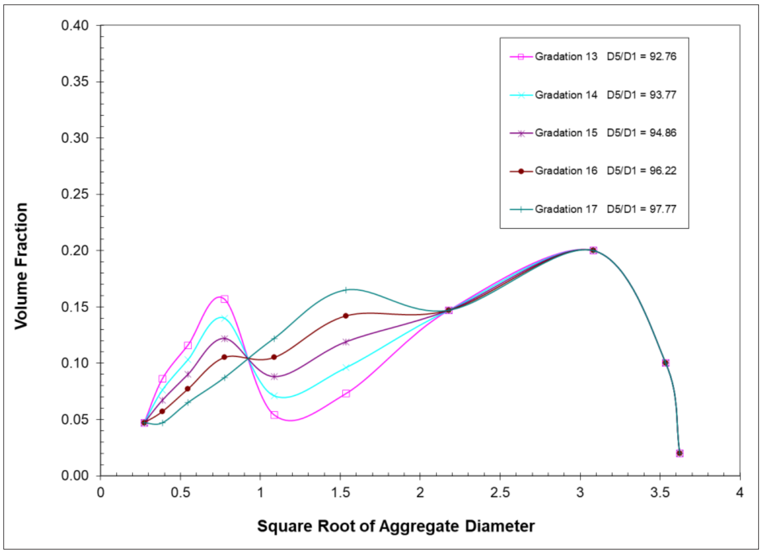


Figure 23: Volume fraction vs Square Root of Aggregate Diameter (Group C – G&L Gradations 13-17)

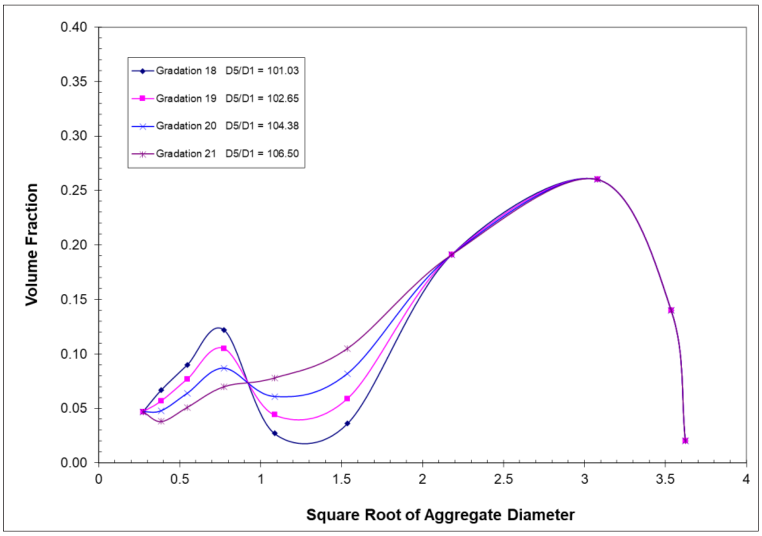


Figure 24: Volume fraction vs Square Root of Aggregate Diameter (Group D – G&L Gradations 18-21)

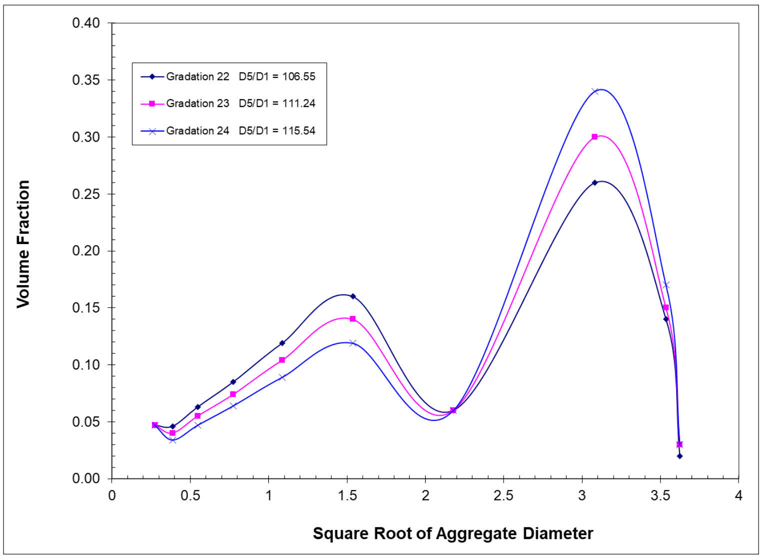
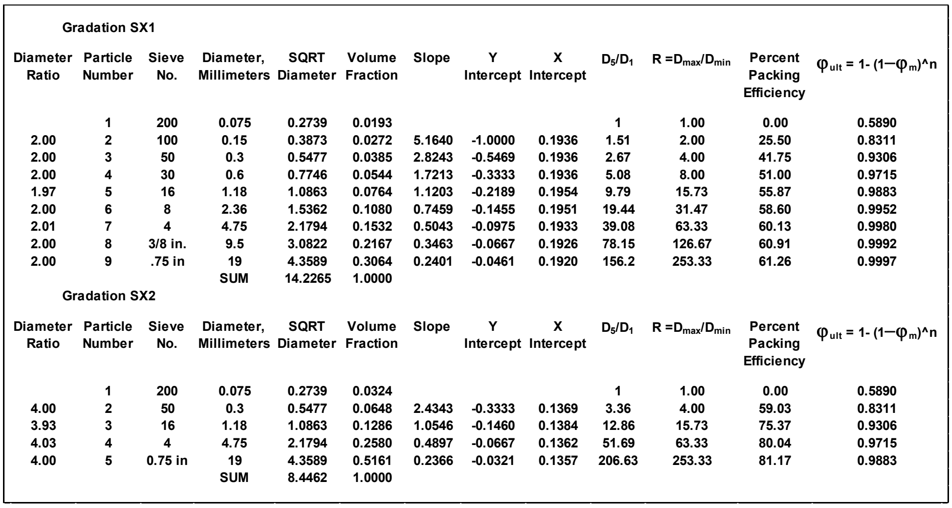


Figure 25: Volume fraction vs Square Root of Aggregate Diameter (Group E – G&L Gradations 22-24)

Characteristics of two Optimum Particle Size Distributions with a Constant Ratio Between Particles for Z=2 and Z=4

Several sets of optimum aggregate gradations with constant ratios between particle diameters for both Z=2 and Z=4 have been summarized in Table 8 and Figures 26 and 27

Table 8: Gradation SX1 (Z=2 with up to 9 Particles and Gradation SX2 (Z=4 with up to 5 Particles) from this Study



Gradation SX1

Diameter Ratio	Particle Number	Sieve No.	Diameter, Millimeters	SQRT Diameter	Volume Fraction	Slope	Y Intercept	X Intercept	D_5/D_1	R =D_{max}/D_{min}	Percent Packing Efficiency	φ_{ult} = 1- (1−φ_m)^n
	1	200	0.075	0.2739	0.0193				1	1.00	0.00	0.5890
2.00	2	100	0.15	0.3873	0.0272	5.1640	-1.0000	0.1936	1.51	2.00	25.50	0.8311
2.00	3	50	0.3	0.5477	0.0385	2.8243	-0.5469	0.1936	2.67	4.00	41.75	0.9306
2.00	4	30	0.6	0.7746	0.0544	1.7213	-0.3333	0.1936	5.08	8.00	51.00	0.9715
1.97	5	16	1.18	1.0863	0.0764	1.1203	-0.2189	0.1954	9.79	15.73	55.87	0.9883
2.00	6	8	2.36	1.5362	0.1080	0.7459	-0.1455	0.1951	19.44	31.47	58.60	0.9952
2.01	7	4	4.75	2.1794	0.1532	0.5043	-0.0975	0.1933	39.08	63.33	60.13	0.9980
2.00	8	3/8 in.	9.5	3.0822	0.2167	0.3463	-0.0667	0.1926	78.15	126.67	60.91	0.9992
2.00	9	.75 in	19	4.3589	0.3064	0.2401	-0.0461	0.1920	156.2	253.33	61.26	0.9997
			SUM	14.2265	1.0000							

Gradation SX2

Diameter Ratio	Particle Number	Sieve No.	Diameter, Millimeters	SQRT Diameter	Volume Fraction	Slope	Y Intercept	X Intercept	D_5/D_1	R =D_{max}/D_{min}	Percent Packing Efficiency	φ_{ult} = 1- (1−φ_m)^n
	1	200	0.075	0.2739	0.0324				1	1.00	0.00	0.5890
4.00	2	50	0.3	0.5477	0.0648	2.4343	-0.3333	0.1369	3.36	4.00	59.03	0.8311
3.93	3	16	1.18	1.0863	0.1286	1.0546	-0.1460	0.1384	12.86	15.73	75.37	0.9306
4.03	4	4	4.75	2.1794	0.2580	0.4897	-0.0667	0.1362	51.69	63.33	80.04	0.9715
4.00	5	0.75 in	19	4.3589	0.5161	0.2366	-0.0321	0.1357	206.63	253.33	81.17	0.9883
			SUM	8.4462	1.0000							

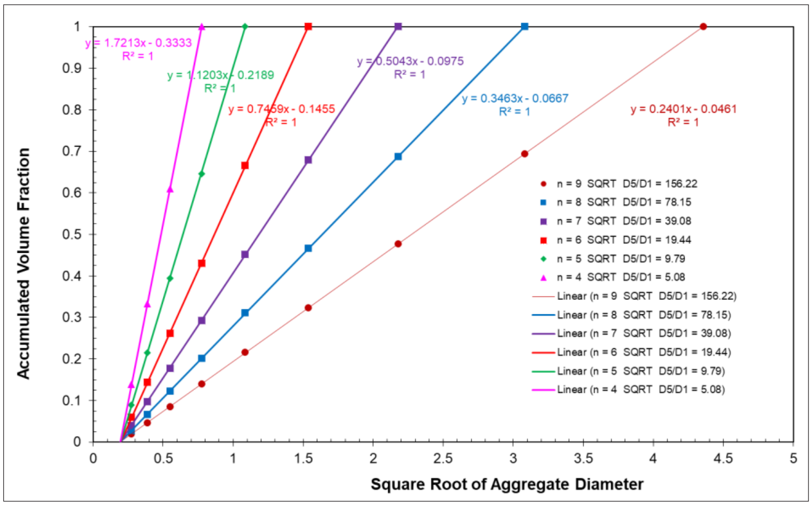


Figure 26: Accumulated Volume Fraction vs Square Root of Particle Diameter (Z = 2)

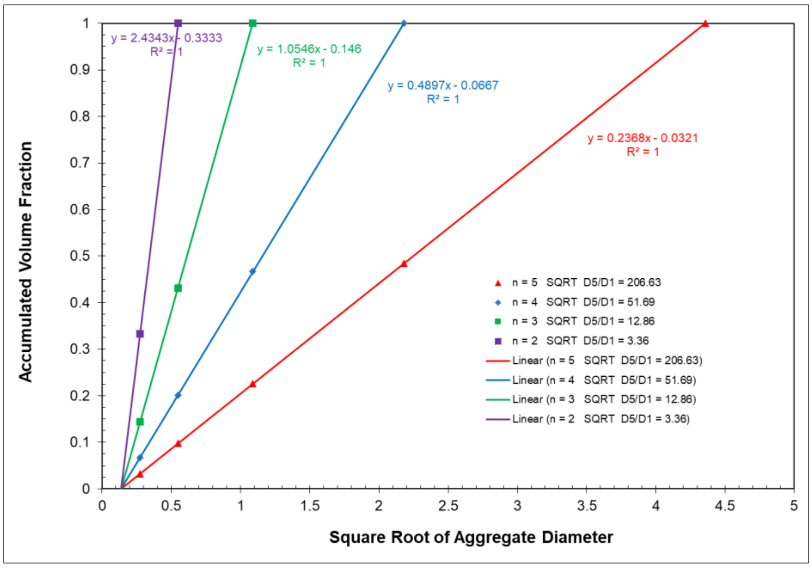


Figure 27: Accumulated Volume Fraction vs Square Root of Particle Diameter (Z = 2)

For each of the Z calculated straight line results in Table 8 the ratio of $\overline{D}_5/\overline{D}_1$ and the Percent Packing Efficiency increased as the number of larger particles were increased. In addition, as the value of $\overline{D}_5/\overline{D}_1$ and the Percent Packing Efficiency increased the maximum possible packing fraction also increased. It is also clear that the value of $\overline{D}_5/\overline{D}_1$ and the Percent Packing Efficiency increased as the value of Z increased. Table 8 also indicates that when a limited number of particle sizes are available, then as the value of Z increases the number of particles must necessarily decrease. As the available particles decreases the maximum possible ultimate packing fraction must also necessarily decrease.

Therefore, a balance needs to be established between the maximum value of Z and the maximum number of particle sizes. Consequently, for the results indicated in Table 8 the preferred maximum value of Z would appear to be Z=4. However, since more particles are available for Z=2 it is possible that having a larger maximum packing fraction available might make the value of Z=2 more desirable.

The aggregate gradations in Table 8 have also been plotted in Figures 26 and 27. Note that the graphs in Figures 26 and 27 do not go through the origin as indicated by equation 1 as originally proposed by Fuller and Thompson and continued by Nijboer Furnas, as well as Goode and Lufsey [1-5]. In fact, each of the graphs in Figures 26 and 27 converge to a point on the X-axis defined by equation 19 that is different for each value of Z and/or the initial particle size, D_1. This result is in direct conflict with the results indicated by Goode and Lufsey.

A more detailed summary of the formulation characteristics for Gradations SX1 and SX2 have been summarized in Table 9.

Table 9: Summary of Formulation Characteristics for Gradations SX1 (Z=2) and SX2 (Z = 4)

SX1

Particles = 9
Max D5/D1= 253.3333
Max Particle Size= 19
Min. Particle Size= 0.075

% Packing Efficiency = 61.2712
Actual D5/D1= 156.2204
Actual D5 = 14.9243
Actual D1 = 0.0955

Diameter Ratio	Particle Number	Sieve No.	Diameter, Millimeters	SQRT(Dia)	Optimum Volume Fraction	Accumulated Volume Fraction
	1	200	0.075	0.2739	0.0193	0.0193
2.00	2	100	0.15	0.3873	0.0272	0.0465
2.00	3	50	0.3	0.5477	0.0385	0.0850
2.00	4	30	0.6	0.7746	0.0544	0.1394
1.97	5	16	1.18	1.0863	0.0764	0.2158
2.00	6	8	2.36	1.5362	0.1080	0.3238
2.01	7	4	4.75	2.1794	0.1532	0.4770
2.00	8	3/8 in.	9.5	3.0822	0.2167	0.6936
2.00	9	.75 in	19	4.3589	0.3064	1.0000
			Sum	14.2265	1.00000	

SX2

Particles = 5
Max D5/D1= 253.3333
Max Particle Size= 19
Min. Particle Size= 0.075

% Packing Efficiency = 81.1713
Actual D5/D1= 206.6339
Actual D5 = 17.1633
Actual D1 = 0.0831

Diameter Ratio	Particle Number	Sieve No.	Diameter, Millimeters	SQRT(Dia)	Optimum Volume Fraction	Accumulated Volume Fraction
	1	200	0.075	0.2739	0.0324	0.0324
4.00	2	50	0.3	0.5477	0.0648	0.0973
3.93	3	16	1.18	1.0863	0.1286	0.2259
4.03	4	4	4.75	2.1794	0.2580	0.4839
4.00	5	.75 in	19	4.3589	0.5161	1.0000
			Sum	8.4462	1.00000	

A direct comparison of the volume fractions of Gradation SX1 and Gradation SX2 in Table 9 and the volume fractions from Goode and Lufsey's Gradation 3 in Table 5 are compared directly in Figure 28. Note that the volume fractions for Gradation SX1 and SX2 form straight lines going through the origin. In addition, each of the volume fractions for both Gradation SX1 and SX2 increased progressively as the square root of the particle diameter increased. In other words for each of these two Gradations the volume fractions for both small particles and the large particles increased proportionally with their particle size. Since Gradation SX2 had 5 particles and Gradation SX1 had 9 particles then Gradation SX2 yielded a volume fraction for the largest particle that was larger than the volume fraction for the largest particle size for Gradation SX1.

However, the most amazing observation indicated in Figure 28 involves the comparison of the straight lines for Gradation SX1 and Gradation 3. Note that the straight-line slopes for both Gradation SX1 and the middle particles for Gradation 3 were essentially identical. However, the straight line for Gradation SX1 goes through the origin whereas the middle particles for Gradation 3 yielded an intercept that was shifted by only 0.0097, which is next to nothing. Consequently, it is amazing that Goode and Lufsey found their volume fractions for Gradation 3 essentially by trial and error whereas the volume fractions for Gradation SX1 were calculated using the procedure described in this study.

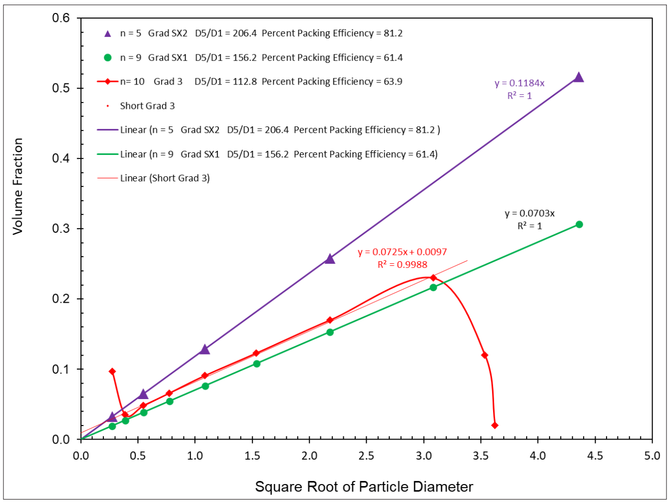


Figure 28: Volume Fraction Comparison between Gradation 3, Gradation SX1(Z=2) & Gradation SX2 (Z=4) vs Square Root of Particle Diameter

Since both Gradations SX1 and SX2 have significantly higher values of $\overline{D}_5/\overline{D}_1$ than Goode and Lufsey's Gradation 3, then based on experience from the coating industry it is strongly expected that both of these gradations will have higher maximum packing fractions than Gradation 3. While Gradation 3 had a higher Percent Packing Efficiency = 63.9 than Gradation SX1 which had a Percent Packing Efficiency = 61.3, these two values are close enough that the higher value of $\overline{D}_5/\overline{D}_1$ for Gradation SX1 should preferentially accentuate the generation of a higher maximum packing fraction.

Modifications of Gradation SX1 to Generate Two Modified Gradations of SX1 with Increased Void Content

According to the discussion included in the Goode and Lufsey study, it is apparently desirable for certain applications to have some gradations with higher void content to allow more asphalt matrix to be infused into the asphalt/Aggregate formulation. Therefore, using the approach recommended in the Goode and Lufsey study, Gradation 13 was used as a guideline to modify the lower portion of Gradation SX1 while attempting to retain as much of the upper portion of Gradation SX1 as possible to try to minimize the loss of stability. The two straight lines added to the volume fraction of Gradation 13 as indicated in Figure 29 have been used to characterize the lower part of Gradation 13. The characterization of the lower portion of Gradation 13 have then been used to modify the lower portion of Gradation SX1 as indicated in Figures 30 &31.

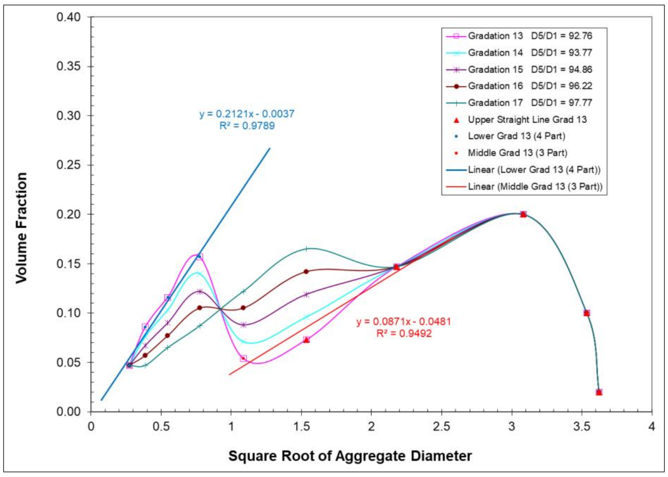


Figure 29: Straight Line Characterizationos of the Lower Portion of Gradation 13 to Modify Gradation SX1

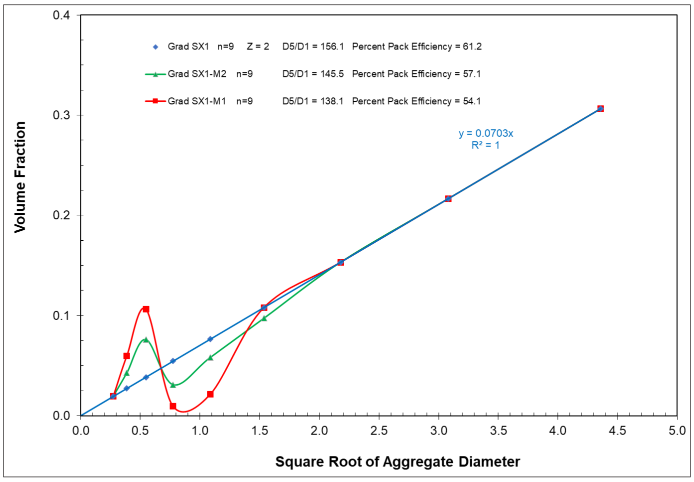


Figure 30: Volume Fraction Modifications for Gradations SX1-M1 and SX1-M2

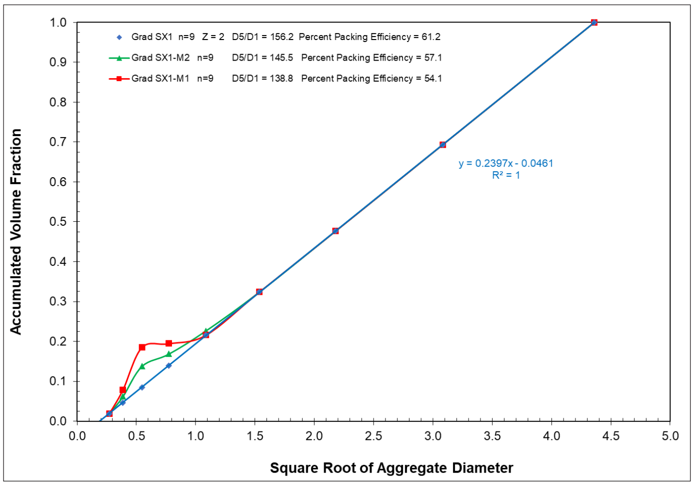


Figure 31: Accumulated Volume Fraction Modifications for Gradations SX1-M1 and SX1-M2

Table 10: Summary of Gradations SX1-M1 and SX1-M2 (Modified SX1 with Z=2 and n = 9)

SX1-M1

Particles =	9	Actual D5/D1=	138.0875
Max D5/D1=	253.3333	Percent Packing Efficiency =	54.1135
Max Particle Size=	19	Actual D5 =	15.0263
Min. Particle Size=	0.075	Actual D1 =	0.1088

Diameter Ratio	Particle Number	Sieve No.	Diameter, Millimeters	SQRT(Dia)	Optimum Volume Fraction	Calculated Volume Fraction	Accumulated Volume Fraction
	1	200	0.075	0.2739	0.0193	0.0193	0.0193
2.00	2	100	0.15	0.3873	0.0272	0.0595	0.0788
2.00	3	50	0.3	0.5477	0.0385	0.1063	0.1851
2.00	4	30	0.6	0.7746	0.0544	0.0094	0.1945
1.97	5	16	1.18	1.0863	0.0764	0.0213	0.2158
2.00	6	8	2.36	1.5362	0.1080	0.1080	0.3238
2.01	7	4	4.75	2.1794	0.1532	0.1532	0.4770
2.00	8	3/8 in.	9.5	3.0822	0.2167	0.2167	0.6936
2.00	9	.75 in	19	4.3589	0.3064	0.3064	1.0000
			Sum	14.2265	1.00000	1.0000	

SX1-M2

Particles =	9	Actual D5/D1=	145.4622
Max D5/D1=	253.3333	Percent Packing Efficiency =	57.0246
Max Particle Size=	19	Actual D5 =	14.9939
Min. Particle Size=	0.075	Actual D1 =	0.1031

Diameter Ratio	Particle Number	Sieve No.	Diameter, Millimeters	SQRT(Dia)	Optimum Volume Fraction	Calculated Volume Fraction	Accumulated Volume Fraction
	1	200	0.075	0.2739	0.0193	0.0193	0.0193
2.00	2	100	0.15	0.3873	0.0272	0.0425	0.0618
2.00	3	50	0.3	0.5477	0.0385	0.0760	0.1378
2.00	4	30	0.6	0.7746	0.0544	0.0308	0.1686
1.97	5	16	1.18	1.0863	0.0764	0.0580	0.2266
2.00	6	8	2.36	1.5362	0.1080	0.0972	0.3238
2.01	7	4	4.75	2.1794	0.1532	0.1532	0.4770
2.00	8	3/8 in.	9.5	3.0822	0.2167	0.2167	0.6936
2.00	9	.75 in	19	4.3589	0.3064	0.3064	1.0000
			Sum	14.2265	1.00000	1.0000	

The results for Gradations SX1-M1 and SX1-M2 in Table 10 clearly indicate that these modification of SX1 have significantly lowered the Percent Packing Efficiency of these Gradations. Therefore, based on the results summarized in Table 7 this would appear to indicate that Gradations SX1-M1 and SX1-M2 would both be expected to have significantly to have larger void contents than an unmodified Gradation SX1.

Comparison of the Goode & Lufsey's Gradation 3 with a Superpave Gradation, A Stone Matrix Gradation and a Bailey Gradation with Several Gradations Developed in This Study

Since Gradation 3 from the Goode and Lufsey study has been considered to be one of the standards for preparing aggregate Gradations in asphalt, it was compared directly in Figure 32and Tables 11 & 12 with three other industry aggregate/asphalt gradation standards [5]. The Superpave Gradation in Figure 32 and Table 11 was from a document published by the Asphalt Institute [25]. The Stone Matrix Gradation in Figure 32 and Table 11 was from a published document by the National Asphalt Pavement Association [26]. The Baily Gradation in Figure 32 and Table 11 was from a document published by the Transportation Research Board [27]. Based on the results indicated in Figure 32 and Tables 11 and 12 it appears that these four different gradations can initially be separated based on their plots of accumulated volume fraction vs the square root of their particle diameters.

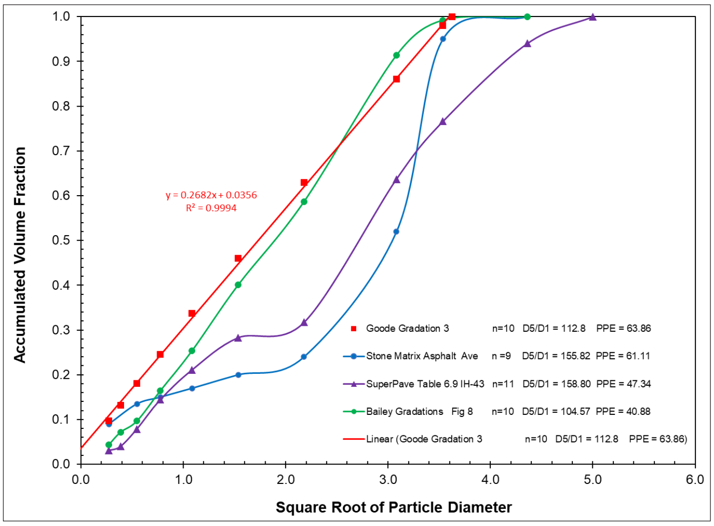


Figure 32: Accumulated Volume Fraction Comparison Between Gradation 3, Superpave, Baily and SMA Asphalt Gradations vs Square Root of Particle Diameter

However, the levels of Percent Packing Efficiency does seem to be the first consideration to separate these four different gradations with the ratio of $\overline{D}_5/\overline{D}_1$ playing a secondary role. Note that the Goode and Lufsey's Gradation 3 had the largest Percent Packing Efficiency = 63.9 with the Stone Matrix coming in second with a Percent Packing Efficiency = 61.1. However, the Superpave Gradation had a Percent Packing Efficiency = 47.3 and the Bailey Gradation had a Percent Packing Efficiency = 40.9. In general, the lower the Percent Packing Efficiency the higher the expected void content.

It is also important to note that only Gradation 3 in Figure 32 had an obvious straight-line plot of the accumulated volume fraction vs the square root of the particle diameter.

A comparison of the volume fractions for these four gradations is shown in Figure 33. Earlier in this study it has been shown that in order for the Goode and Lufsey study to get their desire straight line with maximum density for Gradation 3 it was necessary to add an additional particles to the smallest particle size and unfortunately to subtract a significant amount of particles from the largest particles to get their classic straight line. Consequently, as indicated in Figure 33 in appears that each of the other aggregate/asphalt blending standards found it desirable to utilize the Goode and Lufsey model concepts to create their preferred aggregate/asphalt blends. Therefore, essentially all of the standard blends in Figure 33 appear to have added extra particles to the smallest particles and subtracted a significant amount of volume fraction from the largest particles to generate plots of the accumulation of volume fractions vs the square root of particle diameter to get their optimum blends without necessarily generating a straight line.

Table 11: Aggregate Calculations for Goode and Lufsey's Gradation 3 and One Superpave Gradation

Gradation = 3
Goode and Lufsey
D_{max}/D_{min} = 175.0667
n = 10

Percent Packing Efficiency = 63.8624
Actual D5/D1= 112.8017
Actual D5 = 9.1262
Actual D1 = 0.0809

Ratio Between Particle Sizes	Particle Number	Sieve Number	Diameter, Millimeters	SQRT(Dia)	Volume Fraction	Accumulated Volume Fraction
	1	200	0.0750	0.2739	0.0970	0.0970
2.00	2	100	0.1500	0.3873	0.0350	0.1320
2.00	3	50	0.3000	0.5477	0.0480	0.1800
2.00	4	30	0.6000	0.7746	0.0660	0.2460
1.97	5	16	1.1800	1.0863	0.0910	0.3370
2.00	6	8	2.3600	1.5362	0.1230	0.4600
2.01	7	4	4.7500	2.1794	0.1700	0.6300
2.00	8	3/8 in	9.5000	3.0822	0.2300	0.8600
1.32	9	1/2 in	12.5000	3.5355	0.1200	0.9800
1.05	10	0.525	13.1300	3.6235	0.0200	1.0000
			Sum =	17.0267	1.0000	

Gradation = Blend 1
SuperPave Table 6.9 IH-43
D_{max}/D_{min} = 333.3333
n = 11

Percent Packing Efficiency = 47.3396
Actual D5/D1= 158.7986
Actual D5 = 13.4316
Actual D1 = 0.0846

Ratio Between Particle Sizes	Particle Number	Sieve Number	Diameter mm	SQRT(SS)	Calculated Volume Fraction	Accumulated Volume Fraction
	1	200	0.0750	0.2739	0.0310	0.0310
2.00	2	100	0.1500	0.3873	0.0090	0.0400
2.00	3	50	0.3000	0.5477	0.0390	0.0790
2.00	4	30	0.6000	0.7746	0.0650	0.1440
1.97	5	16	1.1800	1.0863	0.0670	0.2110
2.00	6	8	2.3600	1.5362	0.0720	0.2830
2.01	7	4	4.7500	2.1794	0.0340	0.3170
2.00	8	3/8 in	9.5000	3.0822	0.3200	0.6370
1.32	9	1/2 in	12.5000	3.5355	0.1290	0.7660
1.52	10	3/4 in	19.0000	4.3589	0.1740	0.9400
1.32	11	1 in	25.0000	5.0000	0.0600	1.0000
			SUM=	22.7621	1.0000	

Table 12: Aggregate Calculations for a Stone Matrix Gradation and a Bailey Gradation

Gradation = Ave Values
Stone Matrix 12.5mm NMAS
D_{max}/D_{min} = 253.3333
n = 9

Percent Packing Efficiency = 61.1114
Actual D5/D1= 155.8156
Actual D5 = 12.0033
Actual D1 = 0.0770

Ratio Between Particle Sizes	Particle Number	Sieve Number	Diameter mm	SQRT(SS)	Calculated Volume Fraction	Accumulated Volume Fraction
	1	200	0.0750	0.2739	0.0900	0.0900
4.00	2	50	0.3000	0.5477	0.0450	0.1350
2.00	3	30	0.6000	0.7746	0.0150	0.1500
1.97	4	16	1.1800	1.0863	0.0200	0.1700
2.00	5	8	2.3600	1.5362	0.0300	0.2000
2.01	6	4	4.7500	2.1794	0.0400	0.2400
2.00	7	3/8 in	9.5000	3.0822	0.2800	0.5200
1.32	8	1/2 in	12.5000	3.5355	0.4300	0.9500
1.52	9	3/4 in	19.0000	4.3589	0.0500	1.0000
			SUM=	17.3748	1.0000	

Gradation = Figure 8
Bailey
D_{max}/D_{min} = 253.3333
n = 10

Percent Packing Efficiency = 40.8819
Actual D5/D1= 104.5676
Actual D5 = 8.8449
Actual D1 = 0.0846

Ratio Between Particle Sizes	Particle Number	Sieve Number	Diameter, Millimeters	SQRT(Dia)	Volume Fraction	Accumulated Volume Fraction
	1	200	0.0750	0.2739	0.0440	0.0440
2.00	2	100	0.1500	0.3873	0.0280	0.0720
2.00	3	50	0.3000	0.5477	0.0250	0.0970
2.00	4	30	0.6000	0.7746	0.0670	0.1640
1.97	5	16	1.1800	1.0863	0.0900	0.2540
2.00	6	8	2.3600	1.5362	0.1470	0.4010
2.01	7	4	4.7500	2.1794	0.1860	0.5870
2.00	8	3/8 in	9.5000	3.0822	0.3270	0.9140
1.32	9	1/2 in	12.5000	3.5355	0.0780	0.9920
1.52	10	3/4 in	19.0000	4.3589	0.0080	1.0000
			Sum =	17.7621	1.0000	

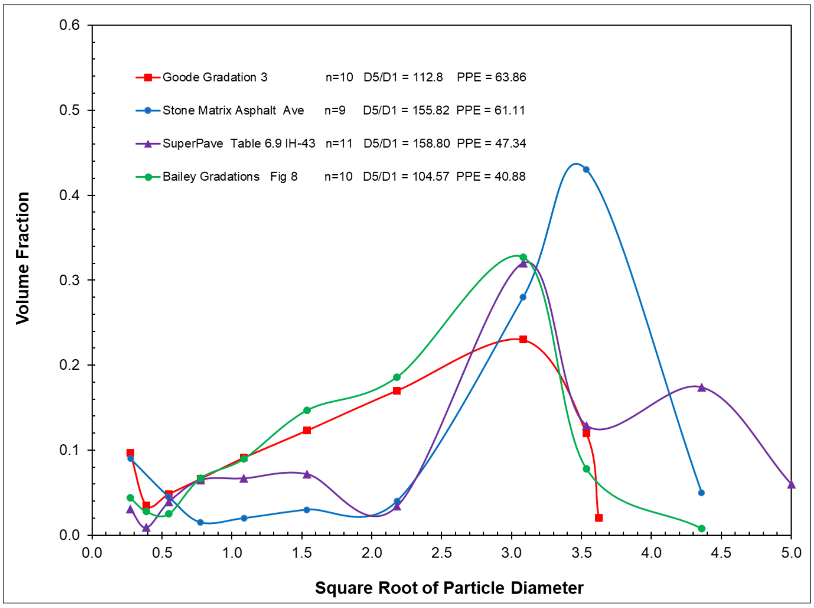


Figure 33: Volume Fraction Comparison between Gradation 3, a Superpave, a Bailey and an SMA Asphalt Gradation vs Square Root of Particle Diameter

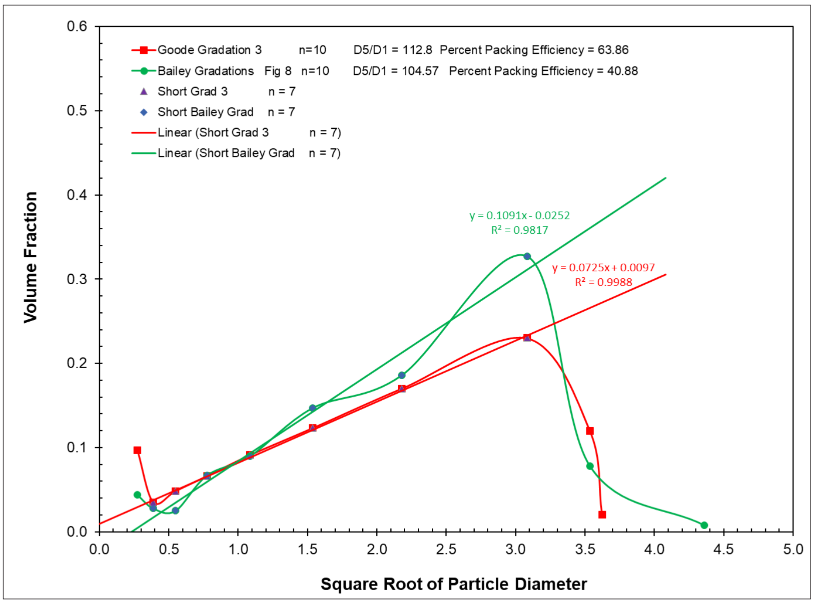


Figure 34: Volume Fraction Comparison between Gradation 3 and a Bailey Gradation vs Square Root of Particle Diameter

A comparison of Gradation 3 with the Baily Gradation from Figure 33 is shown again in Figure 34 where a least square straight line has been applied to the center section of both of these gradations. As indicated in Figure 34 the same process that was utilized to create gradation 3 appears to have been applied to this Bailey Gradation.

While the formulation logic of the gradations in Figure 34 is understandable, the logic for the Superpave and Stone Matrix gradation in Figure 33 is not obvious and does not appear to be consistent with the understanding developed in this study.

Gradation Property Predictions Versus Percent Packing Efficiency as Generated from the Goode and Lufsey Gradations 7-21

The estimated average property values for gradations generated in this study have been summarized in Table 13. The properties indicated in Table 13 have been estimated from straight line average property extrapolations vs Percent Packing Efficiency utilizing the results indicated in Figures 16- 20. The estimated property values in Table 13 have been generated from measured property values reported by Goode and Lufsey for Gradations 7-21[5].

Table 13: Estimated Average Property Values for Gradations Reported or Generated in This Study

Gradation	Number of Particles	fextra	Bulk Gravity	Mineral Voids Per Cent	Air Voids Per Cent	Marshall Stability, Pounds	Marshall Flow	Calculated D_5/D_1	D_{max}/D_{min}	Percent Packing Efficiency	$1-(1-\varphi_m)^n$
SX2	5	0	2.639	4.090	-9.356	2711.006	13.158	206.63	253.3	81.17	0.98827
SA2	9	0.1346	2.466	11.186	-1.419	1790.476	10.408	120.50	175.1	68.26	0.99967
SA2	10	0.1346	2.444	12.096	-0.401	1672.375	10.055	117.60	175.1	66.60	0.99986
Grad 3	10	0	2.407	13.603	1.284	1476.967	9.471	112.8	175.1	63.86	0.99986
Grad 3 - G&L	10	0	2.392	14.400	2.100	1650.000	10.000	112.8	175.1	63.86	0.99986
SA2	9	0	2.374	14.953	2.794	1301.782	8.948	108.50	175.1	61.41	0.99967
SX1	9	0	2.373	15.027	2.877	1292.230	8.919	156.22	253.3	61.27	0.99967
Stone Matrix	9	0	2.370	15.115	2.975	1280.839	8.885	155.8	253.3	61.11	0.99967
SX1-M2	9	0	2.316	17.353	5.478	990.529	8.018	145.5	253.3	57.04	0.99967
SX1-M1	9	0	2.280	18.806	7.104	801.973	7.454	138.8	253.3	54.39	0.99967
Grad 13	10	0	2.254	19.896	8.322	660.640	7.032	92.8	175.1	52.41	0.99986
Grad 13 - G&L	10	0	2.240	20.500	9.000	570.000	7.000	92.8	175.1	52.41	0.99986
Superpave	11	0	2.186	22.684	11.440	298.975	5.952	158.8	333.3	47.34	0.99994
Bailey	10	0	2.099	26.233	15.410	-161.422	4.576	104.6	253.3	40.88	0.99986
Slope			0.0134	-0.5496	-0.6147	71.2950	0.2130				
Intercept			1.5516	48.7015	40.5400	-3076.1000	-4.1318				
R^2			0.8792	0.9045	0.9059	0.8297	0.4804				

The results in Table 13 clearly indicate that of the four standard gradations evaluate that Gradation 3 from the Goode and Lufsey study continues to be the gradation with the possibility of being able to achieve the highest maximum density. As anticipated, the Goode and Lufsey maximum density gradation did yield the maximum Percent Packing Efficiency followed in order by the Stone Matrix Gradation, the Superpave Gradation and finally the Bailey Gradation. Gradation SA2 shifted as generated in this study not only successfully duplicated Gradation 3 but it appears to have the probability of being able to show an improvement over Gradation 3 relative to maximum density and other properties as well.

Since Gradation SX1 and the Stone Matrix Gradation in Table 13 appear to be nearly identical relative to Percent Packing Efficiency, they have been projected to yield nearly the same properties. However, since SX1 was calculated as an optimum particle size distribution and the Stone Matrix gradation was not this may give some advantage to the SX1 gradation.

The Superpave Gradation was found to have a slightly improved Percent Packing Efficiency over the value generated for the Bailey Gradation. However, both of these standard gradations have been projected to provide properties that would not be superior to those of Gradation 3 and the Stone Matrix Gradation.

The modified SX1 Gradations SX1-M1 and SX1-M2 were deliberately designed to increase their void content to allow more asphalt matrix to be infused into their asphalt/Aggregate formulations. Since it has been found that the Percent Packing Efficiency is inversely proportional to the void content, then these modified SX1 Gradations have been projected to have a higher void content than Gradation 3 but a lower void content than the Superpave Gradation. Both modified Gradations SX1-M1 and SX1-M2 appear to have achieved their expected increase in voids. In addition, Gradations SX1-M1 and SX1-M2 also appear to have an increased capability to improve Marshall Stability over the Superpave and Bailey Gradations.

Finally, it is apparent that several new gradations have been identified in this study that could potentially offer significant property improvements over both Gradation 3 from the Goode and Lufsey study as well as the other current aggregate gradation standards for the asphalt industry.

Conclusions

This study extended a previous publication that introduced the concept of the Percent Packing Efficiency to optimize aggregates in asphalt to achieve an improved property balance. Based on previous publications the importance of initiating an aggregate formulation based on an optimum particle size distribution was found to be very important to achieve the full benefit of the Percent Packing Efficiency. In general, an optimum particle size distribution is characterized with volume fractions being generated as the square root of a particle diameter divided by the sum of the square roots of all the particle diameters in the distribution. An optimum distribution was also found to achieve an excellent duplication of the volume fractions that McGeary achieved when he maximized the packing fractions for his binary, tertiary and quaternary particle size distributions for application in a nuclear reactor. In addition, McGeary's maximum packing fractions were very close to the ultimate packing fractions that could be achieved for binary, tertiary and quaternary particle size distributions.

It has also been found that the center particles in the formulation of Goode and Lufsey's maximum density line was also found to be well characterized using an optimum particle size distribution. In addition, it has also been found that when an optimum particle size distribution is generated with a constant ratio, Z, between particle sizes that the accumulated volume fraction vs the square root of the particle diameters must necessarily yield a straight line. It was also found that when a value of Z is increased but maintained as a constant for an optimum particle size distribution then the Percent Packing Efficiency approaches an upper limit as the number of particles in the distribution are increased. It has also been found that it takes fewer particles to achieve the upper limit for Percent Packing Efficiency as the value of Z increases. The upper limit of Percent Packing Efficiency for each value of Z has ironically been found to be only a function of Z. In addition, there is both a theoretical and a practical limit for the value of Z. The theoretical limit of Z has been shown in previous publication to be between 5.92 and 7.67. McGeary also found experimentally that the upper limit of Z to be approximately 7. However, the practical limit of Z depends largely on the largest particle in the distribution which for aggregates in asphalt is typically a particle size of approximately 1inch. Therefore, the value of Z would be dependent primarily on the smallest particle size available.In addition, if the number of particles in an optimum particle size distribution is small then the ultimate packing fractions would be expected to be significantly reduced. In general, a value Z of 2 or 4 would appear to be the most practical for most aggregate asphalt applications.

While the maximum Percent Packing Efficiency can be quite low for particulate fillers in a coating application, the packing fraction can still continue to increase as long as the ratio of $\bar{D}_5/\bar{D}_1$ continues to increase independent of the value of the Percent Packing Efficiency and as long as Z >1.

This study also generated a critical evaluation of all 24 gradations from the original Goode and Lufsey study to extend the characteristics of the Percent Packing Efficiency. The Percent Packing Efficiency did successfully separate all 24 different sets of gradation mixtures generated by Goode and Lufsey into five clearly defined separate groups that each formed an approximate straight line for the relationship between the Percent Packing Efficiency and the Goode and Lufsey gradation number. Gradation 3 was identified by Goode and Lufsey as the standard for their recommended maximum density line since it had the best balance of properties. Gradation 3 also yielded a maximum value of Percent Packing Efficiency.

Even though Goode and Lufsey generated Gradation 3 through trial and error it was useful to attempt to duplicate Gradation 3 to be able to generate some concepts of why this gradation was successful to achieving a formulation with a successful balance of properties. Since the ratio of Z was approximately 2 for all but the final two particles in all of Goode and Lufsey's trial and error gradations, it would seem prudent to first approach a constant ratio by eliminating the ½ inch particle and calculate an optimum particle size distribution. When this distribution was plotted as a function of the accumulated volume fraction vs the square root of the particle diameter all of the points except the last one formed a straight line. The shifting of this straight line so that all particles fell on the same straight line involved adding a specific volume fraction to the smallest particle diameter and subtracting this same amount of volume fraction from the largest particle diameter. The volume fraction of all 10 particles for SA2 could now be calculated from the new accumulated volume fraction straight line. Both of the new duplicated aggregate gradations for SA2 with 9 or 10 particles were then found to have values of the Percent Packing Efficiency that were improvements over that of Gradation 3.

A comparison of the volume fractions for the duplicated gradation SA2 were then compared with the results for Gradation 3 and both sets of volume fractions were found to be nearly duplicated. Also, the middle particles for both sets of data were nearly duplicated indicating that the middle values for Gradation 3 appear to be essentially comparable to the calculated values for an optimum particle size distribution.

At this point it is useful to note that the volume fraction for all 24 of the Goode and Lufsey particle size distributions appeared to be similar to Gradation 3 in that they all had a significant drop in the volume fractions for the largest particles and many had added volume fractions to the smallest particle sizes. While the justification of this result is not apparent, Goode and Lufsey may have modeled the other gradations in their study from the successful development of Gradation 3.

The results generated for Gradation 3 were found to be in direct contrast to the diagram of three interpenetrating particles where the smaller particles were added to the spaces in between the larger particles. Three ways were then used to calculate the volume fractions for the three-particle blend; volume, area and optimum Particle Size distribution. In each of these volume fraction evaluations the largest particles always had the largest volume fractions and the volume fractions were all essentially proportional to their particle diameters. This result is in direct conflict with the results obtained for Gradation 3.

A property analysis of the Goode and Lufsey study data found that the minimums for the Mineral Voids and the Air Voids corresponded with the maximums for Bulk Specific Density as well as the maximums for the Percent Packing Efficiency. These results strongly indicate that the Percent Packing Efficiency is a significant indicator of the packing efficiency of the aggregate blends. Within the Goode and Lufsey study, four of the five properties measured appeared to form effective linear relationships with each other. However, the Marshall Flow property did not appear to yield a consistent relationship with the other measured properties within the Goode and Lufsey study.

At this point two optimum particle gradations were generated using a constant ratio between particles that was again found to be characterized with volume fractions being generated as the square root of the particle diameter divided by the sum of the square roots of all the particle diameters in the distribution. The constant ratio Gradations SX1 (Z = 2) with 9 particles and constant ratio Gradation SX2 (Z = 4) with 5 particles were both series with optimum particle distributions that had the same initial particle and the same maximum 0.75 inch particle diameter. The upper two particles for Gradation 3 were replaced with the 0.75 particle size to allow the generation of Gradations SX1 and SX2. Since Gradation SX2 had a Percent Packing Efficiency that was greater than that for Gradation SX1, then gradation SX2 was predicted to have higher bulk specific gravity and lower void content than Gradation SX1.

A comparison of the volume fractions of Gradation 3 and Gradation SX1 appears to indicate that they are part of the same straight line. This is particularly true of the middle part of the volume fractions of Gradation 3 with those of Gradation SX1. This again indicated that the middle particles of Gradation 3 were also calculated as part of an optimum particle size distribution.

A significant difference was identified between the Percent Packing Efficiency for the four standard gradations evaluated in this study. As anticipated, the Goode and Lufsey maximum density gradation did yield the maximum Percent Packing Efficiency followed in order by the Stone Matrix Gradation, the Superpave Gradation and finally the Bailey Gradation. As a result of the much lower Percent Packing Efficiency calculated in this study for the Bailey Gradation Standard, the Bailey standard would be expected to have a lower bulk specific gravity and a higher void content than the other gradation standards. As a result of a higher void content the Bailey Standard would be expected to be able to accommodate a larger asphalt matrix content which could be very desirable for some specific applications.

Since a certain amount of void content can be desirable in some instances, Gradation 13 from the Goode and Lufsey study was used to modify Gradation SX1 to obtain modifications, SX1-M1 and SX1-M2. Both modified Gradations SX1-M1 and SX1-M2 appear to have achieved their expected increase in voids. In addition, Gradations SX1-M1 and SX1-M2 also appear to have an increased capability to improve Marshall Stability over the Superpave and Bailey Gradations.

The five properties measured by Goode and Lufsey for Gradation 7-21 have been plotted vs the values of their Percent Packing Efficiency and these relationships have been used to predict the properties of all 24 of the Gradations generated by Goode and Lufsey as well as all of the gradations generated in this study. Note that both Mineral Voids and Air Voids decreased with and increase in the Percent Packing Efficiency. Conversely both the Bulk Gravity and the Marshall Stability increased with an increase in the Percent Packing Efficiency.

Finally, it is apparent that several new gradations that have been identified in this study that could potentially offer significant property improvements over both Gradation 3 from the Goode and Lufsey study as well the other current aggregate gradation standards for the asphalt industry.

Conflicts of Interest

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