

Dynamics of Rigid Bodies with Intrinsic Rotation Subjected to Non-Coaxial Actions

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ABSTRACT

This article presents the **Theory of Dynamic Interactions (TDI)** as a framework for extending the analysis of bodies with intrinsic rotation subjected to additional non-coaxial excitations within classical rotational mechanics. The proposed formulation introduces transverse coupling terms intended to describe possible real dynamic effects.

In particular, we examine the hypothesis that a constant non-coaxial excitation may induce an additional transverse contribution to the evolution of both translational velocity and angular momentum, which could lead to trajectory curvature or supplementary precession in systems with intrinsic angular momentum.

The proposal is formulated in terms of distributed rotational fields and includes a hypothesis of finite propagation for rotational interactions. The model is not intended to replace classical mechanics within its established domain of validity, but rather to define a specific regime in which distinguishable predictions may be formulated and tested.

Finally, we describe the requirements for validation, including quantitative comparison with classical rigid-body dynamics, reproducible numerical simulations, trajectory measurements, uncertainty analysis, and independent experimental verification. This is therefore a propositional and testable contribution, not a conclusive one.

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Introduction

Rotating bodies subjected to non-coaxial excitations arise in several areas of physics, including gyroscopic systems, rigid-body dynamics, spacecraft attitude dynamics, and celestial mechanics. The Theory of Dynamic Interactions (TDI) is a proposal for extending classical rotational dynamics to the analysis of systems with intrinsic angular momentum subjected to non-coaxial actions [1].

The central hypothesis states that, when a rotating body receives an additional excitation whose axis does not coincide with that of its prior spin, dynamic coupling terms may arise that are not explicitly included in the usual classical formulation. Within this framework, rotation ceases to be considered exclusively as a kinematic descriptor and is instead treated as an active physical magnitude capable of coupling with translation or with new rotational excitations.

The proposal is articulated through a formulation in which new mathematical terms appear, describing in a unified way phenomena such as induced precession and trajectory curvature in systems with intrinsic rotation.

TDI is not intended to replace classical mechanics within its domain of validity, but rather to propose an extended framework for specific dynamic regimes. We consider that the model may be of interest in rotational dynamics and astrophysics. The Theory of Dynamic Interactions (TDI) has been set out in several texts, the most recent being the recently published book entitled *New Celestial Mechanics* [2].

Statement of The Problem and Motivation

Classical mechanics has provided an extraordinarily effective basis for describing translational and rotational phenomena, especially in ideal systems or in those in which the applied actions are coaxial with the principal axes of motion [3]. However, in many situations, configurations arise in which a body with prior rotation is subjected to new non-coaxial actions [4].

In such scenarios, the classical analysis is usually formulated by means of non-inertial frames, apparent terms, or indirect treatments of precession. TDI arises as a proposal aimed at describing more explicitly the coupling between intrinsic rotation, non-coaxial excitation, and trajectory evolution [5].

The central hypothesis of TDI states that, when a body with intrinsic angular momentum L receives an additional rotational excitation characterized by an angular velocity Ω , non-coaxial with the prior rotation, an effective dynamic interaction may arise, with two main consequences: a modification in the evolution of angular momentum, and a coupling between rotation and translation capable of inducing curvature in the trajectory. Our proposal and its mathematical treatment have been developed within the framework of distributed vector fields [6].

Table 1: Schematic Comparison Between the Classical Model and TDI in A System with Intrinsic Rotation Subjected to A Constant Non-Coaxial Excitation

Analytical framework	Description of the expected behavior	Verifiable differential prediction
Classical mechanics	The evolution of the system is described through rigid-body dynamics, attributing the observed precession and deviation to the balance of moments, the inertia tensor, and the geometric and kinematic conditions of the setup.	Under identical initial conditions, geometry, and applied excitation, the trajectory and precession should agree with the magnitudes predicted by the classical formulation.
Theory of Dynamic Interactions (TDI)	In addition to the classical balance, the non-coaxiality between the intrinsic rotation and the new excitation generates an additional dynamic coupling term, capable of inducing effective trajectory curvature and supplementary precession of the system.	In a setup with a rotor and non-coaxial excitation, TDI predicts a transverse deviation with a radius of curvature systematically different from that calculated exclusively by the classical model, while keeping the initial conditions, geometry, and applied torque constant.

Table 1 presents, in schematic form, the conceptual difference between the classical treatment and the formulation proposed by TDI, highlighting an experimental prediction that may be subjected to quantitative testing.

The objective is not to replace the Newton–Euler framework, but to examine whether an additional effective coupling term can be formulated and tested in a clearly defined regime of non-coaxial rotation.

In Table 2, we propose a comparative analysis of both dynamic hypotheses, indicating their differentiating attributes.

Table 2: Comparative Analysis of Both Frameworks

Aspect	Newton–Euler	TDI
Variables	r, v, ω, L	r, v, ω, L, Ω
Translation–rotation coupling	It does not appear as a direct physical term in the translational equation of the center of mass.	It is introduced as an effective transverse term.
Differential term	None.	$m(\Omega \times v)$
Observable magnitude	Classical trajectory/precession	Additional transverse deviation
Verification criterion	Complete agreement with Newton–Euler	Reproducible systematic difference

Note: The proposed comparison is experimentally testable through measurements of trajectory, radius of curvature, and precession velocity under reproducible conditions.

Proposal for a Dynamic Coupling Framework

In our proposal, the induced transverse term is the dominant additional contribution. The magnitude of the translational velocity is conserved while its direction changes.⁷

The equation of motion in TDI will be determined by the translational velocity of the body's center of mass, whose magnitude does not change; therefore, it will be equal to the initial translational velocity of the rotating body:

$$\vec{v} = \Psi \vec{V}_0 = \begin{pmatrix} \cos\left(\frac{M't}{I\omega}\right) & -\sin\left(\frac{M't}{I\omega}\right) & 0 \\ \sin\left(\frac{M't}{I\omega}\right) & \cos\left(\frac{M't}{I\omega}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} \vec{V}_0$$

where:

v is the final translational velocity of the moving body's center of gravity.

Ψ is the operator that transforms the initial velocity, V_0 into the final velocity.

M' is the applied moment. t is the time variable.

I is the body's moment of inertia. ω is its intrinsic rotational velocity.

Ψ is introduced as a rotation operator acting on the translational velocity vector, under persistent non-coaxial excitation.

The non-discriminating coupling proposed in our hypothesis should be identified as a spatial rotation of the translational velocity vector, resulting in the above equation of motion.⁸

Therefore, we deduce a formulation for bodies with translational motion and intrinsic rotation when they are subjected to new torques that are non-coaxial with that rotation.

At the beginning of our project, we arrived at the hypothesis that there could be a nomological, physico-mathematical correlation between simultaneous rotation and the orbital motions observed in bodies with intrinsic rotation and orbital motion.

In our studies, it was confirmed that simultaneous intrinsic rotation and orbital motions can be observed in nature. However, as far as our analysis has shown, we have not found an explicit formulation equivalent to the one obtained, nor a physical or mathematical model establishing a scientific correlation between the two motions, nor one that has been systematically explored within the standard Newton–Euler framework.

TDI proposes a dynamic framework for analyzing the possible relationship between intrinsic rotation, non-coaxial excitations, and trajectory curvature. In certain systems, our proposal makes it possible to justify this constant coincidence between orbit and intrinsic rotation, and to develop a specific dynamics for rotating bodies subjected to non-coaxial excitations.

Mathematical formulation

The system is written in terms of continuous fields. We may propose the following minimal logical mathematical development for rigid bodies, in an inertial frame, with intrinsic rotation and subjected to non-coaxial actions:

Equation of motion:

$$\vec{v} = \Psi \vec{V}_0.$$

The following equations were derived by Arturo Rodríguez Palenzuela and published in his book *Rotational Field Theory: The Theory of Dynamic Interactions*:⁹

Rotational field:

$$\mathbf{R} = \nabla \times \mathbf{v}$$

Rigid-rotation guide:

$$\mathbf{v}_{rot} = \boldsymbol{\omega} \times \mathbf{r} \Rightarrow \mathbf{R} = 2\boldsymbol{\omega}$$

Angular momentum (continuous):

$$\mathbf{L} = \int \mathbf{r} \times \mathbf{v} dm$$

Generalized angular momentum equation:

$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau}_{ext} + \boldsymbol{\Omega} \times \mathbf{L}$$

Induced transverse acceleration:

$$\mathbf{a}_{ind} = \boldsymbol{\Omega} \times \mathbf{v} = \frac{d\mathbf{v}}{dt}$$

Newton's Second Law:

$$m \frac{d\mathbf{v}}{dt} = \mathbf{F}_{ext} + m(\boldsymbol{\Omega} \times \mathbf{v})$$

Rotational wave equation:

$$\nabla^2 \mathbf{R} - \frac{1}{c_r^2} \frac{\partial^2 \mathbf{R}}{\partial t^2} = 0$$

Lagrangian equation:

$$\mathcal{L} = \frac{1}{2} m \mathbf{v}^2 + \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{I} \boldsymbol{\omega} + m \mathbf{v} \cdot (\boldsymbol{\Omega} \times \mathbf{r}) - V$$

Bold vectors: $\mathbf{v}$, $\mathbf{a}$, $\boldsymbol{\omega}$, $\boldsymbol{\Omega}$, $\mathbf{L}$, $\boldsymbol{\tau}$, with their usual physical dimensions. Vector product: $\times$; scalar product: $\cdot$; gradient: ∇ ; curl: $\nabla \times$; divergence: $\nabla \cdot$.

$\mathbf{r}$: position vector; $\mathbf{v}(\mathbf{r}, t)$: velocity field; $\mathbf{a}(\mathbf{r}, t)$: acceleration field. c_r is an additional hypothesis that depends on the propagation medium.

In this set of formulas for rotational dynamics, as derived in TDI, new terms appear due to the dynamic interactions generated in these cases. Thus, an effective induced transverse acceleration appears:

$$\mathbf{a}_{ind} = \boldsymbol{\Omega} \times \mathbf{v},$$

and in the calculation of the Lagrangian the term also appears:

$$m \mathbf{v} \cdot (\boldsymbol{\Omega} \times \mathbf{r}).$$

A further term must also be considered:

$$\boldsymbol{\Omega} \times \mathbf{L},$$

which represents a transverse coupling between the imposed non-coaxial angular velocity and the body's intrinsic angular momentum.

The formulation is completed by an interpretation of rotation as a distributed physical field, in which the rotational field may be written as $\mathbf{R} = \nabla \times \mathbf{v}$ and, under certain hypotheses, may admit a variational description through an extended Lagrangian.

This proposed theory generates the differential prediction expressed in Table 1.

Newton's Second Law

TDI does not deny Newton's Second Law, $\mathbf{F} = m\mathbf{a}$. Instead, it proposes that, for our specific regime, the total acceleration may include a transverse contribution induced by a non-coaxial rotational excitation.

Unlike inertial terms introduced by a change of reference frame, the term proposed here is intended to represent a physical coupling associated with the non-coaxial rotational excitation. This distinction must be established by formulating the equations in an inertial frame and comparing the predicted transverse deviation with the corresponding Newton–Euler solution, since the new term $m(\boldsymbol{\Omega} \times \mathbf{v})$ is not a fictitious force due to a rotating reference frame.

In material systems endowed with intrinsic rotation and subjected to a new non-coaxial rotational action, TDI proposes extending Newton's Second Law by means of a dynamic coupling term between the induced rotation and the translational velocity:

$$m \frac{d\mathbf{v}}{dt} = \mathbf{F}_{ext} + m(\boldsymbol{\Omega} \times \mathbf{v})$$

Where $\boldsymbol{\Omega}$ represents the angular velocity associated with the new non-coaxial rotation imposed on the system, and $\mathbf{v}$ is the body's translational velocity. The term $m(\boldsymbol{\Omega} \times \mathbf{v})$ expresses a transverse dynamic interaction, perpendicular to the velocity, capable of modifying the direction of motion without necessarily modifying the magnitude of the velocity.

In classical mechanics, Newton's laws are applied directly in inertial systems; in rotating systems, additional apparent or reference-frame transformation terms appear. Furthermore, in classical rotational dynamics, Euler's equations already introduce cross terms of the type $\boldsymbol{\omega} \times \mathbf{L}$ when the motion is expressed in the body-fixed frame.

The difference introduced by TDI is more profound: it interprets this transverse term as a real physical coupling between translation and rotation in bodies with intrinsic angular momentum. This interpretation is consistent with the presentation of TDI as a theory directed at bodies subjected to multiple non-coaxial rotations.

The new equation preserves the Newtonian structure of force equal to mass times acceleration, but extends the dynamic formulation by incorporating a transverse contribution produced by the interaction between translational velocity and the new rotation imposed on the system.

All this implies questioning the strict separation that classical mechanics establishes between translational motion and rotational motion, accepting their possible coupling, as we understand occurs in cases of non-coaxial excitations.

Final Observation on The Domain of Validity

Within the framework proposed here, the classical Newtonian and Eulerian formulations are recovered as limiting cases when the coupling disappears, for example when $\boldsymbol{\Omega} \rightarrow 0$, when the rotations are coaxial, or when the system may be treated as ideal, without an active intrinsic rotational structure.

In the limit $\boldsymbol{\Omega} = 0$, TDI recovers the usual Newtonian and Eulerian expressions as limiting cases. Finally, the theory incorporates a finite propagation velocity in space, c_r , since this propagation is not assumed to be instantaneous.

Methodology and State of Validation

TDI has been developed from three complementary lines of work: theoretical analysis, numerical simulation, and experimental testing. AI-assisted tools were also used in drafting this text, but only for language editing, structural organization, and preliminary consistency checks of the human-generated text. They were not used as autonomous authors or as substitutes for mathematical derivation, empirical validation, peer review, or scientific responsibility. TDI was conceived more than thirty years ago, following experimental tests, at a time when such AI tools did not exist.

On the theoretical side, the behavior of bodies with intrinsic rotation subjected to non-coaxial excitations was reviewed, identifying possible descriptive gaps when multiple non-coaxial rotations occur. In parallel, simulations were developed to compare the evolution of the trajectory and rotational state predicted by classical mechanics with that obtained through the extended model [12].

On the experimental side, we have designed prototypes and setups to observe the behavior of rotating systems subjected to new non-coaxial excitations. Preliminary qualitative observations motivated the formulation presented here, and the tests carried out appeared compatible with the proposed dynamic coupling hypothesis. Nevertheless, for this line of work to reach a degree of acceptance comparable to that of other developments in applied dynamics, independent quantitative validation remains essential, incorporating calibration, error analysis, uncertainty propagation, and systematic comparison with reproducible simulations.

The work combines conceptual review, mathematical formalization, numerical simulation, and an extensive experimental line developed over more than forty years.

The present manuscript does not claim definitive experimental validation. Previous exploratory prototypes and qualitative observations motivated the formulation presented here. A rigorous validation program is still required, including calibrated instrumentation, uncertainty propagation, reproducible numerical simulations, and direct comparison with Newton–Euler predictions.

Scope, Limitations, and Technical Discussion

The main strength of the proposal is that it clearly delimits the dynamic regime it aims to study: systems with intrinsic angular momentum subjected to non-coaxial excitations. This approach is relevant for a technical discussion because it transforms a broad hypothesis into a more concrete and verifiable problem. At the same time, TDI requires a more precise experimental definition of its magnitudes, a better delimitation of its domain of validity, and a systematic comparison with predictions derived from classical rigid-body dynamics and the specialized literature on spacecraft attitude dynamics.

For this reason, the version presented here deliberately adopts the tone of a theoretical proposal in development. Its value at the current stage lies in offering a conceptual and mathematical framework capable of producing differential predictions. Its consolidation will depend on the reproducibility of the tests, the quantification of the observed effects, and the opening of verification programs in independent academic or laboratory environments [13].

A theory that proposes a new regime of rotational dynamics must be tested through high-precision independent experiments. The appropriate next step is not rhetorical approval or rejection, but external verification.

A rigorous validation program should include calibrated instrumentation, systematic error analysis, uncertainty propagation, quantitative tracking of motion, controlled comparison with Newton–Euler predictions, and full reproducibility by independent laboratories.

Only through this process will it be possible to determine whether the proposed coupling terms represent genuinely new physics, or whether the observed effects can be absorbed within refined classical models.

Although the validation of a new physical theory is certainly complex, it may be carried out by means of prototypes and experimental tests.

Differential Prediction and Falsifiability

The central differential prediction of the proposed framework is the appearance of an additional transverse contribution proportional to $\Omega \times v$. For a system prepared under identical initial conditions, geometry, mass distribution, and applied torque, the model predicts a measurable deviation with respect to the Newton–Euler solution. Our hypothesis would be unacceptable if calibrated experiments and reproducible simulations showed that the complete observed trajectory, the precession velocity, and the transverse acceleration are fully explained by the standard Newton–Euler formulation within experimental uncertainty [14].

Potential Applications to Celestial and Space Systems

This proposed conceptual framework could be applied to the analysis of [15]:

- bodies with intrinsic rotation subjected to non-coaxial gravitational moments;
- planet–satellite system dynamics;
- orbital or axial precession;
- proper rotation and trajectory curvature;
- dynamics of minor bodies, asteroids, comets, or artificial satellites.

The proposed formulation may be relevant in systems in which a body with intrinsic rotation is subjected to constant or recurrent non-coaxial rotational excitations. Such cases may include planetary bodies with axial precession, satellites subjected to gravitational moments not aligned with their axis of rotation, minor bodies with irregular rotation, asteroids under cumulative torque effects, or artificial attitude-control systems in an orbital environment.

These applications are not considered validated consequences of the model, but rather possible contexts in which the proposed rotation–translation coupling term could be tested.

The main differential prediction would be the appearance of an additional transverse component in the evolution of the trajectory or angular momentum, which should manifest itself as a measurable deviation from the solutions obtained exclusively through the classical Newton–Euler formulation.

Conclusions

This text proposes the Theory of Dynamic Interactions as a framework for describing intrinsically rotating bodies subjected to non-coaxial rotational excitations. The central hypothesis is that, under specific conditions, an additional transverse coupling term may contribute to the evolution of translational velocity, while a corresponding term may contribute to the balance of angular momentum.

The proposed formulation is not intended to replace the Newton–Euler framework within its established domain of validity [16]. Rather, it defines a specific regime in which distinguishable predictions can be formulated and tested. Whether the proposed terms represent genuinely new physics or an alternative parametrization of classical effects must be determined through quantitative comparison with standard rigid-body dynamics, reproducible simulations, and independent experimental validation [17].

Potential applications include gyroscopic systems, rotating rigid bodies, spacecraft attitude dynamics, and certain celestial systems in which intrinsic rotation and non-coaxial excitations are dynamically relevant [18].

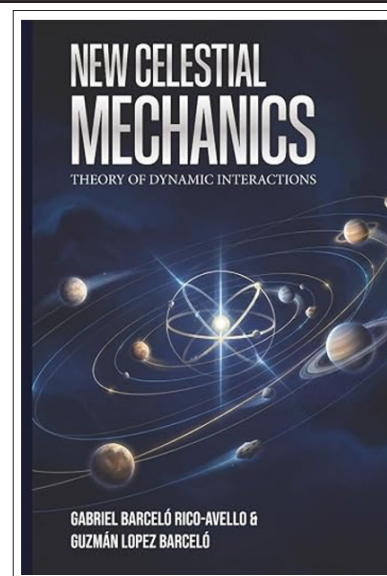


Figure 1: Cover of the book *New Celestial Mechanics* (19)

Declarations

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Competing interests

The authors declare that they have no financial or non-financial competing interests that are relevant to the content of this article.

Author contributions

Gabriel Barceló Rico-Avello: conceptualization, theoretical formulation, original research, methodology, supervision.

Guzmán López Barceló: manuscript organization, editorial development, AI-assisted workflow management, review and editing.

All scientific claims, equations, interpretations, and conclusions are the sole responsibility of the authors.

Data availability

No new experimental dataset is presented in this theoretical manuscript. Where appropriate, simulation scripts, supplementary calculations, or experimental videos supporting the discussion will be made available upon reasonable request or deposited in a public repository.

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