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Comparative Study of Tidal Evolution of Mars-Phobos-Deimos Based on Kinematic Model and Based on Seismic Model

Bijay Kumar Sharma

Indian Institute of Technology Patna, India

ABSTRACT

Phobos, a moon of Mars, is in sub-synchronous orbit and hence it is in-spiraling towards Mars to its complete destruction on a gravitational runaway course. On the other hand Deimos, the second moon of Mars is in extra-synchronous orbit and almost stay put in the present orbit. My results predict that Phobos is losing its altitude at a rate of 21cm/yr and is likely to crash with Mars in 10My whereas recent Mars Express (Burns 1978, Witasse et.al 2013) results show that the altitude loss is at 1.8cm/yr and the doomsday will occur in 100My. Bills et.al.(2005) and Ramslay & Head III(2013) have reported altitude loss rate at 4cm/yr and remaining life-time for Phobos as 30-50My. The author had proposed a planetary-satellite dynamics based on detailed study of Earth-Moon [personal communication: http://arXiv.org/abs/0805.0100] which he calls the Kinematic Model. Based on this Kinematic Model, 1.8cm/yr and 4cm/y approach velocity leads to the age of Phobos to be 53 Gyrs and 24 Gyrs which is physically untenable since our Solar System's age is 4.567Gyrs. Assuming that Phobos is co-accreted body along with Mars or formed from impact generated debris, the age of Mars-Phobos-Deimos system should be 4.5Gy. Within this constraint, the present altitude loss of Phobos is 21 cm/y and the dooms day of Phobos is predicted to be much shorter at 10My. Deimos is also assumed to be a co-accreted body with an age of 4.5Gyrs and launched in super-synchronous orbit hence it is on an expanding spiral path but its insignificant mass ratio with respect to Mars makes it almost stay put in its present orbit and it has negligible tidal evolution history. Considering that Phobos is trapped in a gravitational runaway death spiral the rapid decay of Phobos orbit at 21cm/y and its early doom seems to be reasonable but we do not have a conclusive proof .The conclusive validation of Kinematic Model will be made by Phobos Laser Ranging(PLR) Mission to be set up at Phobos. Though one thing is very clear that results obtained by Bills et.al.(2005) and Ramslay & Head III(2013) depend on several elastoviscous properties of the planetary bodies which are hard to obtain for the variety of Planetary Systems being encountered in this era of Exo-Planet hunting whereas the Kinematic Model analysis depends only on the age of the secondary component which is readily available. Hence this study definitely establishes that the tidal evolutionary history of Planet-Satellite systems can be easily arrived at through Kinematic Model. Regarding its accuracy, we have to wait for the results from PLR Mission.

***Corresponding author**
Bijay Kumar Sharma, Indian Institute of Technology, India

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In this paper I have utilized Planetary Satellite Dynamics as developed in my personal communication http://arXiv.org/abs/0805.0100 and in my two papers to calculate the rate of altitude loss of Phobos which is in sub-synchronous orbit. Section 1 gives the work done on Phobos till date. Section 2 gives the recent rethinking about the birth of Phobos and Deimos and hence its probable age. Section 3 gives Seismic Model for Tidal Evolutionary Analysis. Section 4 gives the planetary satellite dynamics as developed through the rigorous analysis of Earth-Moon System and is referred to as Kinematic Model in contrast to the Seismic Model or Elastoviscous Model which is generally used for tidal evolutionary studies. Sub-sections of Section 4 do the tidal evolutionary study of Mars-Phobos-Deimos based on Kinematic Model. Section 5 gives the discussion and Section 6 gives the conclusions.

Section 1: A brief history of Mars-Phobos-Deimos. (Sangdeev & Zakharov 1989)

The observed present-day orbital decay of Phobos suggests that Phobos once orbited just below the synchronous altitude of Mars early in the geological history of Phobos, though the rate of decay is not well-understood, and estimates of the orbital history of Phobos have only recently been made based on Seismic/Elastoviscous Models of Mars-Phobos system.

Table 1: History of the studies of Mars and its moons

Year	Person or Spacecraft	Work done.
1659	Christian Hugens	Drew the first sketch of the dark and bright side
1780	William Herschel	Noted thin Martian Atmosphere
1877	Giovanni Schiapaprelli	Drew first detailed map of Martian surface.
1900	Percival Lowell	Used Lowell Telescope to make drawing of the canals on Martian Surface.
1965	Mariner 4	Beamed back 20 photos from first flyby of Mars.
1971	Mariner 9	Sent back 7300 images from first ever orbital mission. An interlocking grid covered Video Frame 4209-75 was one of the images.
1976	Viking 1 & 2	First probes to land on Martian Surface and photograph the terrain.
July 7,1988	Phobos 1	It failed enroute. On September 2, 1988, it lost its lock on Sun due to software glitch and hence it lost its power source.
July 12, 1988	Phobos 2	It became Mars Orbiter on January 29,1989, and sent 38 images with a resolution of 40m. It has gathered data on Sun, Interplanetary Medium, Mars & Phobos. A base station and a Mars rover was to be released but the all contact was lost on March 29,1989. One of the images are similar to Frame 4209-75 sent by Mariner 9.
1998	Mars Global Surveyor	It is mapping the whole surface of Mars
2002	Mars Odyssey	It took night time I.R. pictures of Martian Crater called Hyataspis Chaos.
2003	European Space Agency Mars Express	(1) It has revealed the volcanic past of Mars; (2) Icy Promethei Planum , the icy south pole of Mars , has been photographed; (3) In 2008 Atmosphere stripping on Mars and Venus are being simultaneously studied by Mars Express and Venus Express.
Jan 4, 2004	Mars Exploration RoverA	It did extensive geological analysis of Martian Rocks and Planetary Surface Features.
Jan 25, 2004	Mars Exploration RoverB called OPPORTUNITY	It landed on the opposite side. Active from Jan 25,2004 to March 22,2010 it remained active.
August 12, 2005	Mars Reconnaissance Orbiter launched by Jet Propulsion Lab,USA.	Launched by JPL and it monitors daily weather And surface conditions on Mars
27 May,2008	Phoenix launched by NASA	Soft Landed on North Pole of Mars in search of extraterrestrial life
Nov 26,2011	Mars Science Laboratory CURIOSITY launched. Landed on Mars 6.8.2012	Curosity established that Martian environment was favourable to Microbial life in the past.
September 22, 2014.	In Gale Crater MAVEN(Mars Atmos- phere and Volatile Evolution Mission) Launched by NASA	It has to keep communicating with Mars Rovers Namely "Opportunity" and "Cutiosity".
24th Sept.,2014	Mars Orbit Spacecraft (Mangalyan) Launched by Indian Space Research Organiz. (ISRO)	Captures first images of Mars. Mangalyan is in Elliptical orbit, 150 degree inclined to the equatorial plane of Mars. The nearest point (Periopsis) is 421.7Km and farthest point on the Elliptical orbit(Apopsis) is 76,993.6Km.

1. Spirit Team, " Special Issue-SPIRIT at Gusev Crater", Science, 305 (5685), 737-900, August 6, 2004.
2. Webster, Guy; Brown, Dwayane; "NASA's Mars Curiosity Rover Marks First Martian Year", NASA retrieved June 23,2014.

Section 2. The probable origin and ages of Phobos and Deimos
Phobos and Deimos are the two moons of Mars. They were discovered by Asaph Hall in 1877. The history of the studies of Mars and its moons are given in Table 1. Grey coloured Phobos and Deimos are quite unlike ruddy, pink-skied planet Mars. The two natural satellites are pitted and like drought-state potato. Their surfaces are seared by meteorites and raked by solar wind. They have much lighter density and are probably formed of carbonaceous chronditic material found in outer part of the asteroid belt. The central force of these lilliputian natural satellites are weak hence the constituent materials have not undergone compaction. These natural satellites have escaped the deeper trauma of heating and inner shifting that have occurred in the formation of Planets.

Researchers have made a wide range of assumptions regarding models of dissipation by anelastic tidal deformation within Mars and satellites to test the Capture Hypothesis. Szeto has proposed that Capture would have led to collision but no collision seems to have occurred in last 1.5Gy. Also Capture could not have resulted into near circular orbit of Deimos though it could have led to gravitationally runaway orbit of Phobos. Hence by general consensus of the older researchers, the capture origin is discarded.

By the study of Mars impact ejecta in the regolith of Phobos it has been concluded that the bulk concentration of Mars-like material in the regolith of Phobos greatly exceeds the upper predicted range of 1250 ppm for Mars ejecta in the regolith of Phobos. This indicates an interior of Phobos that has a mineralogy similar to that of Mars. This may provide strong evidence that Phobos originated either from a primordial impact on Mars or coaccreted with Mars.

Because of these new researches I assume the age of Phobos and Deimos to be 4.5 Gy. **Section 3.Seismic/Elastoviscous Model of Mars-Phobos**.

From George Howard Darwin's time it is recognized that planets raise body tides in their natural satellites and natural satellites raise body tides in their host planets. It is also recognized that planets and satellites are anelastic bodies (elastoviscous bodies). Hence tidal deformation (tidal stretching and squeezing) leads to dissipation of energy called tidal dissipation. By assuming different Love Numbers(kj) and different Q parameter, different rate of tidal dissipation can be incorporated in the tidal interaction. Tidal interaction inevitably leads to tidal drag (or secular deceleration) or spin down of the primary component if the satellite is above synchronous orbit or tidal secular acceleration or spin-up of the primary component if the satellite is in sub-synchronous orbit and zero tidal interaction if the two bodies are tidally interlocked. When the primary and secondary are tidally interlocked, the lag angle in case of sub-synchronous orbits and lead angle in case of super-synchronous orbits become zero. Here lag/lead angle refer to the angular

separation between the radius vector, joining the planet and its moon, and planet's tidal bulge. Meaningby in perfect tidal lock-in position, the long axis of the tidal bulge of primary and secondary components are exactly aligned and both the components orbit the barycenter as one single body. The tidal stretching and squeezing completely stops and hence tidal dissipation is zero. This perfect lockin occurs when the two components are synchronized , the orbit of each component around the barycenter are circularized and the orbital planes of the two components are co-planer. This observation in reference to stellar binaries had been made by Zahn(1992) in 1975:

"Eventually the Binary may settle in its state of minimum kinetic energy, in which the orbit is circular , rotation of both stars is synchronized with the orbital motion and the spin axis are perpendicular to the orbital plane. Whether the system actually reaches this state is determined by the strength of tidal interaction, thus by the separation of the two components, equivalently the orbital period. But it also depends on the efficiency of the physical process which are responsible for the dissipation of the kinetic energy."

Mars-Phobos is the example of sub-synchronous Satellite where Mars-Phobos Radius vector leads the tidal bulge in Mars , Phobos spins-up Mars and Phobos approaches Mars because of transfer of angular momentum and orbital energy from Phobos to Mars. Earth-Moon is the example of super-synchronous Satellite where Earth-Moon Radius vector lags the tidal bulge in Earth, Moon is spinning down Earth4 and Moon is receding at 3.8cm/y presently. Here angular momentum is being transferred from the Earth to our Moon. Pluto-Charon is the example of tidally interlocked orbital configuration where the tidal bulge of both the components are aligned and both the components are orbiting the barycenter as one body in a perfect circle.

X. Shi, K. Willner and J. Oberst (2013) give the following tidal evolution equation:

$$a_t = \left[a_0^{\frac{13}{2}} - \frac{13}{2} \times \frac{3k_2}{Q} \sqrt{\frac{G}{M}} \times mR^5 \Delta t \right]^{\frac{2}{13}} \qquad (1)$$

$where\ k_2 = Love\ Number\ of\ Mars, \quad Q\ is\ the\ quality\ factor,$

$M\ and\ R\ are\ the\ mass\ and\ radius\ of\ of\ Mars,$

$m = mass\ of\ Phobos, a_0 = current\ semi-major\ axis.$

This equation gives the orbital radius = a_t at a time of Δt seconds ago.

In Equation (1), Love Number and Quality Factor depend upon density, rigidity, viscosity and rate of periodic forcing. These parameters are known with large uncertainties for different Planets and their Satellites and hence their Tidal Evolutionary History will be arrived at with equal uncertainty in Seismic Model based analysis.

It is estimated that Phobos from the present orbit of 9830 km from the center of Mars will spirally collapse to an orbital radius of 3397km (the martian surface) in about 100My. Altitudinal loss rate is estimated as 1.8cm/y. and Ramsley and Head III (2013) have estimated Altitudinal loss rate as 4cm/y and the future date of catastrophic collision between Mars and Phobos as 30 to 50My.

Section 4. The Kinematic Model of Mars-Phobos-Deimos for tidal evolutionary history.

In Kinematic Model, any binary system has two triple synchrony orbits which I refer to as inner and outer Clarke's Orbits and in Earth-Moon system they are referred to as inner geo-synchronous orbit(a_{G1}) and outer geo-synchronous orbit (a_{G2}). Triple synchrony orbit is defined as:

$$\omega(spin\ angular\ velocity\ of\ the\ primary) = \Omega(orbital\ angular\ velocity) = \Omega'(spin\ angular\ velocity\ of\ the\ secondary \qquad (2)$$

From the rigorous analysis of Earth-Moon System in my personal communication as cited above the following scenario has emerged:

Secondary tidally evolves out of inner Triple Synchrony State which is called Inner Clarke's Orbit (a_{G1}). If it tumbles short of a_{G1}, secondary rapidly spirals-in to its certain destruction and if it tumbles long of a_{G1} then through Gravitational Sling Shot secondary is launched on an outward spiral path by a powerful Impulsive Torque. But as the differential between orbital velocity and spin velocity of primary grows, tidal stretching and squeezing sets in the primary body which leads to tidal dissipation which causes a rapid exponential decay of the impulsive torque. Tidal dissipation causes primary's tidal bulge to lead the radius vector joining primary and secondary. This 'lead angle' causes secular deceleration of the primary and angular momentum transfer from primary to secondary for angular momentum conservation. From then onward the secondary coasts on its own until it locks into the outer Triple Synchrony State called Outer Clarke's Orbit (a_{G2}). But through out this tidal evolutionary history the Total Angular Momentum is conserved hence we have the following Conservation of Momentum equation:

$$J_T = C\omega + (m^* a_{present}^2 + I)\Omega = [C + (m^* a_{G1}^2 + I)]\Omega_{aG1} = [C + (m^* a_{G2}^2 + I)]\Omega_{aG2} \qquad (3)$$

In (3):

C = Moment of Inertia of the Primary around its spin axis.
I= Moment of Inertia of the Secondary around its spin axis.
And m*= reduced mass of the secondary = m/(1+m/M) where m = the mass of the
secondary and M= mass of the primary.

From Kepler's Third Law:

$$\Omega_{aG1} = \frac{B}{a_{G1}^{3/2}} \quad and \quad \Omega_{aG2} = \frac{B}{a_{G2}^{3/2}} \quad where\ B = \sqrt{G(M+m)} \tag{4}$$

Substituting (4) in (3) we get:

$$J_T = C\omega + \left(m^* a_{present}^2 + I\right)\Omega = [C + (m^* a_{G1}^2 + I)]\frac{B}{a_{G1}^{3/2}}$$
$$= [C + (m^* a_{G2}^2 + I)]\frac{B}{a_{G2}^{3/2}} \tag{5}$$

Solving (5) we get the two roots of the Binary System namely a_{G1} and a_{G2}. In classical Newtonian Mechanics two triple synchrony orbits donot exist. Hence I call this Post-Newtonian Kinematic Model.

From Classical Mechanics the Synchronous Orbit is the same as the Inner Clarke's Orbit calculated in Kinematic Framework. In Classical Mechanics, the synchronous orbit is defined as:

$$a_{sync}^{3/2}\Omega_{orb} = a_{sync}^{3/2}\omega_{primary} = B \tag{6}$$

In Classical Mechanics there is no outer Clarke's Orbit. For vanishingly small values of 'q' where q = m/M , the outer Clarke's Orbits are too large to be perceptible but in Earth-Moon system or in Pluto-Charon system where mass ratios are 1/81 and 1/8 respectively, the outer Clarke's Orbit are finite and perceptible as can be seen in the Table2.

Table 2: Comparative Study of Triple Synchrony Orbits of Earth-Moon, Mars-Phobos-Deimos , Pluto-Charon Systems,Sun-Jupiter and two stellar binaries (NN-Serpentis and RW Lac) from Classical Newtonian Mechanics and Kinematic Model.[The Globe-Orbit Parameters based on which the calculations have been made are given in Appendix A in SOM]

Planet-Sat	Massratio (q)	a (present) (m)	B (m3/2/s)	a_{G1} (m)	a_{G2} (m)	Ω=ω (radians/s)	async (m) from (E)
Earth-Moon	1/81	3.84400×10^8	2.00873×10^7	1.46×10^7	5.53×10^8	7.2722×10^{-5}	4.23362×10^7
Mars-Phobos	10^{-8}	9.378×10^6	6.54×10^6	2.04×10^7	7.46×10^{18}	7.08824×10^{-5}	2.04×10^7
Mars-Deimos	10^{-9}	23.459×10^6	6.54×10^6	2.04×10^7	1.69×10^{20}	7.08824×10^{-5}	2.04×10^7
Pluto-Charon	1/8	19.600×10^6	9.88×10^5	1.37672×10^6	1.95579×10^7	1.13859×10^{-5}	1.96133×10^7
Sun-Jupiter	9.55×10^{-4}	19600	1.15256×10^{10}	1.06889×10^9	7.92465×10^{11}	2.86533×10^{-6}	2.53×10^{10}
NNSerpent is	0.2074	6.49597×10^8	9.25989×10^9	4.44958×10^7	6.4986×10^8	5.594×10^{-4}	6.49514×10^8
RWLac	0.9375	1.69267×10^{10}	1.54426×10^{10}	4.08908×10^8	1.69314×10^{10}	7.01327×10^{-6}	1.69252×10^{10}

In Paper No. B0.3-0011-12 presented at 39th COSPAR Scientific Assembly, Mysore, India from 14th July to 20th July 2012, the correspondence between Newtonian Formalism of Synchronous Orbit and Kinematic Formalism was found as shown in Figure 1.

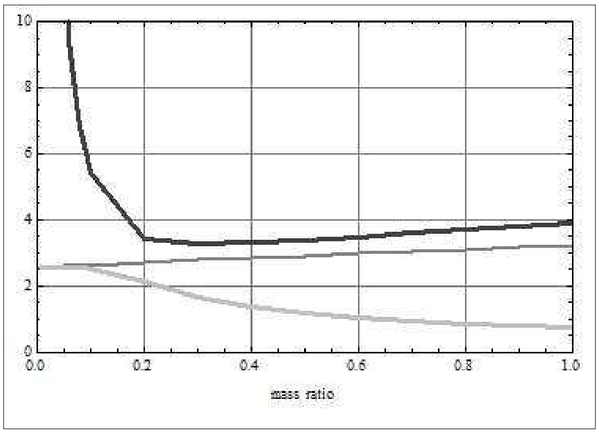

Figure 1: Plot of a_{synSS} ($\times R_{lap}$)[thin gray], a_{G1} ($\times R_{lap}$)[thick gray] and a_{G2} ($\times R_{lap}$)[thick black] as a function of 'q'.

Inspection of Figure 1, tells us that at infinitesimal values of 'q' a_{synSS} is the same as a_{G1} and only one Clarke's Orbit is perceptible. But at larger mass ratios the two (classical and kinematic formalism for a_{G1}) rapidly diverge. My analysis till now has confirmed that a_{G1} is the correct formalism for predicting the inner triple synchrony orbit in a binary system at q < 0.2..

At mass ratios greater than 0.2, a_{G1} is physically untenable and only a_{G2} is perceptible. Outer Triple Synchrony Orbit seems to converge but does not actually converge to the classical formalism but remains offsetted right till the limit of q =1. Here again only outer Clarke's Orbit is perceptible but the actual Star pairs satisfy the Kinematic formalism and not the classical formalism.

So Kinematic Formalism, though satisfies the correspondence principle at q ~ 0, is a theory in its own right. Till date there exists no formalism for two triple synchrony orbits in Classical Newtonian Mechanics in the mass ratio range 0.0001 to 0.2.

For mass ratio less than 0.0001, binaries remain in inner Clarke's Configuration stably which is predicted by Classical Newtonian Formalism also.

At mass ratios greater than 0.2 right up to unity, star pairs remain in outer Clarke's Configuration stably and its magnitude is more than Newtonian prediction.

For mass ratios 0.0001 < q < 0.2, Outer Clarkes configuration is the only stable orbit and secondary is catapulted from aa_{G1} by Gravitational Sling Shot mechanism and it migrates out of that configuration. If it is at $a > a_{G1}$ the pair spirals out with a time constant of evolution and if $a < a_{G1}$ then the pair spirals-in on a collision course again with a characteristic time constant of evolution.

Time Constant of Evolution is in inverse proportion of some power of mass ratio.

For q = 0.0001, it is Gy and as q increases , time-constant decreases from Gy to My to kY to years. This is valid for mass scale encountered in Solar and Exo-Solar Systems. Between 0.2 to 1, a solar nebula falls into outer Clarke's Configuration by hydro-dynamic instability within months/years.

For q being vanishingly small, the calculation of the man-made Geosynchronous Satellite's orbit of 36,000Km above the equator has been done by Kinematic Formalism. This calculation has been done by me in my personal communication: http://arXiv.org/abs/0805.0100

In Table 2, all cases are consistent with Kinematic Formalism except Pluto-Charon (case no.4). This exception is due to large uncertainty in the Globe-Orbit parameters of Pluto-Charon.

Case 1: Moon is a significant fraction of Earth (1/81) hence our Moon has a definite Tidal Evolution History. It started its journey about 4.5Gya just beyond Roche's Limit 15,000Km. By gravitational sling shot it was launched on an expanding spiral orbit from inner geo-synchronous orbit of 15,000Km orbital radius towards the outer geo-synchronous orbit of 5.53×10^8 m = 553,000Km. At the inner geo-synchronous orbit, the length of day = length of month = 5 hours and at the outer geo-synchronous orbit, the length of day = length of month = 47 days. Presently the lunar orbital radius is 384,400Km with sidereal length of day = 23.9344 hours and length of Sidereal Month = 27.32 Earth days. Earth-Moon started from geo-synchrony and will end in geo-synchrony. As predicted in Figure 1, for mass ratio = 1/81 the classical synchronous orbit is less than the outer geosynchronous orbit.

Case 2 and 3: In case of Mars-Phobos-Deimos, since the mass ratio is insignificant hence Deimos launched on an orbit long of inner Clarke's Orbit has hardly evolved from its point of inception which is inner Clarke's Orbit. But Phobos is launched on an orbit short of inner Clarke's orbit hence it is on a gravitational runaway orbit, trapped in a death spiral. Deimos is stay-put in its orbit of inception which is 20,400Km but Phobos has lost altitude from its point of inception of 20,400Km to the present altitude of 9,378Km. Since the mass ratio is insignificant hence the classical synchronous orbit is the same for both Phobos and Deimos equal to 20,400Km same as the inner Clarke's Orbit. This is in exact correspondence with Figure 1.

Case 4.Pluto-Charon's classical synchronous orbit should be smaller than Outer Clarke's Orbit as required by Kinematic Analysis but the former is 0.28% larger.

This is due to the uncertainty in Globe-Orbit parameters of Pluto-Charon. Case 5.Mass ratio of Jupiter to Sun is 10^{-3} hence according to Primary-centric analysis Jupiter-Sun has a tidal evolutionary history with a rapid Time-constant of evolution of 4.275My. It has evolved from inner Clarke's Orbit 3.7859×10^9m to the present orbit of 778.3×10^9m where its evolution factor is 0.893 and eventually it will lock into second triple-synchrony state in the outer Clarke's Orbit of 871.161×10^9m. The classical synchronous orbit is at 25.3×10^9m, 97% smaller than outer Clarke's Orbit, as predicted by Figure 1 also.

Case 6 and Case 7: These are stellar non-relativistic binaries. I call them nonrelativistic because the mean apisidal motion is negligible. Here since the mass ratio is greater than 0.2, hence the original molecular cloud settles into a binary in Months-Years and gets locked-into outer Clarke's Orbit. In both cases the synchronous orbit is shorter than the Outer Clarke's Orbit by 0.05% and 0.04% respectively. This is consistent with Kinematic Analysis.

4.1. Kinematic Analysis of Mars-Phobos-Deimos.

Table 3: Globe and Orbit Parameters of Mars-Phobos-Deimos

Parameters	Mars	Phobos	Deimos	Source
Mass(Kg)	0.64174×10^{24}	10.7046×10^{15}	2.24888×10^{15}	Ref 1,2
GM(Km^3/s^2)	0.042828382×10^{6}	$(7.14\pm0.19)\times10^{-4}$	$(1.5\pm0.11)\times10^{-4}$	Ref 2
Volumetric Mean Radius Or Median Radius ($\times10^3$m)	3389.5	11.2	6.1	Ref.1
Flattening	0.00589	irregular	irregular	Ref 1
Mean Density(Kg/m^3)	3933	1900	1750	Ref 1
Moment of Inertia(I/(MR^2))	0.366	0.4	0.4	Ref 1
Sidereal Spin period	24.6229h	0.31891d	1.26244d	Ref 1
Sidereal Orbital period(d)	-	0.31891d	1.26244d	Ref 1
a*(semi-major axis)($\times10^6$m)	-	9.378	23.459	Ref 1
Orbital eccentricity	-	0.0151	0.0005	Ref 1
Orbital inclination w.r.t. The equatorial plane of Mars(deg)	-	1.08	1.79	Ref 1

*Mean Orbital Distance from the center of Mars.

Reference 1. http://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html

Reference 2. Bills ,Bruce G.; Neumann,Gregory A.; Smith,David E. and Zuber, Maria T. "Improved estimate of tidal dissipation within Mars from MOLA observations of the shadow of Phobos", JOURNAL OF GEOPHYSICAL RESEARCH, 110, E07004, doi:10.1029/2004JE002376, 2005

Inspection of the Table clearly establishes that Phobos and Deimos are tidally locked with Mars. They present the same face to Mars all the time.The two satellites are moving in nearly circular orbits and are in nearly coplanar orbital plane. The orbital plane of the natural satellites are coplanar with the equatorial plane of Mars.

Table 4: Derived Kinematic Parameters needed in Kinematic Model

Parameters	Mars	Phobos	Deimos	Source
Moment of Inertia around the spin axis(Kgm^2)	C= 2.69843×10^{36}	I1= 5.37114×10^{23}	I2= 3.34723×10^{22}	Calculated
Reduced Mass m*=m/(1+m/M)_($\times10^{15}$Kg)		10.704599821	2.24887999212	calculated
Θ1 (I/C)_($\times10^{-14}$)		19.9047	1.24044	calculated
Θ2 ꞌ(m*/C)_($\times10^{-22}$)		39.6697	8.33403	calculated
B=√[G(M+m)]_($\times10^6$)m 3/2/s		6.54248	6.54248	calculated
Present Spin Angular Velocity of Mars(radians/s)	7.08824×10^{-5}			
Present Orbital/Spin Angular Velocity of Phobos(radians/s)	2.28033×10^{-4}			
Present Orbital/Spin Angular Velocity of Deimos(radians/s)	5.76044×10^{-5}			
JT(total ang.momentum) ($\times10^{32}$Kg-m^2/s)		1.912715482	1.9127140479	Calculated from (3)
Inner Clarke's Orbit(m) a_{G1}		2.04238×10^{7}	2.04238×10^{7}	From (5)
Outer Clarke's Orbit(m) a_{G2}	-	7.4589×10^{18}	1.68998×10^{20}	From (5)

Inspection of Table 4 tells us that both Phobos and Deimos have the same Inner Clarke's Orbit that is they both have originated from the nearly the same orbital region but have taken very different evolutionary paths:

Phobos by some perturbation, solar wind-cosmic particle perturbation or photoradiation pressure perturbation, tumbled short of aG1 falling into a subsynchronous orbit where it got trapped in a contracting spiral which is called death spiral. In this death spiral it is launched on a gravitational runaway trajectory and because of this runaway condition it should be rapidly losing its altitude though its Time Constant of Evolution is inordinately large for Phobos ($\tau=10^{18}$y) (Sharma et.al. 2004). Because of this runaway orbital collapse it is doomed for an early collision with Mars or for early pulverization even before it collapses. Even the present orbit of 9378 Km is within Roche's Limit. Our analysis says that Roche's Zone lies within 8000km to 14,000km but Phobos is intact. Hence the question of Phobos being pulverized by primary tides does not arise. This is because Phobos is a captured asteroid with high tensile strength
though it lacks compaction hence primary tides cannot pulverize it.

On the other hand Deimos by some perturbation mechanism tumbled long of aG1 and got launched on an expanding spiral orbit by Gravitational Sling Shot but with an inordinately long Time Constant of Evolution ($\tau= 10^{20}$y) (Sharma et.al.2004) . Hence at the present orbital radius of 23,459Km, Deimos is practically *stay-put* in inner Clarke's Orbit region of 20,423.8Km.

4.2 Stability Analysis based on Energy Budget of Phobos and Deimos in its tidally evolving path.

In this section we will study the energy profile of Phobos and Deimos during its tidally evolving trajectories.

From (3):

$$J_T = C\omega + (m^* a_{present}^2 + I)\Omega$$

At Triple-synchrony where ω=Ω at a_{G1} and at a_{G2} we get the following relations:

$$J_T = C\omega + (m^* a_{present}^2 + I)\Omega = [C + (m^* a_{present}^2 + I)]\Omega = [1 + (\theta_2' \times a_{G1}^2 + \theta_1)]\frac{CB}{a_{G1}^{3/2}} \quad 7$$

$$In\ Eq. 7, \quad \theta_1 = \frac{I}{C} \ and\ \theta_2' = \frac{m^*}{C};$$

Solution of (7) gives the two Triple Synchrony Orbits defined as Clarke's Orbits:

Inner Clarke's Orbit = a_{G1} and Outer Clarke's Orbit = a_{G2} Rewriting (3) we get:

$$\frac{J_T}{C\Omega} = \left[\frac{\omega}{\Omega} + \left(\left(\frac{m^*}{C}\right) a_{present}^2 + \frac{I}{C}\right)\right] = \left[\frac{\omega}{\Omega} + \theta_2' a_{present}^2 + \theta_1\right] \quad (8)$$

Substituting (6) in (8) we get :

$$\left(\frac{J_T}{CB}\right) a^{\frac{3}{2}} = \left[\frac{\omega}{\Omega} + \theta_2' a_{present}^2 + \theta_1\right] \quad (9)$$

Rearranging the terms of (9) we get:

$$\frac{\omega}{\Omega} = \left(\frac{J_T}{CB}\right) a^{\frac{3}{2}} - (\theta_2' a^2 + \theta_1) = E a^{\frac{3}{2}} - F a^2 \quad (10)$$

$$where\ E = \frac{J_T}{BC}\ and\ F = \left(\theta_2' + \frac{\theta_1}{a^2}\right)$$

At a_{G2} ,

$$\omega = \Omega = \frac{B}{a_{G2}^{3/2}} \quad (11)$$

In (11) we could as well have taken a_{G1} in place of a_{G2}.

Substituting (11) in (10) we get:

$$\frac{\omega}{\Omega} = 1 = \left(\frac{J_T}{CB}\right) a_{G2}^{3/2} - (\theta_2' a_{G2}^2 + \theta_1) \quad (12)$$

Rearranging (12) we get:

$$E = \frac{J_T}{CB} = [1 + (\theta_2' a_{G2}^2 + \theta_1)]\frac{1}{a_{G2}^{\frac{3}{2}}} = [1 + (\theta_2 + \theta_1)]\frac{1}{a_{G2}^{\frac{3}{2}}}\ where\ \theta_2' a_{G2}^2 = \theta_2 \quad (13)$$

Therefore:

$$E = [1 + (\theta_2 + \theta_1)]\frac{1}{a_{G2}^{\frac{3}{2}}} = \frac{k_1}{a_{G2}^{\frac{3}{2}}}\ where\ k_1 = [1 + (\theta_2 + \theta_1)] \quad (14)$$

Now (10) can be rewritten as:

$$\frac{\omega}{\Omega} = \left(\frac{J_T}{CB}\right) a^{\frac{3}{2}} - (\theta_2' a_{present}^2 + \theta_1) = E a^{\frac{3}{2}} - F a^2 = k_1 x^{3/2} - \theta_2 x^2 - \theta_1 \quad where\ x = \frac{a}{a_{G2}};$$

Therefore:

$$\frac{\omega}{\Omega} = k_1 x^{3/2} - \theta_2 x^2 - \theta_1 \quad (15)$$

Now Total Energy (TE) = Kinetic Energy (KE)+Potential Energy(PE)

$$KE = rotational\ KE\ of\ the\ Primary + rotational\ KE\ of\ the\ Secondary + Orbital\ KE = \frac{1}{2}C\omega^2 + \frac{1}{2}I\Omega^2 + \frac{1}{2}m^* a^2 \Omega^2 = \frac{C\Omega^2}{2}\left[\left(\frac{\omega}{\Omega}\right)^2 + \theta_1 + \theta_2' a^2\right] \quad (16)$$

Substituting (6) and (15) in (16) we get:

$$KE = \frac{CB^2}{2a^3}\left[(k_1 x^{\frac{3}{2}} - \theta_2 x^2 - \theta_1)^2 + \theta_1 + \theta_2' a^2\right] \quad (17)$$

Normalizing (17) with respect to aG2 we get:

$$KE = \frac{CB^2}{2{a_{G2}}^3} \times \frac{1}{x^3}\left[(k_1 x^{\frac{3}{2}} - \theta_2 x^2 - \theta_1)^2 + \theta_1 + \theta_2 x^2\right] \quad (18)$$

Let:

$$K = \frac{CB^2}{2{a_{G2}}^3} \quad and\ K_1 = \frac{GMm}{a_{G2}} \quad (19)$$

Substituting (19) and (18) in (16) we get:

$$TE = \frac{K}{x^3}\left[(k_1 x^{\frac{3}{2}} - \theta_2 x^2 - \theta_1)^2 + \theta_1 + \theta_2 x^2\right] - \frac{K_1}{x}\ where\ x\ normalized\ orbital\ radius \quad (20)$$

Differentiation and solving the Derivative = 0 will give the maxima Energy and minima Energy points.

The kinematic parameters of Phobos and Deimos is given in Table 5.

Table 5: Kinematic parameters needed for Stability Analysis

Parameters	Phobos	Deimos
JT(total ang. mom.)) ($\times 10^{32}$Kgm^2/s)	1.912715482	1.9127140479
C(moment of inertia of Mars) ($\times 10^{36}$Kg-m^2)	2.69843	2.69843
B($\sqrt{(G(M+m))}$) ($\times 10^6$m$^{3/2}$/s)	6.54248	6.54248
Θ_1(Dimensionless) ($\times 10^{-14}$)	19.9047	19.9047
Θ_2(Dimensionless) ($\times 10^{-7}$)	16.5475	3.47639
k_1(Dimensionless)	1.0000016547487678	1.0000003476390125
K=(CB^2)(2$a_{G2}{}^3$) ($\times 10^{27}$Joules)	6.77885	6.77885
K1=(GMm)/ a_{G2}($\times 10^{21}$Joules)	22.4346	4.71319
a_{G1}($\times 10^7$m)	2.04238	2.04238
a_{G2}($\times 10^{18}$ m)	7.4589	168.997
E($\times 10^{-11}$m$^{-3/2}$)	1.0834199115213353	1.0834190992037116
F($\times 10^{-22}$m^{-2})	39.6697	8.33403

Setting up Equation (20) we get the Total Energy of the Binary-System as a function of x where x is the normalized orbital radius and normalization is with respect to a_{G1} in case of Phobos and Deimos because inner Clarke's Orbits are perceptible and outer Clarke's Orbit are inordinately large.

Total Energy Function (20) is differentiated with respect to 'x' and equated to Zero. This gives the extremum points. We obtain 3 extremum points as tabulated in Table 6.

Table 6: The Energy Extremum Points of Mars-Phobos and Mars-Deimos

	Mars-Phobos	Mars - Deimos	Nature of Extremum	Comment
1st Extremum	0.00060072	0.000327178	Minima	Towards the center
2nd Extremum	1	1	Maxima	aG1
3rd Extremum	3.65205×10^{11}	8.27453×10^{12}	Minima	aG2

Section 4.2.1. Energy Profile around these three extremum points for Phobos

For Mars-Phobos between 0.0005 and 0.00065:

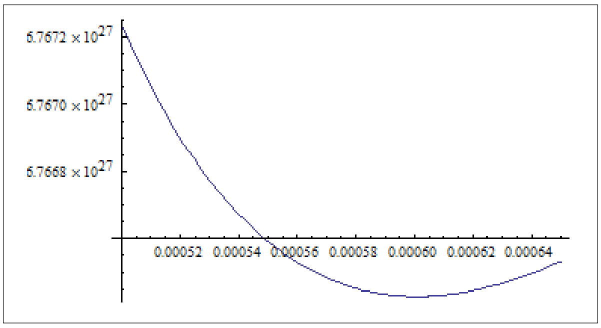

Figure 2: Energy Profile of Mars-Phobos between x = 0.0005 to 0.00065.

By inspection of Figure 2 we see that the first Energy minima occurs at x=0.00060072 this corresponds to 12.269Km from the center of Mars. Hence it is a stable point. An orbit of 12.269Km is physically untenable.

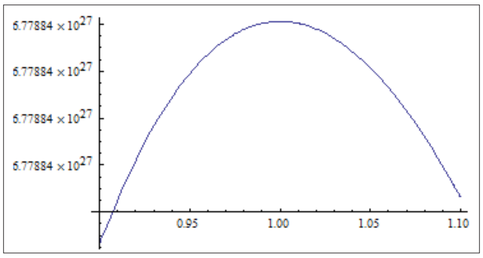

Figure 3: Energy Profile of Mars-Phobos between x = 0.9 to 1.1

By inspection of Figure 3 we see that the Energy Maxima occurs at x = 1 which corresponds to a_{G1}= 2.04238×10^{7}m. Hence inner Clarke's Orbit is an unstable point.

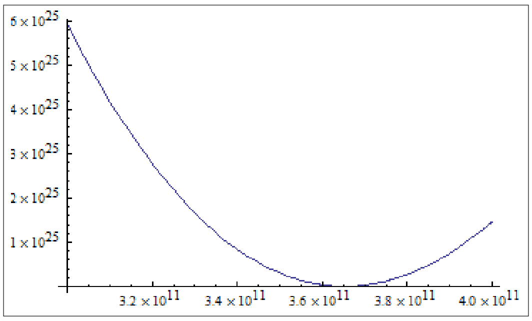

Figure 4: Energy Profile of Mars-Phobos between x = 3×10^{11} to 4×10^{11}.

By inspection of Figure 4 we see that Energy Minima occurs at x = 3.65205×10^11 which corresponds to a_{G2} = 7.4589×10^18m. Hence Outer Clarke's Orbit is a stable point.

Similar profiles are obtained for Mars-Deimos. This Energy Profile study clearly establishes that the secondary tumbles out of the Inner Clarke's Orbit at the slightest perturbation. If the secondary tumbles short of aG1= 2.04238×10^7m, it gets trapped in a death spiral and if it tumbles long of aG1= 2.04238×10^7m, it is launched on an outward expanding spiral path until it gets tidally locked into the Outer Clarke's Orbit. The time-constant of evolution is a strong function of 'q'=mass ratio. If q is vanishingly small, the time constant of evolution is practically infinite and the secondary hardly evolves out of its orbit of inception as is the case with our geo-stationary satellites. But as q exceeds 10^{-4}, time constant of evolution becomes perceptible. At solar system or exo-solar system mass scale time scale of tidal evolution is scaled down from Gy to My to Ky to Y until beyond q = 0.2 up to q =1 in months and days the secondary component settles into Outer Clarke's Orbit configuration where it tends to get tidally interlocked with the primary.

Section 4.2.2. Calculation of the spiral trajectory of Phobos and Deimos.

For the calculation of the spiral trajectory we need the radial velocity of recession in case of super-synchronous configuration and velocity of approach in case of sub-synchronous configuration. The radial integration of the reciprocal of radial velocity gives the non-Keplerian Transit time from its inception to the present orbit. This transit time should be equal to the age of the secondary. The starting point of this radial integral will be the tidal torque.

The Tidal Torque of Satellite on the Planet and of Planet on the Satellite = Rate of change of angular momentum hence

$$Tidal\ Torque = T = \frac{dJ_{orb}}{dt} \qquad (21)$$

But Orbital Angular Momentum:

$$J_{orb} = m^* a^2 \times \frac{B}{a^{3/2}} = m^* B\sqrt{a} \qquad (22)$$

Time Derivative of (22) is:

$$T = \frac{dJ_{orb}}{dt} = \frac{m^* B}{2\sqrt{a}} \times \frac{da}{dt} \qquad (23)$$

In super-synchronous orbit, the radius vector joining the satellite and the center of the planet is lagging planetary tidal bulge hence the satellite is retarding the planetary spin and the tidal torque is BRAKING TORQUE..

In sub-synchronous orbit, the radius vector joining the satellite and the center of the planet is leading planetary tidal bulge hence the satellite is spinning up the planet and the tidal torque is ACCELERATING TORQUE..

These two kinds of Torques are illustrated in Figure B1 and Figure B2 in Appendix B.

I have assumed the empirical form of the Tidal Torque as follows:

$$T = \frac{K}{a^Q}\left[\frac{\omega}{\Omega} - 1\right] \qquad (24)$$

(24) implies that at Inner Clarke's Orbit and at Outer Clarke's Orbit, tidal torque is zero and (23) implies that radial velocity is zero and there is no spiral-in or spiral-out.

At Triple Synchrony, Satellite-Planet Radius Vector is aligned with planetary tidal bulge and the system is in equilibrium. But there are two roots of ω/Ω=1: Inner Clarke's Orbit and Outer Clarke's Orbit. As already shown in Total Energy Profile, Inner Clarke's Orbit a_{G1} is unstable equilibrium state and Outer Clarke's Orbit a_{G2} is stable equilibrium state. In any Binary System, secondary is conceived at a_{G1}. This is the CONJECTURE assumed in Kinematic Model. From this point of inception Secondary may either tumble short of aG1 or tumble long of a_{G1}. If it tumbles short, satellite gets trapped in Death Spiral and it is doomed for destruction. If it tumbles long, satellite gets launched on an expanding spiral orbit due to gravitational sling shot impulsive torque which quickly decays. After the impulsive torque has decayed, the satellite coasts on it own toward final lock-in at a_{G2}.

Equating the magnitudes of the torque in (23) and (24) we get:

$$\frac{m^* B}{2\sqrt{a}} \times \frac{da}{dt} = \frac{K}{a^Q}\left[\frac{\omega}{\Omega} - 1\right] \qquad (25)$$

Rearranging the terms in (25) we get:

$$V(a) = Velocity\ of\ recession = \frac{2K}{m^* B} \times \frac{1}{a^Q}\left[Ea^2 - Fa^{2.5} - \sqrt{a}\right] m/s \qquad (26)$$

The Velocity in (26) is given in m/s but we want to work in m/y therefore (26) R.H.S is multiplied by 31.5569088×10^6s/(solar year).

$$V(a) = \frac{2K}{m^* B} \times \frac{1}{a^Q}\left[Ea^2 - Fa^{2.5} - \sqrt{a}\right] \times 31.5569088 \times 10^6 m/y \qquad (27)$$

In (27) 'a' refers to the semi-major axis of the evolving Satellite. There are two Unknowns exponent 'Q' and structure constant 'K'. Therefore two unequivocal boundary conditions are required for the complete determination of the Velocity of Recession.

First boundary condition is at a = a_2 which is a Gravitational Resonance Point where ω/Ω = 2 (Rubicam 1975),

i.e. $(Ea^{3/2} - Fa^2) = 2$ has a root at a2.

In Mars-Phobos case, a_2 = 3.24207×10^7 m.

At a_2 the velocity of recession maxima occurs. i.e. $V(a_2) = V_{max}$.

Therefore at $a = a_2$, $(\delta V(a)/\delta a)(\delta a/\delta t)|_{a2} = 0$.

On carrying out the partial derivative of V(a) with respect to 'a' we get the following:

$$At\ a_2,\quad (2-Q)E \times a^{1.5} - (2.5-Q)F \times a^2 - (0.5-Q) = 0 \qquad (28)$$

Now structure constant (K) has to be determined . This will be done by trial error so as to get the right age of Phobos i.e. 4.5Gy.

We will assume the age of Mars and Deimos as 4.5Gy as already mentioned in Section 2. The Transit Time from a_{G1} to the present 'a' is given as follows:

$$Transit\ Time = \int_{a_{G1}}^{a} \frac{1}{V(a)} da \qquad (29)$$

The results of the calculations of spiral trajectory for Phobos (collapsing Spiral) and for Deimos (expanding spiral) are tabulated in Table 7 and Table 8.

Table 7: Kinematic Parameters of the spiral trajectory of Phobos and Deimos

Parameters	Phobos	Deimos
a_2 ($\times10^7$m)	3.24207	3.24207
Q(exponent)	3.5	3.49999
B ($\times10^6 m^{3/2}/s$)	6.54248	6.54248
Vmax(m/y)	0.00743	0.0087
K(structure constant) [$\times10^{35}$]	2.80961	0.691082

Table 8: The Transit Time, Dooms day expected for Phobos and Approach Velocity for Phobos and Recession Velocity for Deimos

Parameters	Phobos	Deimos
Time Constant of evolution (τ)[1] [$\times10^{22}$y)	0.100389	1.94251
Evolution Factor(€)2	-1.48088×10-12	1.796×10-14
Transit time	4.50293Gy	4.50734Gy
Expected Dooms day of Phobos	9.79742My in future	Not applicable
Radial Velocity (Approach or Recession)	-0.211473m/y	+0.00530499m/y

1. Time Constant of Evolution = $\tau = (a_{G2} - a_{G1})/$ Vmax;
2. Evolution Factor = € $= (a - a_{G1})/(a_{G2} - a_{G1})$

The analysis based on Kinematic Model but assuming the altitude decay rate as derived by Johnson(1972) and Bills(2005) based on Seismic Model give the same time for dooms day as the estimation based on Seismic Model but give technically untenable age of Phobos. These results are tabulated in Table 9.

Table 9.Transit Time and Dooms day estimate by Kinematic Model assuming the altitude decay rate as calculated by Burns(1978) and Bills et.al (2005)

Altitude decay rate	-0.018m/y (Johnson 1972)	-0.0398858m/y (Bills 2005)
Transit Time from Kinematic Model	52.9027Gy	23.8744Gy
Dooms day estimate from	115.105My	51.9455My

In Kinematic Model analysis, if the altitude decay rate is assumed as calculated by Burns(1978) and Bills et.al.(2005) then we arrive at the same dooms day time table as estimated by Burns(1978) and Bills et.al.(2005) based on Seismic Model which is 100My and 50 to 30My respectively but the transit time is 53Gy and 24Gy respectively from Kinematic Model. The transit time is inordinately large. Hence Seismic Model does not seem to be giving realistic results.

Discussion

Analytical results based on Kinematic Model stand in sharp contrast to that obtained from Seismic Model. In fact former is a time scaled version of tidal evolution of Phobos by one order of magnitude in comparison to that obtained from Seismic Model predicting a much earlier catastrophic impact (within 10My) with Mars with grave implications for Human-kind. This scaled up tidal evolution seems to be reasonable considering the fact that Phobos is caught in a gravitational runaway in-spiral. Since the mass ratio is vanishingly small, the time constant of evolution of both Phobos and Deimos are inordinately large (10^{21}y and 2×10^{22}y respectively) and if both were in super-synchronous orbits Phobos would also have had negligible evolutionary history just as Deimos. But since Phobos is in sub-synchronous orbit it is exhibiting a significant tidal evolution to the extent that it has descended from 20,042Km synchronous orbit to the present 9378Km well within Roche's limit of Mars. It is moving towards a head-on collision with Mars. But even before head on collision takes place, the primary tides should have smashed it and converted it into annular ring of dust which will eventually spiral into Mars. According to our analysis Roche's Zone[5] lies within 8000km to 14,000km but Phobos is intact. Hence the question of Phobos being pulverized by primary tides does not arise. May be Phobos is an accreted body with high tensile strength though it lacks compaction.

Conclusion

The results in this paper seems to be reasonable considering the fact that Phobos is in a gravitationally runaway in-spiral path but the ultimate validation or invalidation of these results will come from future Interplanetary Laser Ranging Missions(ILRM) notably from Phobos Laser Ranging Mission(PLRM) [Appendix C]

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Supplementary on-line materials for **"Comparative study of Tidal Evolution of Mars-Phobos-Deimos based on Kinematic Model and based on Seismic Model".**

APPENDIX A.

Fact Sheet of Earth-Moon : http://nssdc.gsfc.nasa.gov/planetary/factsheet/moonfact.html

Parameters	Earth	Moon
Mass(Kg)	5.9726×10^{24}	0.07342×10^{24}
GM(Km^3/s^2)	0.3986×10^6	0.0049×10^6
Volumetric Mean Radius Or Median Radius($\times10^3$ m)	6371	1737
Flattening (ellipticity)	0.00335	0.0012
Mean Density(Kg/m^3)	5514	3344
Moment of Inertia($I/(MR^2)$)	0.33086	0.394
Sidereal Spin period	23.9344h	27.322d
Sidereal Orbital period(d)	-	655.7208h (27.3217d)
a*(semi-major axis)($\times10^8$m)	-	3.84400
Lunar Orbit eccentricity	-	0.0549

Lunar Orbital inclination w.r.t.Ecliptic	-	5.145 degrees
B=√(G(M+m)) ($m^{3/2}$/s)		2.00873×10^{7}

*Mean Orbital Distance from the center of Earth.

Fact Sheet of Pluto-Charon:

http://nssdc.gsfc.nasa.gov/planetary/factsheet/plutofact.html

Parameters	Pluto	Charon
Mass(Kg)	13.1×10^{21}	1.62×10^{21}
GM(Km^3/s^2)	0.00087×10^{6}	?$\times10^{6}$
Volumetric Mean Radius Or Median Radius($\times10^3$ m)	1195	593
Flattening(ellipticity)	0.00	0.00
Mean Density(Kg/m^3)	1830	1850
Moment of Inertia($I/(MR^2)$)	0.4	0.4
Sidereal Spin period	6.3872d	6.3872d
Sidereal Orbital period(d)	-	6.3872d
a*(semi-major axis)($\times10^3$m)	-	19600
Charon Orbit eccentricity	-	0.00
Charon Orbital inclination w.r.t. Pluto's Orbit	-	118 degrees
Pluto's orbit inclination to Ecliptic	17.2 degrees	
Spin and Orbit	retrograde	retrograde
B=√(G(M+m)) ($m^{3/2}$/s)		988966

*Mean Orbital Distance from the center of Pluto.

Fact Sheet of Sun-Jupiter

http://nssdc.gsfc.nasa.gov/planetary/factsheet/jupiterfact.html
http://nssdc.gsfc.nasa.gov/planetary/factsheet/sunfact.html

Parameters	Sun	Jupiter
Mass(Kg)	1.9885×1030	1.8983×1027
GM(Km^3/s^2)	132,712	126.687
Volumetric Mean Radius Or Median Radius($\times10^3$ m)	696,000	69,911
Flattening(ellipticity)	0.00005	0.06487
Mean Density(Kg/m^3)	1408	1326
Moment of Inertia($I/(MR^2)$)	0.059	0.254
Sidereal Spin period	609.12h	9.9250h
Sidereal Orbital period(d)	-	4,332.589d
a*(semi-major axis)(×109m)	-	778.57
Jupiter Orbit eccentricity	-	0.0489
Jupiter's Orbit inclination w.r.t. Ecliptic	-	1.304deg
Obliquity to Orbit	-	3.13deg
Obliquity to Ecliptic	7.25deg	
Spin and Orbit	prograde	prograde
B=√(G(M+m)) ($m^{3/2}$/s)		1.15256×10^{10}

*Mean Orbital Distance from the center of Sun.

The Globe-Orbit Parameters of NN Serpentis. [Parsons et.al. (2009)]

Parameter	Rel.magnitude	Abs. magnitude
a(semi-major axis)	$0.934\pm0.009R_O$	6.49597×10^{8}m$\pm6.2595\times10^{6}$m
R_1(rad.ofWD)	$0.0211\pm0.0002R_O$	14.67505×10^{6}m
R_2(Rad. of M Dwarf)	$0.149\pm0.002R_O$	1.036295×10^{8}m
M_1(mass of WD)	$0.535\pm0.0121M_O$	1.06465×10^{30}Kg
M_2(mass of M Dwarf)	$0.111\pm0.004M_O$	2.2089×10^{29}Kg
$P_{orb} = P_{spin1} = P_{spin2}$	0.13days	11232s

The Globe-Orbit Parameters of RW-Lac.

Parameters	Primary		Secondary	Ref.
Age		11Gy		
Distance d (pc)		190±10		1
Spectral type	G5		G7	1
Stellar mass($\times M_0$)	0.928±0.006		0.87±0.004	1
Stellar Radius ((×R0)	1.186±0.004		0.964±0.004	1
Orbital period(d)		10.3692046±0.0000017 =895899.2774 s		1
Semi-major axis((×R0)		24.32±0.05		1
Rotational Vel (Km/s)	5.8±0.1		4.7±0.1	1
Stellar Spin Period(s)	893580.53		896305.742	1
eccentricity		0.0098±0.0010		1
Angle of inc.(i)		89.45°		1
Tidal locking time	2.92Gy		3.18Gy	1
Apsidal Motion		Undetectable.		
Argument of Periastron		183±11°		

1. Lacy , Claud H. Sandberg; Torres, Guillermo: Claret, Antonio; and Vaz, Luiz Paulo Ribeiro; Absolute Properties of the Eclipsing Binary "tar RW Lacertae'', The Astronomical Journal, 130 : 2838 -2846, December, (2005);

APPENDIX B.

The concept of Tidal Torque as seen in Earth-Moon and in Mars-Phobos.

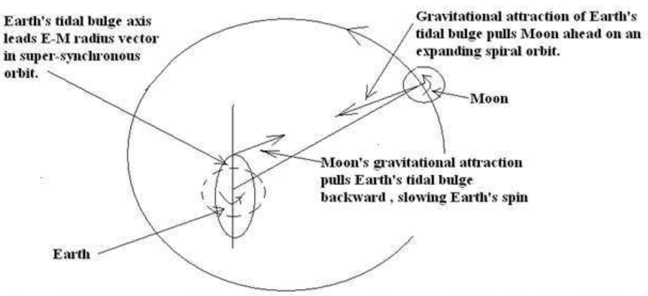

Figure A.1. In Earth-Moon System, Moon is in super-synchronous orbit. The off-setting of the line of bulge in Earth with respect to E-M radius vector creates a tidal drag and de-spinning of Earth leading to secular lengthening of day. The de-spinning of Earth leads to increased angular momentum of Moon . During the conservative phase of the evolutionary phase of E-M System, by gravitational sling shot impulsive torque Moon is launched on an expanding spiral path around Earth. After the conservative phase, Earth coasts on its own towards the outer Clarke's orbit where it terminates its non-keplerian journey.

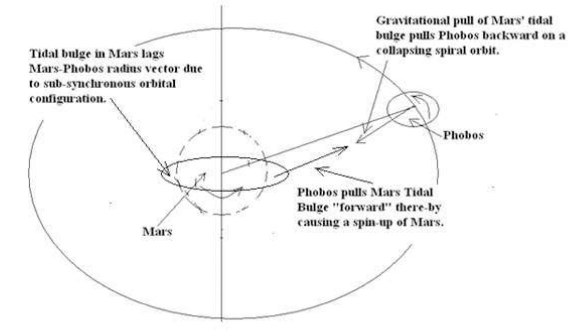

Figure A.2. In Mars-Phobos System, Phobos is in sub-synchronous orbit and is speeding up Mars spin and losing angular momentum to conserve total angular momentum. In the process it is launched on a gravitational runaway collapsing spiral orbit. The offset of the tidal bulge axis of Mars with respect to Mars-Phobos radius vector creates a tidal acceleration of the spinning Mars.

APPENDIX C.
Interplanetary Laser Ranging(Turyshev et.al.2010)

With recet successful Laser Transponder Experiments conducted with MLA(Mercury Laser Altimeter) and MOLA(Mars Orbiter Laser Altimeter) instruments (Smith et.al. 2006; Sun et.al. 2005; Abshire et.al. 2006; Degnan 2008)15-18), Interplanetary Laser Ranging (ILR) is rapidly becoming a mature technology. A mm-level ranger precision over inteplanetary distances is within our reach thus opening a way for significant advances in the tests of gravity on Solar System Scale(Degnan 2007)19). ILR allows for a very precise trajectory estimation to an accuracy of less than 1cm at a distance of ~2AU. One of these missions being planned is Phobos Laser Ranging (PLR) Mission which is expected to be set up by 2016. In this mission a Laser Ranging Transponder Instrument will be deployed on Phobos. This Transponder will enable measurements of distances from Phobos to Earth with 1-mm accuracy during daily hour long passes(Murphy et.al 2009)25). Precision Laser Ranging to Phobos could measure the distance between an observatory on the Earth and a terminal on the surface of Phobos to an accuracy of 1-mm in less than 5 minutes of Integration Time.Phobos shows a large secular acceleration in orbital longitude. Recent fits by Bill et.al.(2005), Lainey et.al.(2007) and Jacobson (2010) give an acceleration in the forward orbital longitude = a(dn/dt)= 416m/y^2. This secular acceleration can be easily detected by PLR giving refined accuracy. The cause of this acceleration is Phobos-raised tides on Mars perturbing Phobos. The tidal bulge in Mars is behind (in time and longitude) Phobos position radius vector as a result Phobos is accelerating Mars spin and in the process sapping energy from the orbit which consequently shrinks by 4cm/y as estimateby Bill et.al(2005) and by Ramslay & Head III(2013). Phobos will eventually impact Mars(Efronsky et.al 2010). The most important of the tidal components for the secular acceleration should be the second degree M2 tide of period 5.55h on Mars. The small eccentricity (0.015) and inclination (1.1^O) tend to reduce the influence of other degree 2 tides by ~ 3 order of magnitude or more. The influence of tides of higher degree fall off as even powers of (R/a)=(1/2.76) about an order of magnitude per degree so the degree 3 tide of 3.7h-period on Mars is a small contribution to tidal secular acceleration.Yoder (1982) has placed an upper limit on Phobos k_2(Love Number)/Q= 2×10^{-7}. Which would make dissipation in Phobos a minor contributor of the order 10^{-3} relative to the overall tidal acceleration of Phobos. Periodic tidal displacement on Phobos might reach 1mm. Meteroic Impact are not a concern for the dynamics of PLR mission. Once the secular acceleration measurement is made by high confidence level in PLR mission the altitude loss can be accurately ascertained and this will provide the ultimate validation or invalidation of the kinematic model and this model based analysis.

References

1. Bills BG, Neumann GA, Smith DE, Zuber MT (2005) Improved estimate of tidal dissipation within Mars from MOLA observations of the shadow of Phobos. J. Geophys. Res 110.
2. Jacobson RA (2010) The Orbits and masses of the Martian satellites and the libration of Phobos. Astron J 139: 668-679.
3. Abshire JB, Sun X, Neumann G, McGarry J, Zagwodzki T, et al. (2006) Laser pulses from Earth detected at Mars. In: Proc. of the Confer-ence on Lasers and Electro-Optics (CLEO). Paper CThT6, May 25, 2006, Long Beach, CA. Optical Society of America http://www.opticsinfobase.org/abstract.cfm?URI=CLEO-2006- CThT6.
4. Degnan JJ (2007) Asynchronous laser transponders: a new tool for improved fundamental physicsexperiments. Int. J. Mod. Phys D 16: 2137-2150.
5. Degnan JJ (2008) Laser transponders for high accuray interplanetary laser ranging and time transfer. Lasers, Clocks and Drag-Free Control 349: 231-242.
6. Dittus H, Lämmerzahl, C., Turyshev, S.G. (eds.) Lasers, Clocks, and Drag-Free: Technologies for Future Exploration in Space and Tests of Gravity Astrophysics and Space Science Library. Springer, Berlin
7. Efroimsky M, Lainey V (2077) Physics of bodily tides in terrestrial planets and the appropriate scales of dynamical evolution. J. Geophys Res 112: E12003.
8. Lainey V, Dehant V, Pätzold M (2007) First numerical ephemerides of the Martian moons. Astron. Astrophys 465: 1075-1084.
9. Murphy Jr TW, Farr W, Folkner WM, Girerd AR, Hemmati H, et al. (2009) Testing fundamental gravity via laser ranging to Phobos. In: Schilliak, S. (eds.) Proc. 16th International Workshop on Laser Ranging 675-681.
10. Smith DE, Zuber MT, Sun X, Neumann GA, Cavanaugh JF, et al. (2006) Two-way laser link over interplanetary distance. Science 311: 53.
11. Sun X, Neumann GA, McGarry JF, Zagwodzki TW, Cavanaugh JF, et al. (2005) Laser ranging between the mercury laser altimeter and an Earth-based laser satellite tracking station over a 24 million kilometer distance. In: OSA Annual Meeting Abstracts, Tucson AZ 16-20.
12. Turyshev SG, Farr W, Folkner WM, Girerd AR, Hemmati H, et al. (2010) “Advancing tests of relativistic gravity via laser ranging to Phobos”, Exp. Ast 28: 209-249.
13. Yoder CF (1982) Tidal rigidity of Phobos. Icarus 49: 327-346.