

Modeling of Average Global Climate Change by 2200

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ABSTRACT

According to a predictive global climate model described and used in a recent study (April 2025), future climate change is relatively insensitive to various greenhouse gas (GHG) emission scenarios, despite their significant differences. The author concluded that it is likely necessary to move even closer to pre-industrial CO₂ concentrations to truly reverse the current trend. This work aims to verify this hypothesis using a "280 ppm" scenario and to reproduce the 2025 study with a more powerful tool. To do so, the article begins by rigorously justifying the innovative principles of the model, improving its reliability, and expanding the scope of the simulations. Validated by comparison with data covering the period 1870-2025, the improved model simulates six CO₂ emission scenarios for the period 2026-2200. The results obtained are then compared with others described in the Sixth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC). Finally, the scenario offering the best compromise between feasibility and climate effectiveness is identified. Thus, the scientific community, policymakers and more generally all those interested in the problems caused by climate change will have an easily accessible tool and new perspectives on climate strategies to adopt for the benefit of future generations and biodiversity.

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Introduction

Global Climate Models (GCMs) are generally used to assess climate change, but their simulations involve significant uncertainties [1]. They do indeed encounter fundamental limitations due to factors such as model structure, calibration processes, stochastic complexity, dependence on empirical parameterizations, the strong non-linearity of many phenomena involved, etc. All of this makes it difficult, if not impossible, for CGMs to obtain deterministic and physically consistent projections beyond 2050 for key global climate parameters [2]. Following recommendations to address this major problem, a deterministic climate model, based on a mean-field approach and described in detail in, uses an innovative approach to understand and predict long-term global climate trends [3,4].

Unlike traditional GCMs, this model does not incorporate a multitude of local interactions or their feedbacks. It operates directly at the global level of the world's major energy exchanges—making it an Energy Balance Model (EBM) and focuses on the main global mean parameters, as suggested for other climate-related issues by [5,6]. This approach significantly reduces complexity and allows for an analysis of long-term trends in the global averaged climate system.

To achieve good predictive accuracy, this EBM benefits from several important innovations recently developed by its author:

The detection of hidden interactions between climate time series is performed using the Method of Average Lengths (MAL), described in detail in [7]. The interactions identified by the MAL are translated into "mean-field" relationships [8,9] that represent the average effects of the components on the global climate system. Each of the recurrence relationships thus obtained, giving the evolution of a given average climate parameter, captures as accurately as possible the average effect of the other parameters on it. The effects of fluid dynamics (turbulence, currents, stratification, etc.) are taken into account thanks to the concept of Dynamic Heat Capacity (DHC), already introduced in and used in [10,7,11]. The CO₂ emission altitude is obtained simply and reliably using the following property: at the emission altitude z_e , the product of [CO₂] by the air density ρ and by the infrared emission I is constant with respect to [CO₂] [4]. Using this property, it is easy to obtain a reliable estimate of the altitude at which CO₂ exerts its radiative forcing. The model thus made it possible to simulate the evolution of the main global climate parameters: atmospheric temperature, temperature of the Upper Oceanic Stratum (UOS), cloud cover, mixing layer depth, UOS stratification, and atmospheric albedo. The simulations were given up to 2100, with a deterministic rather than stochastic approach, and indicated that the average warming by 2100 would be moderate for the lower atmosphere and fairly insensitive to the emission scenarios considered [4].

However, some key aspects were not explored in depth or sufficiently clarified in and the primary objective of this work is to fill these gaps [4]. To this end, we begin by providing a robust justification for the constancy property of the product [CO₂] $\times$ ρ $\times$ I , drawing on the laws of radiative equilibrium and atmospheric

physics. This property allows us to easily and reliably determine the CO₂ emission height, a key parameter for radiative forcing in the model. We will compare this method with those derived from the work of and with those based directly on the Beer-Lambert law (for the law itself: [15]; for a method using it: [12]). Next, we provide the mathematical definition of C_{dyn}(t), the DHC at time t, and explain its relevance to climatology in general, and to the model in particular. We then improve and adapt the model, notably to simulate a return to 280 ppm, by recalibrating the parameters and feedbacks detected by the MAL. We validate the resulting model by comparing its simulations with historical data from the period 1870-2025. We then obtain and analyze the simulations obtained under six different scenarios of CO₂ concentration evolution, ranging from a very moderate slowdown of the current rate to that corresponding to a 280-ppm scenario of a return to pre-industrial CO₂ concentrations. Finally, among the six scenarios considered, we choose the one that offers the best compromise between the climate improvement it would bring and its practical applicability.

The article is divided into six sections and a final conclusion. **Section 1** states and justifies the Principle of Radiative Invariance (PRI). It establishes this principle for all CO₂ concentrations, not just those above 300 ppm, where, according to, infrared saturation of CO₂ is largely achieved. It then explains how this principle allows us to determine the CO₂ emission height [16]. Finally, it compares this new method with previous techniques and results concerning emission heights. Section 2 mathematically defines the concept of CTD, interprets it physically and explores its relevance to the model itself and, more generally, to climatology. After reviewing the principle of MAL, Section 3 explains the interactions that this statistical method has detected and that are taken into account by the model. Section 4 begins by describing the model and the improvements made compared to the one used in notably through a more precise calculation of the forcing caused by infrared radiation captured in the atmosphere and by taking into account the significant convective transfers to the troposphere and space [4]. This section then concludes with a validation of the finalized model by comparing its simulations with historical atmospheric temperature data for the period 1870–2025. Section 5 first presents the six CO₂ emission scenarios considered in this study and then describes in detail, with confidence intervals, the results obtained for the five climate parameters studied. Finally, it concludes with a presentation of tests carried out on the most important constants of the model. Section 6 first compares the results obtained with those described in the report and then indicates which of the six evaluated scenarios, in our opinion, offers the best compromise between its feasibility and the corresponding improvement in the global climate [16]. Finally, the Conclusion summarizes the contributions of this work and specifies its relevance to understanding future climate challenges.

Principle of Radiative Invariance The Principle

Let us first clarify the notation and concepts involved: The concentration of carbon dioxide in the atmosphere is denoted [CO₂]. It is expressed in ppm and is independent of altitude z and homogeneous throughout the atmosphere (an approximation valid for well-mixed greenhouse gases like CO₂ with negligible vertical variations on a global scale). z_e represents the emission altitude, that is, the altitude at which the atmosphere becomes optically thin for the infrared radiation emitted by CO₂. It is therefore also the altitude at which CO₂ effectively emits into space. ρ(z) represents the air density at altitude z in the troposphere. The temperature T(z) decreases with altitude z at a rate of approximately 6.5 K

per km a value consistent with [17]. Taking T(0) = 287 K (or 14 °C), we therefore have T(z) = 287 – 0.0065z, which will be used subsequently. I(z) represent the infrared emission of CO₂ at altitude z and temperature T(z), expressed in K. It therefore measures the amount of energy radiated by CO₂ into space at this altitude. And according to Kirchhoff's law of radiation, the absorption and emission fluxes are identical for each frequency and each direction. Thus, if we denote A(z) as the average absorptivity of CO₂ at temperature T(z), we have: A(z) = I(z). The product [CO₂] × ρ(z_e) × I(z_e) therefore measure the energy radiated per unit volume at altitude z_e, where CO₂ effectively emits into space. It represents the overall efficiency of infrared CO₂ emission into space at altitude z_e.

Let us now state the Principle of Radiative Invariance (PRI)

Principle: In thermal equilibrium and in an atmosphere where CO₂ is the main infrared absorber/emitter, the product [CO₂] × ρ(z_e) × I(z_e) is invariant with respect to the [CO₂] concentration.

Justification:

Beer-Lambert's law states that the fraction of radiation passing through a layer of optical thickness τ = 1 is equal to e⁻¹, or approximately 37%. (See Section 4.1 of) [18]. Therefore, at the emission altitude z_e, where the optical thickness τ is close to 1, only 37% of the infrared radiation emitted by CO₂ escapes into space [18,13,19,20]. Thus, to maintain radiative equilibrium and conserve the 37% fraction of radiation escaping into space, and since it precisely measures the radiative efficiency of CO₂, the product [CO₂] × ρ(z_e) × I(z_e) must remain approximately constant. Without this constancy, the condition τ ≈ 1 could not be met simultaneously for different CO₂ concentrations, and radiative equilibrium would be disrupted. The emission altitude z_e therefore adjusts so that the product [CO₂] × ρ(z_e) × I(z_e) remain constant, compensating for any variation in the [CO₂] concentration due to changes in density and temperature at z_e. In disequilibrium, the principle does not apply instantaneously, but it describes the state toward which the system tends. In practice, once this constant (denoted C) has been determined from historical data, the emission height z_e for a given concentration [CO₂] is obtained very simply by solving the equation with unknown z_e.

$$[\text{CO}_2] \times \rho(z_e) \times I(z_e) = C$$

Determination of the Radiative Invariance Constant

To estimate this constant C, we will use the Tropospheric Energy-Flux Attenuation Coefficient (TEFAC), which measures the ratio between outgoing radiative energy and radiative energy emitted at the surface. According to the data shown in Figure 7.2 of the TEFAC at the beginning of the century was 0.477 (90% confidence interval: [0.468; 0.486]) [21]. Furthermore, using their formula (25), established, based on the fundamental laws of physics, a relationship between the TEFAC and the emission height z_e [22]:

$$T_{\text{EFAC}}(z_e) = 0.794 \left(\frac{T(0)}{T(0) + Hz_e} \right)^4 \frac{Mg(m-1)}{RHn}$$

We apply this using typical values for the current Earth's atmosphere and the following physical constants: T(0) = 287 K; M = 0.0029 kg/mol; g = 9.8 m/s²; H = -0.0065 K/m and m = 1.235, the associated polytropic index; R = 8.314 J/(mol·K). The equation: T_{EFAC}(z_e) = 0.477 then has a solution z_e ≈ 5400 m, a value that therefore provides an estimate of the CO₂ emission height at the beginning of the century. Since the atmospheric CO₂

concentration was estimated at 369 ppm in 2000, we therefore obtain $C = 369 \times \rho(5400) \times I(5400)$. To estimate $I(5400)$, we use the affine approximation: $I(z) = 0.008T(z) - 1.78$ obtained by the least squares method from estimates of CO_2 emissivity at different temperatures and using Planck's law. see Figure 3 in [4]. Consistent with the previous expression for $T_{\text{EFAC}}(z_e)$, we use a vertical gradient of -6.5 K per km of altitude for the expression of $T(z)$, valid for an atmosphere with standard humidity and independent of radiative exchange and CO_2 concentration [23].

We therefore take $T(z) = 287 - 0.0065z$. For the density $\rho(z)$ of air, expressed in kg/m^3 , according to the data from, we obtain by the least squares method: $\rho(z) = -0.000038z + 1.198$ [23].

Therefore, $C \approx 369 \times (-0.000038 \times 5400 + 1.198) \times 0.008 (287 - 0.0065 \times 5400) - 1.78$.

And finally, $C \approx 86.164$.

Specifics of the PRI

The PRI builds upon the foundational work on atmospheric radiative equilibrium: First, that of who introduced the concept of radiative equilibrium between the flux absorbed by an atmosphere and the infrared flux it emits into space [24]. Then, that of who showed, using of radiative transfer equations, that effective emission into space occurs at an altitude where the optical thickness is close to 1 (a necessary condition for equilibrium between absorption and emission) [15, 24]. Next, that of who clarified the link between emission height and radiative equilibrium [20]. provide a very clear explanation for the increase in emission height when CO_2 concentration increases [12]. Link the emission height to the emission pressure, using the fact that if the absorption coefficient is constant, the optical thickness increases linearly with pressure [12]. Finally, and as already seen established a relationship between the TEFAC and the emission height based on the fundamental laws of physics [22].

But unlike existing approaches, the PRI unifies mechanisms previously treated separately, thus offering a synthetic and predictive view, whereas previous methods addressed them separately and failed to determine the emission altitude (z_e). Once the radiative invariance constant is determined, the PRI allows the emission height to be directly calculated for any concentration carbon dioxide concentration. And unlike Moderate Resolution Atmospheric Transmission (MODTRAN) models, which require complex numerical calculations, this principle offers a simple analytical law to describe the dynamic adjustment of the atmosphere, whether in equilibrium or not [25]. The PRI does not describe a static equilibrium state, but rather a dynamic mechanism for adjusting the emission altitude (z_e). Whether the climate system is in equilibrium or in transient disequilibrium (as today, with a surplus of +1 W/m^2), this principle remains valid: $[\text{CO}_2] \times \rho(z_e) \times I(z_e)$ is invariant because z_e adjusts to maintain $\tau(\lambda)(z_e) \approx 1$. Moreover, it can likely be adapted to any physical medium where Beer-Lambert's law applies. For example, in a turbid lake, the depth of light penetration z_p decreases as the particle concentration $[P]$ increases, keeping the product $[P] \times \rho(z_p) \times \varepsilon(z_p)$ invariant, where ρ is the density of water and ε is the extinction coefficient of light. This suggests a universal law for adjusting critical layers in climate or physical systems, where each reservoir (atmosphere, oceans, lakes) regulates energetic or physical perturbations via an adjustment variable. For example, in the oceans, the mixing depth adjusts to maintain constant thermal efficiency in the face of disturbances. Or more generally, for any natural system that develops adjustable critical layers to regulate energy flows.

This principle thus fills a gap between detailed models (e.g., MODTRAN, the kspectrum code, etc.) and popularized explanations, offering a powerful simplification for teaching, modeling, and understanding climate equilibria [26]. It offers an elegant alternative to existing methods for estimating z_e . Unlike the spectral approach, which requires detailed data on vertical temperature and absorption profiles, or the method derived directly from Beer-Lambert's law, which overlooks the near-saturation of CO_2 at concentrations above 300 ppm, our method is based on a universal physical principle: the constancy of the product $[\text{CO}_2] \times \rho(z_e) \times I(z_e)$ [12]. This approach is sufficient to predict z_e values consistent with complex models (Current value (400–420 ppm): 6–8 km. - CO_2 doubling (800 ppm): 8–10 km (+1.5 to 3 km [26,11]), while offering analytical simplicity and broad applicability, for example to greenhouse gases other than CO_2 . PRI thus constitutes a highly relevant additional tool for modeling climate simply and effectively.

Dynamic Heat Capacities

Concept Overview

The model uses the concept of Dynamic Heat Capacity (DHC), which varies over time according to local climatic and physical conditions. This concept, introduced in [10] and refined in [7], allows for a better capture of the transient responses of the climate system. To illustrate the idea behind this concept, consider two streams located at the same high altitude in the mountains, both with the same temperature in the early morning (5 °C) and exposed to intense sunlight throughout a summer day. The first is a turbulent mountain stream. Its water, constantly churned by rapids and waterfalls, quickly distributes heat throughout its volume and is rapidly replenished with fresh water. Its temperature will remain relatively stable despite the sunlight and will barely reach 6 or 7 °C by the end of the day. The second is a calm stream flowing through a very gently sloping area. Its relatively still water accumulates heat on the surface and is replenished very slowly with fresh water. Its temperature will rise significantly: it will likely reach 10 or 15 °C by the end of the day. This example illustrates how the internal dynamics of a system modulate its thermal response, a principle we apply to the atmosphere and oceans in our climate model.

Mathematical Definition and Usefulness of the Concept

Let Q be the heat flux for a climate system (in our case, the atmosphere or UOS) and T its mean temperature. $C_{\text{dyn}}(t)$, the

instantaneous DHC, is defined by: $C_{\text{dyn}}(t) = \frac{dQ}{dT} \mid_t$ where dQ

and dT are differentials linked by instantaneous physical processes (turbulence, stratification, exchanges).

Unlike a classical heat capacity, this formulation explicitly incorporates the dynamic effects of the system. Previous work already considered the variability of thermal properties. For example: Increase in the "effective heat capacity" of the atmosphere due to an increase in air humidity [27]. Stochastic climate modeling showing the key role of ocean heat capacity in the temporal response to forcings, without, however, considering its explicit variability [28]. Introduction of effective thermal capacities dependent on ocean depth, but without coupling with atmospheric dynamics [29]. Two-layer approach with variable thermal capacities, but without monthly calibration [30]. The study of the proportionality between global warming and cumulative CO_2 emissions highlights the importance of long-term feedback mechanisms (implicitly including ocean-atmosphere exchanges), but without explicitly modeling dynamic

thermal capacities [31].

The concept of DHC goes further and offers two key advantages: First, it provides a very simple way to integrate most of the local interactions and variations into the model in an averaged manner, unlike GCMs, which are forced to parameterize with maximum precision. Second, it is a particularly effective tool for appropriately simulating the scenarios considered by linking atmospheric DHC to CO₂ emission levels. As we will see later, atmospheric and oceanic DHCs will allow the model to benefit from these powerful advantages after calibration using global historical data (1955-2025).

Heat Conservation Formulated using the CTD

Consider a physical domain Ω filled with a fluid and subjected to an external heat source or sink. The classical heat conservation equation is expressed within Ω as follows [32]:

$$(1) \rho C \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) + \nabla \cdot \mathbf{q} = Q_{\text{ext}}$$

where:

- ρ = density of the medium
- C = classical heat capacity of the medium (constant)
- T = temperature at time t
- $\mathbf{v}$ = fluid velocity
- $\mathbf{q}$ = heat flux (via conduction, convection, radiation)
- Q_{ext} = external heat sources or sinks (e.g., solar radiation, external cold) as a function of t .

In this study, T represents the average temperature value at time t within domain Ω . Therefore, $\nabla T = 0$ (no spatial gradients), and equation (1) effectively becomes:

$$(1) \rho C \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = Q_{\text{ext}}$$

The term $\rho C \frac{\partial T}{\partial t}$ represents the rate of heat storage within domain Ω .

We will now rewrite the heat conservation equation by introducing the dynamic heat capacity (DHC).

$C_{\text{dyn}}(t) = \frac{dQ}{dT}$ captures all dynamic effects (turbulence,

stratification, rapid exchanges, etc.) that influence the relationship between dQ and dT .

Thus, the rate of heat storage can be expressed as:

$$(2) \frac{dQ}{dt} = C_{\text{dyn}}(t) \frac{dT}{dt}$$

In terms of volumetric density, this yields:

$$(3) \rho \frac{dQ}{dt} = \rho C_{\text{dyn}}(t) \frac{dT}{dt}$$

Consequently, the heat conservation equation is written as:

$$(4) \rho C_{\text{dyn}}(t) \frac{dT}{dt} + \nabla \cdot \mathbf{q} = Q_{\text{ext}} \text{ throughout domain } \Omega$$

In this study, domains Ω_1 (lower troposphere) and Ω_2 (upper ocean mixed layer, soils, snow, and ice) are defined as sufficiently large to encompass all internal dynamics—specifically, for Ω_1 , variations in GHG emission heights, and for Ω_2 , variations in the depth of the upper ocean mixed layer. Furthermore, the parameters considered are globally averaged, and since T is spatially uniform,

the internal heat fluxes $\nabla \cdot \mathbf{q}$ are zero (there are no spatial gradients). Consequently, $\rho C_{\text{dyn}}(t)$ captures all the physical processes influencing the relationship between dQ and dT :

For Ω_1 : humidity, clouds, soils, the biosphere, ice sheets, greenhouse gas effects, etc., as well as horizontal and vertical atmospheric circulations, Hadley cells, hurricanes, etc.

For Ω_2 : variable depth, stratification, salinity, etc., as well as currents, thermohaline circulation, storm-driven mixing, etc.

Thus, for the two domains considered, the term $\nabla \cdot \mathbf{q}$ vanishes, and the heat conservation equation is therefore written as:

$$(5) \rho C_{\text{dyn}}(t) \frac{dT}{dt} = Q_{\text{ext}}$$

A key advantage of the CTD becomes apparent here: it allows internal feedbacks and transformations to be captured in a single term, thereby simplifying the modeling while maintaining a complete and rigorous description of the system. Moreover, equation (5) can be interpreted as a direct application of the first law of thermodynamics for a closed system, where C_{dyn} incorporates the dynamic effects of internal properties on the thermal response.

To solve this nonlinear differential equation, we assume that $C_{\text{dyn}}(t)$ can be expressed as a function of $T(t)$. In other words, there exists a function $\tilde{C}_{\text{dyn}}$ of the variable T such that $C_{\text{dyn}}(t) = \tilde{C}_{\text{dyn}}(T(t))$.

This allows us to define an antiderivative β de $C_{\text{dyn}}(T)$ with respect to T :

$$\beta(T) = \int_{T_0}^T \tilde{C}_{\text{dyn}}(T') dT'.$$

$$\text{Thus, } \frac{d\beta}{dt} = \frac{d\beta}{dT} \frac{dT}{dt} = \tilde{C}_{\text{dyn}}(T) \frac{dT}{dt} = C_{\text{dyn}}(t) \frac{dT}{dt}$$

The previous equation can then be written in this linear form:

$$(6) \rho \frac{d\beta}{dt} = Q_{\text{ext}}$$

After integration, we obtain:

$$(7) \beta(T(t)) = \beta(T_0) + \frac{1}{\rho} \int_{t_0}^t Q_{\text{ext}}(t') dt'$$

Assuming β is invertible, we deduce the temperature:

$$(8) T(t) = \beta^{-1}(\beta(T_0) + \frac{1}{\rho} \int_{t_0}^t Q_{\text{ext}}(t') dt')$$

We will see later that the relationship in the model used to simulate the temperature in Ω_1 can be considered an explicit discretized form equivalent to (6).

Adaptation to the model

We use a discreet model where the increment is the month. Hence the need to introduce monthly definitions of dynamic capacities for the two media considered (UOS and atmosphere):

For a given month n , the monthly DHC is theoretically obtained by:

$$CD_n = \int_n^{n+1} C_{\text{dyn}}(t) dt \text{ (for the UOS)}$$

$$CD'_n = \int_n^{n+1} C'_{\text{dyn}}(t) dt \text{ (for the atmosphere)}$$

In the model, we use the more practical discrete form:

$$CD_n \approx \frac{\Delta Q_n}{\Delta T_n} \text{ (for the UOS)}$$

$$CD'_n \approx \frac{\Delta Q'_n}{\Delta T_n} \text{ (for the atmosphere)}$$

where ΔQ_n , $\Delta Q'_n$ et ΔT_n are the monthly variations. These approximations effectively capture the integrated effects of the UOS and atmospheric thermal dynamics over the monthly period. This formulation allows for efficient modeling of energy transfers while remaining well-suited to the overall model approach.

Interactions Detected by MAL and used in the Model Principle of MAL

The MAL method, introduced in, used, and presented in detail in, uses Markov chains to analyze climate time series [33,7,10,12,34,35]. We summarize it below: Starting with the data series under study (e.g., monthly temperature), we construct a series of its successive deviations. Then we verify that this series can be modeled by "white noise" [35]. If this step is successful, the method can continue. A third series is created by assigning 0 to each decrease and 1 to each increase. We then verify the approximate equiprobability of these 0s and 1s. If this second condition is satisfied, all the conditions are met for modeling by a Markov chain to be possible for the series of 1s and 0s. The method can continue by calculating the average lengths of successive 0s and successive 1s. If these lengths deviate significantly from the expected theoretical value under a binomial distribution (≈ 1.94 per 100 terms), this reveals a probable interaction (the method provides the means to calculate the probability) with another climatic entity. MAL can thus detect nonlinear or lagged mechanisms, invisible to classical methods (correlation R , R^2 , cointegration defined in [36]), and allows these interactions to be integrated into mathematical models for more deterministic simulations.

Interactions Revealed by MAL and Implemented in the Model

Three successive articles have revealed, through MAL, interactions and mechanisms that have ultimately been integrated into the model: In MAL reveals an interaction between monthly atmospheric temperature and the quarterly heat of the upper ocean layer (study period: 1955-2022) [10]. The hypothesis formulated is as follows: Atmospheric warming cools the UOS via evaporation (heat removal), and vice versa. The thermal inertia of the UOS explains the quarterly periodicity present within it. This hypothesis is validated by the Z.1 model (the first version of the model presented here), which simulates energy exchanges and confirms its consistency with observations. In the MAL reveals a strong interaction between oceanic cloudiness (OC) and the heat of the UOS (study period: 1954–2008) [12]. The proposed explanation is as follows: Cloudiness acts as a natural thermostat because its increase cools the UOS, and vice versa. The Z.2 model (an adaptation of Z.1) incorporates the effect of cloudiness, notably the simulated temperature increases of $\sim 1^\circ\text{C}$ (atmosphere) and

$\sim 2^\circ\text{C}$ (UOS). The article demonstrates that data uncertainty (particularly regarding cloudiness) does not affect signal detection by the method. Finally analyzes the stratification of the UOS (study period: 1955–2023) and its link with the El Niño-Southern Oscillation (ENSO) phenomenon [7]. The MAL does not detect the strong correlation between stratification and UOS thermal energy (0.84), but it does isolate the interaction with ENSO.

The Z model (an adaptation of Z.2) incorporates five climatic parameters (lower atmosphere temperature, UOS temperature, ocean cloud cover, oceanic mixing layer sinking, and ocean stratification), and this version reveals the asymptotically limited growth of these five parameters. The MAL has thus proven to be a very well-suited tool for directly accessing interactions at the global level. This was exactly what was needed to build a predictive model based on mean fields. A recent article confirms these results and validates this method using neural networks [37].

Model Description

Fundamental Relationships of the Model

Let us first clarify some essential notations: C_n denotes the amount of heat gained or lost during month n by the surface (UOS + land surfaces and ice). θ_n is its average temperature. The amount of heat gained or lost during month n by the lower atmosphere is denoted T_n , and t_n is its average temperature. In addition to θ_n and t_n , the model allows us to simulate three other parameters crucial to the climate : cl_n , which represents the evolution during month n of ocean cloud cover (OC) anomalies; s_n , that of ocean stratification; and finally, S_n , that of the depth of the mixing layer. For their calculation, see [12] for cl_n and [7] for S_n and s_n . The formulas established in these two articles are reproduced in Table 1 and used by the model presented. These five parameters— θ_n , t_n , a_n , cl_n , s_n and S_n —are associated with their “theoretical” parameters, i.e., those simulated by the model, in which the random coefficients used to account for natural variability have been neutralized. They are denoted θ'_n , t'_n , a'_n , cl'_n , s'_n and S'_n .

During month n , the energy balance of the UOS corresponds to the difference between the energy received and the energy lost during this period. It is expressed by this equation:

$$(9) \quad C_n = (1 - a_n)R_g - b_n R_g - K_n - K'_n - \sigma_n IV_n$$

Similarly, that of the troposphere corresponds to this relationship

$$(10) \quad T_n = b_n R_g + R_{e_n} + K_n - O_n + \sigma_n IV_n$$

It is primarily these relationships (2) and (3) that make the model an Energy Balance Model (EBM). The notations used and the relationships that define them are detailed in Table 1. Sections immediately following this table supplement this point with explanations that are too long to be included directly.

Table 1: Summary of the Model's Foundational Relationships and Supplementary Relationships

N°	Relations	What is at stake.	Details and additional relationships
1	$C_n = (1 - a_n)R_g - b_n$ $R_g - K_n - K'_n - \sigma_n$ $/ V_n$	Monthly energy balance of the UOS per m ² .	R_g : energy from monthly global solar radiation (per m ²). $R_g = 340 \times 3600 \times 24 \times 30 = 881,280,000 \text{ J/m}^2$
2	$T_n = b_n R_g + Re_n +$ $K_n - O_n + \sigma_n / V_n$	Monthly energy balance of the troposphere per m ² .	$l = 2450000 \text{ J/kg}$ (approximate latent heat of seawater at 15°C and atmospheric pressure)
3	$C_n = CD_n (\theta_{n+3} - \theta_{n+2})$	Heat received per m ² column by the UOS during month n.	CD_n : dynamic heat capacity per m ² column of the SOT in month n.
4.	$T_n = CD'_n (t_{n+1} - t_n)$	Heat received per m ² column by the troposphere during month n.	CD'_n : dynamic heat capacity per m ² column of the troposphere in month n. $CD'_1 = (1/30) \times CD_1$ (obtained by comparing the thermal capacities of the UOS and the troposphere)
5.	$a_n = \gamma \times (68 + cl_n) \times 0.01$	Atmospheric albedo	$\gamma = 0.44$ measures the ratio between atmospheric albedo and OC cover.
6.	$b_n R_g = \frac{1}{3} (1 - a_n) R_g$	Proportion of solar origin absorbed by the atmosphere.	
7.	$cl_{n+1} = cl_n + \beta(\theta_{n+1} - \theta_n)$	Percentage change in OC present above the ocean during month n.	$\beta = 1.1$ constant value obtained by calibration with observed data collected in [38]
8.	Si $\theta_n \geq t_n$, $K_n = kT(\theta_n - t_n)$, Si $\theta_n < t_n$, $K_n = k'T(\theta_n - t_n)$	Evolution of sensible heat exchange from the UOS to the troposphere (during month n per m ²).	Effusivities expressed in $\text{JK}^{-1} \text{m}^{-2} \text{s}^{-1/2}$: $k=6$ and $k' = 1340$ (constant values justified after Table 1.) $T = 3600 \times 24 \times 30 = 2,592,000 \text{ s}$.
9.	$K'_n = \frac{\theta_n K'_1}{\theta_1 s_n}$	Evolution of exchange between the UOS and lower ocean layers (during month n per m ²).	$K'_1 = 0.2 \times \sigma_1 / V_1$
10.	$s_{n+1} = s_n + \mu(\theta_{n+1} - \theta_n)$	Evolution of the stratification of the upper oceanic layer (during month n per m ²).	$\mu = 5$, value of the constant obtained by calibration with observed data collected in [39].
11.	$Rs_n = \alpha \times t_n^4$	Infrared radiation emitted by the surface (during month n per m ²).	By denoting $\varepsilon = t_{n+1} - t_n$, the model uses the approximation: $Rs_{n+1} \approx Rs_n (1 + \frac{4\varepsilon}{t_n^4})$
12.	Ra_n	Infrared trapping by the troposphere (during month n per m ²).	Specified following Table 1
13.	$Re_n = Ra_n - Rs_n$	Net infrared radiation retained in the troposphere. (during month n per m ²).	
14.	$O_n = O_1 \times \frac{t_n}{t_1}$	Evolution of heat exchange by convection between the lower atmosphere and the upper troposphere (during month n per m ²).	Specified following Table 1
15.	$V_n = V_1 \left(\frac{\theta_n}{\theta_1} \right) \left(\frac{t_n}{t_1} \right)$	Variation in the amount of water evaporated (during month n per m ²).	$V_1 = 73 \text{ kg}$ (approximate amount of water that evaporates in 1 month on a 1 m ² surface of the ocean subjected to a surface temperature of 13°C and a pressure of 1 bar)
16.	$E_n = l V_n$	Energy of vaporization (during month n per m ²).	
17.	$\sigma_n = -0.1t_n + 0.8 + 0.1\theta_n$	Share of vaporization energy removed from the UOS and troposphere (during month n per m ²).	For justification, see [10]
18.	$S_{n+3} = S_{n+2} + \lambda(\theta_n - \theta_{n+1}) + L_{Th}$	Mean subsidence of the base of the troposphere mixing layer (during month n per m ²)	$\lambda = 100$, constant value obtained by calibration with observed data collected in [40] $L = 0.03 \text{ m}$ and $Th = 2 \times 10^7 \text{ J}$.
19.	$CD_n = CD_1 \times \frac{L_1 + S_n}{L_1}$	Dynamic heat capacity of the UOS (during month n per m ²).	$CD_1 = 6.5 \times 10^{12} \text{ J/K}$ (estimate obtained by calibrating the model with observations). $L_1 = 125 \text{ m}$
20.	CD'_n	Dynamic heat capacity of the troposphere (during month n per m ²).	$CD'_1 = (1/30) \times CD_1 \approx 2.167 \times 10^{11} \text{ J/K}$ Calculation of CD'_n is detailed following Table 1

Emission Height

As explained in Section 1, the emission height z_n in month n for an average CO_2 concentration is obtained using the PRI by solving the equation with unknown z_n :

$$(11) \quad \text{CO}_2 \times \rho(z_n) \times I(z_n) = 86.164$$

Figure 1 shows the emission height for $[\text{CO}_2]$ concentrations above 280 ppm. The variations obtained are consistent with the estimates of, and with satellite observations (AIRS) [12,41,20].

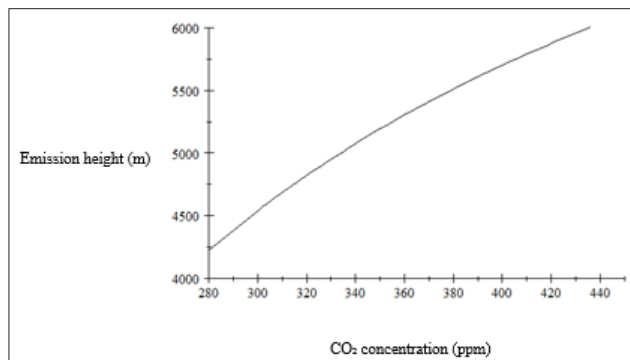


Figure 1: Evolution of the CO_2 Emission Height for Concentrations above 280 ppm

Atmospheric Dynamic Capacity

The atmospheric dynamic capacity CD'_n in month n is estimated using the following relationship:

$$(12) \quad \text{CD}'_n = \text{CD}'_1 \times \frac{z_n}{z_1} \times \frac{V_n}{V_1}$$

The factor $\frac{z_n}{z_1}$ accounts for the contribution of CO_2 to

the greenhouse effect: as the CO_2 concentration increases, the emission height z_n increases, and therefore CD'_n also increases. This increases the heat received in the atmosphere, and thus its temperature, and is consistent with the findings [42,43,20]. As for

the factor $\frac{V_n}{V_1}$, it takes into account the influence of water vapor

V_n released by evaporation into the atmosphere in month n , which, according to, constitutes the main GHG: 60% on its own, 40% for other GHGs (with CO_2 alone accounting for 26%) [11]. This relationship (4) assumes an independence between the effects of water vapor and the emission height, which is validated by CMP6 simulations [13]. Therefore, when evaporation V_n increases, CD'_n increases. This is consistent with, for example, who stated that "the effective heat capacity of air at a given relative humidity is greater than the heat capacity of dry air" [27].

Calculation of Heat Transfer K_n

In this discrete global model, the average thermal effusivity of the Earth's surface (oceans at 70%, land at 25%, ice/snow at 5%) is estimated at $1340 \text{ JK}^{-1} \text{ m}^{-2} \text{ s}^{-1/2}$. This value, calculated as a weighted average of the effusivities of each surface (1590 for water, 800 for land, 500 for ice/snow), reflects the overall capacity of the atmospheric surface to exchange heat with the Earth's surface. For the reverse transfer, the effusivity of air ($6 \text{ JK}^{-1} \text{ m}^{-2} \text{ s}^{-1/2}$) is used. These values allow us to capture the fundamental asymmetry of heat exchange using the following formulas:

If $\theta_n \geq t_n$,

$$K_n = 6 \times T \times (\theta_n - t_n),$$

If $\theta_n < t_n$,

$$K_n = 1340 \times T \times (\theta_n - t_n)$$

where $T = 3,600 \times 24 \times 30 = 2,592,000 \text{ s}$ (the duration in seconds of a 30-day month).

The model thus takes into account not only the ocean surface, but also other surfaces of the globe. That said, seas and oceans play a much more significant role in the Earth's heat balance than their 70% cover suggests. For example, according to, they absorb approximately 93% of the additional heat since 1955, with the remainder shared between the continents, ice sheets, and the atmosphere [44,45]. Therefore, from now on, regarding surfaces, we will only refer to the UOS. This is, of course, a significant approximation, but more than sufficient for the mean-field approach used by our model.

Modeling the Forcing Directly caused by the Presence of CO_2

Studies by highlight an asymmetry: according to these authors, the forcing caused by the increase in CO_2 from 285 to 300 ppm over the period 1850-1950 is approximately 0.8 W/m^2 , while over the period 1950-2020, when the concentration increased from 300 to 420 ppm, this forcing is estimated at 2.2 W/m^2 [46,47]. Therefore, for the period 1950-2020, an additional ppm causes a forcing of 18 mW/m^2 , whereas for the period 1850-1950, it causes a forcing of 53 mW/m^2 . This represents a threefold difference between the two periods. This is true even though the raw figures do not directly reflect this transition, as other phenomena partially mask the effect of CO_2 . For example, cooling by sulfate aerosols or water vapor and cloud feedbacks [16]. Table 2, constructed from data from, confirms this asymmetry [48]. It compares the evolution of the forcing produced by an increase of one ppm, first between 0 and 280 ppm, then between 280 and 300 ppm. It again highlights the strong non-linearity between the concentration of CO_2 in the atmosphere and the associated greenhouse effect.

Table 2: Increase in Temperature and Forcing Per ppm for Two Different CO_2 Variations

CO_2 variation in ppm	Increase in temperature	Forcing
0 – 280	33 °C, i.e. + 0.11 °C per additional ppm	40 W/m^2 , i.e. + 0.14 W/m^2 per additional ppm
280 – 300	0.8 °C, i.e. + 0.04 °C per additional ppm	1 W/m^2 , i.e. + 0.05 W/m^2 per additional ppm

In both cases, the molecules are the same, yet the effects are very different, with ratios ranging from almost 1 to 3. The saturation phenomenon is somewhat similar to that of a sponge with water: when dry, it can absorb a lot of water, but as it becomes saturated, its capacity to absorb more and more water decreases until it is completely saturated. The saturation caused by the influx of CO_2 into the atmosphere for the absorption and re-emission of terrestrial infrared radiation is, of course, of a different type but has similar consequences: the infrared absorption bands of CO_2 become saturated as the CO_2 concentration increases, meaning that each new molecule arriving has a decreasing probability of absorbing and therefore re-emitting infrared photons into the atmosphere.

Consequently, the direct effect on atmospheric warming becomes less and less effective as the concentration $[CO_2]$ increases. To model this nonlinear forcing, the use of a geometric sequence or an exponential function seems appropriate. Since our model is discrete, we opted for the first solution, proceeding as follows: Let (u_m) be the geometric sequence modeling the temperature variations indicated in Table 2. It must therefore satisfy

$$\sum_{i=1}^{280} u_i = 33 \text{ and } \sum_{i=281}^{300} u_i = 0.8. \text{ This imposes } u_1 \approx 0.221$$

as the first term and $q \approx 0.994$ as the common ratio. Figure 2 shows the evolution of the forcing (in $^{\circ}C$) for each additional ppm as a function of the $[CO_2]$ concentration from 1 to 600 ppm.

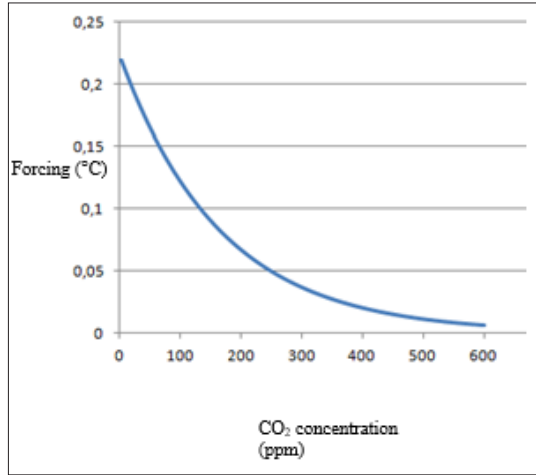


Figure 2 : Forcing (in $^{\circ}C$) Per Additional ppm as a Function of CO_2 Concentration (in ppm)

The temperature change $\Delta t_{[x \rightarrow y]}$ in $^{\circ}C$ caused by the forcing $\Delta W_{[x \rightarrow y]}$, corresponding to a change in the $[CO_2]$ concentration from x to y ppm, can therefore be estimated using this relationship:

$$(13) \quad \Delta t_{[x \rightarrow y]} = u_1 \times \frac{q^{x-1} - q^y}{1-q} \approx 0.221 \times \frac{0.994^{x-1} - 0.994^y}{1-0.994}.$$

And the forcing itself, $\Delta W_{[x \rightarrow y]}$, expressed in W/m^2 :

$$(14) \quad \Delta W_{[x \rightarrow y]} = \frac{40}{33} \times u_1 \times \frac{q^{x-1} - q^y}{1-q} \approx 0.267 \times \frac{0.994^{x-1} - 0.994^y}{1-0.994}$$

Trapping R_{an} of Infrared by the Troposphere

This is defined by $R_{an} = R1a_n + R2a_n$, where $R1a_n$ represents the portion originating from water vapor and $R2a_n$ represents the portion originating from CO_2 and other GHGs, which, for simplicity, are considered equivalent to CO_2 . The recurrence relations used to evaluate these two entities are the following formulas (15) and (16):

$$(15) \quad R1a_{n+1} = R1a_n \times \frac{V_{n+1}}{V_n}.$$

$$(16) \quad R2a_{n+1} = R2a_n + 3.600 \times 24 \times 30 \times \Delta W_{[CO2n \rightarrow CO2n+1]}$$

Equation (15) takes into account the increase in the greenhouse effect due to additional evaporation caused by global warming, which is the main factor controlling $R1a_n$. This is validated by the trends observed in the CERES data [49].

Equation (16) adds the current month's infrared trapping to the previous month's forcing calculated using equation (14).

Calculating Heat Transfer O_n

Atmospheric convection plays a central role in maintaining Earth's dynamic energy balance by ensuring vertical heat transfer from the surface to the upper troposphere [50]. Without this mechanism, the energy trapped by the greenhouse effect near the surface would lead to excessive heat accumulation. By transporting warm, humid air to higher altitudes, convection allows energy to be released as latent heat during condensation and then ultimately radiated into space as infrared radiation from the upper atmosphere. Thus, convection partially compensates for radiative trapping due to greenhouse gases by facilitating energy dissipation into space, maintaining a balanced overall energy budget [51]. The tropopause is the altitude at which regular convection ceases and stratification begins [52].

When the climate system is in equilibrium, the infrared heat radiating from the Earth's surface to the tropopause and then into space is insufficient to maintain this equilibrium, as a large portion of this energy is trapped in the lower layers by the greenhouse effect. Climate equilibrium is nevertheless maintained by the emission of infrared radiation into space, but this emission is only possible thanks to the vertical redistribution of heat by convection, which bypasses the radiative trapping of the greenhouse effect by transporting energy to atmospheric layers capable of radiating efficiently. Therefore, it is primarily the exchange of heat by convection from the lower atmosphere to the tropopause that allows energy equilibrium to be reached between what the Sun provides to the Earth in the form of radiation and what the Earth's climate system radiates back to the tropopause and into space. Convective processes are fundamental to the climate system, as they vertically redistribute heat and moisture, allowing the upper atmosphere to radiate energy into space and thus maintain planetary energy balance. For example, according to: “*Under the tropics, the vertical temperature profile is primarily determined by moist convection and approaches a moist adiabatic pattern. Warming is greater in the upper troposphere than in the lower troposphere, resulting in greater radiative emission into space and, consequently, negative feedback.*” (section 7.4.2.2) [16].

This is not necessarily the case in extratropical regions, but according to the same source, “*the tropical contribution is globally predominant*”, thus leading to negative global average feedback. Convective heat transfer represents the sum of sensible and latent heat transported by convection and is assessed as a global average based on decades of measurements and modeling. Table 3 summarizes data on this global convective heat transfer from several sources

Table 3: Evaluation of Convective Heat Flux over Several Periods and from Several Sources

Source	Period/ Scenario	Sensible heat (W/m ²)	Latent heat (W/m ²)	Total (W/m ²)	Context
[51]	1980- 1990	24	78	102	Reference energy balance
[53]	1990-2000	17	80	97	Updated with CERES satellite data
[54]	2000-2010	20	80	100	Re-evaluation with CERES data and reanalysis
[55]	1985-2010	18	82	100	Synthesis of observations (satellites + ground stations)
[56]	2000- 2018	19	81	100	CERES satellite data (18-year average)
[16]	2000-2020	20	80	100	CMIP6 model consensus and recent observations
[16]	2050 (projections)	21	90	111	Projection for a +1.5 °C warming (SSP1-2.6)
[16]	2050 (projections)	22	93	115	Projection for a +2 °C warming (SSP2-4.5)

We observe that for recent historical periods, the consensus is around an average value for convective heat flux close to 100 W/m². For future projections, an increase in convective heat flux is predicted, modulated by the projected increase in atmospheric temperature according to the scenario considered. Indeed, when this temperature increases with the rise in greenhouse gases, the two main agents of convection become even more active: First, the intensification of evaporation, closely linked to an increase in latent heat [57]. Second, the strengthening of radiative forcing, which traps more heat near the surface and increases vertical temperature gradients, thus stimulating convection [42]. Let O_n be this global heat transfer during month n . Given the above, and since for the model the evolution of temperature is directly linked to this radiative forcing and is closely related to that of evaporation (see rows 15 and 16 of Table 1), we use the following relationship to calculate O_n :

$$(17) \quad O_n = O_1 \times \frac{t_n}{t_1}$$

O_1 represents the convective heat transfer in January 1870, the beginning of the 1870-2025 period used for model validation, which is described in the following section. We take $O_1 = 95 \times T$ (where $T = 3,600 \times 24 \times 30$, the duration in seconds of a thirty-day month). The value of 95 W/m² was chosen so that our model would simulate on values consistent with the estimates shown in Table 3. Thus, for January 1993, the model obtains $O_{1477} \approx 100 \times T$; for January 2000, it obtains $O_{1561} \approx 101.3 \times T$; and for December 2025, the end of the historical data period used for model validation, it obtains $O_{1872} \approx 103.4 \times T$.

Note Regarding de t_n

Based on lines 2, 4, and 15 of Table 1 and equation (12), the temperature t_n of the lower atmosphere satisfies the following equation:

$$(18) \quad t_{n+1} \times t_n = t_n^2 + \frac{T_n}{CD'1 \times \frac{z_n}{z_1} \times \frac{\theta_n}{\theta_1 t_1}}$$

This equation is an explicit discretization equivalent to equation

$$(6), \text{ with } \beta(y) = y^2 \text{ and } Q_{\text{ext}}(n) = \frac{T_n}{CD'1 \times \frac{z_n}{z_1} \times \frac{\theta_n}{\theta_1 t_1}}.$$

This aligns with the model's consistency with the physical theory of heat and further demonstrates the value of introducing the CTD.

Model Validation

We use historical global mean surface temperature (Sourced from the American NOAA website) data to calibrate certain constants in the relationships in Table 1 and to validate the model over the period 1870–2025. Graph 1 in Figure 3 shows a simulation of the global mean surface temperature anomalies, its theoretical curve, and the two curves defining the 95% confidence interval. Graph 2 shows the historical anomaly curve, the theoretical anomaly curve of t_n , and the two curves defining the 95% confidence interval. Graph 3 shows the historical anomaly curve (blue) and that of a simulated anomaly of t_n (red). Finally, Graph 4 juxtaposes all the previous curves from Figure 3. The constants involved were calibrated with observations, specifically to ensure that the regression line of the theoretical anomaly curve (i.e., obtained by the model after neutralizing the random coefficients used to account for natural variability) of t_n and that of the historical anomalies are practically identical. We observe excellent consistency between the model simulations and the historical data.

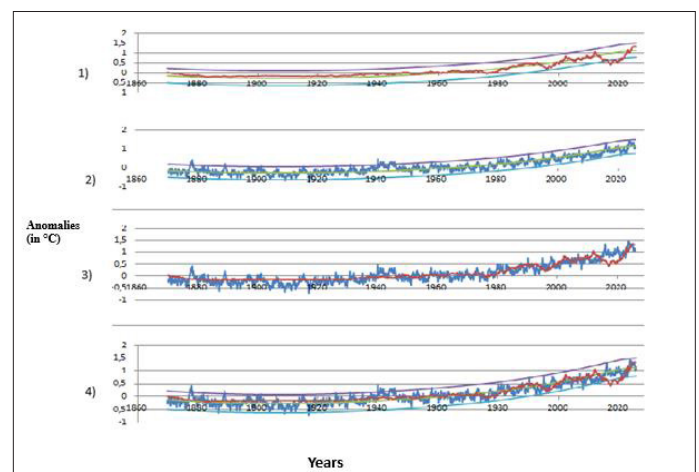


Figure 3: 1) Simulation, Theoretical Curve, and 95% Confidence Interval Curves. 2) Historical Anomalies, Theoretical Curve, and 95% Confidence Interval Curves. 3) Simulation (red) and Historical Anomalies (blue). 4) Juxtaposition of the Previous Curves.

Considered Scenarios and Simulations Obtained

Considered Scenarios

Figure 4 illustrates the six scenarios S₁, S₂, S₃, S₄, S₅, and 280 ppm considered for CO₂ concentrations over the 2026–2200 model forecast period. The six curves corresponding to this period are preceded by the curve obtained from observed data for the period 1870–2025 (Updated for 2021–2025) and supplemented by historical estimates for the earlier period 1870–1955 [16]. The scenarios S₁, S₂, S₃, S₄, S₅, and 280 ppm are arranged from the least stringent in terms of CO₂ emission reductions (S₁) to the most stringent (280 ppm). Even though the correspondence is only approximate, for the common period studied (2026–2100), we can compare them with the Shared Socio-economic Pathways (SSP) scenarios described in [10]. Again, ranked by increasing GHG emission requirements, these are listed as follows: SSP5-8.5, SSP3-7.0, SSP2-4.5, SSP1-2.6, and finally SSP1-1.9.

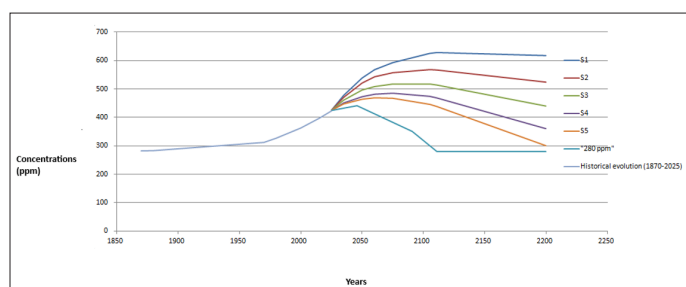


Figure 4: Evolution of CO₂ Concentrations According to the Six Scenarios Considered for the Period 2026–2200, Preceded by that of the Historical Period (1870–2025)

Simulations Obtained by the Theoretical Model

The adaptation of the model for the historical period 1870–2025 and its connection with the future period 2025–2200, used for forecasting, followed the three principles P1, P2, and P3 below. P1: The constants used by the model to simulate anomalies for the period 1850–2025 are the same as those used for the period 2025–2170 (see Table 1). P2: The calibration of the initial temperatures (θ_1 and t_1 in January 1870), the initial CTDs (CD_1 and CD'_1 in January 1870), and the initial values of the other parameters involved was performed primarily by calibration based on observations, or, failing that, on good estimates. P3: The simulations for the period 2026–2200 use as initial values the theoretical final values obtained by the model for the historical period 1870–2025. Figure 5 presents the simulations of the t_n anomalies obtained by the theoretical model for the period 1950–2200 for the six scenarios considered. Table 4 shows the evolution of the mean temperature t_n obtained by the model according to these same scenarios and for five different periods. The results are given as 95% confidence intervals (95% CI) obtained from a series of 250 simulations.

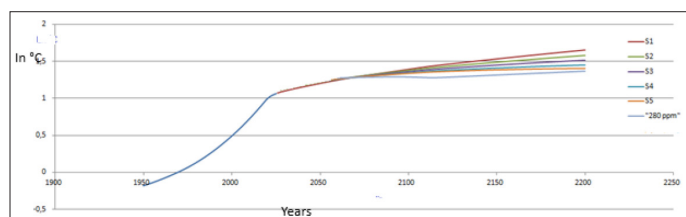


Figure 5: Simulations of the Theoretical Temperature t_n Anomalies (period 1950–2200) for Scenarios S₁, S₂, S₃, S₄, S₅, and 280 ppm (Curves in the same Order from Top to Bottom on the Right of the Graph).

Table 4 : Evolutions of t_n Obtained by the Model for the Six Scenarios Considered and Over Five different Periods. The 95% Confidence Intervals are Specified for each

Period	S ₁	S ₂	S ₃	S ₄	S ₅	280 ppm
1870-2200	1.65±0.40 °C	1.57±0.40 °C	1.49±0.40 °C	1.42±0.40 °C	1.37±0.40 °C	1.32±0.40 °C
1870 - 2100	1.44±0.35 °C	1.41±0.35 °C	1.38±0.35 °C	1.35±0.35 °C	1.33±0.35 °C	1.25±0.35 °C
1870-2025	1.16±0.28 °C	1.16±0.28 °C	1.16±0.28 °C	1.16±0.28 °C	1.16±0.28 °C	1.16±0.28 °C
2026-2200	0.48±0.30 °C	0.40±0.30 °C	0.33±0.30 °C	0.26±0.30 °C	0.21±0.30 °C	0.15±0.30 °C
2026-2100	0.28±0.20 °C	0.24±0.20 °C	0.21±0.20 °C	0.18±0.20 °C	0.16±0.20 °C	0.09±0.20 °C

We observe a clear but slight difference depending on the scenario: For the period 2026–2200, we see differences of approximately one-tenth of a degree Celsius between two adjacent scenarios. The same is true for the period 1870–2200. For the period 2026–2100, the differences are reduced to just a few hundredths of a degree Celsius between two adjacent scenarios. The same is true for the period 1870–2100. Obviously, there is no difference depending on the scenario for the historical period 1870–2025. As for the evolution of 1.16 ± 0.28 °C (95% CI: [0.88 - 1.44 °C]) obtained by the model for this period, it is consistent with the 90% confidence intervals (90% CI) indicated by the IPCC for this period: "The global surface temperature increased by 0.99 °C (90% CI: [0.84 - 1.10 °C]) between 1850–1900 and the first two decades of the 21st century (2001–2020) and by 1.09 °C (90% CI: [0.95 - 1.20 °C]) between 1850–1900 and 2011–2020" [16]. Table 5 presents the central value and 95% confidence interval for the evolution over the period 1870–2200 for the five parameters simulated by the model: atmospheric temperature t_n , UOS temperature θ_n , cloud cover percentage cl_n , ocean stratification percentage s_n , and mixing layer depth S_n . This is provided for each of the scenarios considered.

The central values are those obtained by the theoretical model, and the 95% confidence intervals were obtained for each parameter and each scenario using 250 simulations. Regardless of the scenario, we observe that the amplitudes of the changes in θ_n are approximately four times smaller than those of t_n . This is primarily due to the model's consideration of the very different thermal capacities of the two environments involved, the cooling of the UOS due to evaporation, and the mixing layer depth. That said, the evolutions of t_n and θ_n are closely linked, as clearly shown in Figure 6. This is due to the deep interdependence of equations (9) and (10), which allow us to calculate C_n and T_n , and therefore θ_n and t_n . Another point to note about these simulations is that the variations of the parameters cl_n , s_n , and S_n are directly dependent on those of θ_n through the relationships shown in rows 7, 10, and 18 of Table 1. Therefore, cl_n , s_n , and S_n evolve in a very similar manner to θ_n .

Table 5: 95% Confidence Intervals for the Five Simulated Parameters Over the Period 1870–2200 According to the Six Scenarios Considered

Parameters	S ₁	S ₂	S ₃	S ₄	S ₅	280 ppm
Temp. atm. t_n	1.65 ± 0.4 °C	1.56 ± 0.4 °C	1.49 ± 0.4 °C	1.42 ± 0.4 °C	1.37 ± 0.4 °C	1.31 ± 0.4 °C
Temp. UOS θ_n	0.40 ± 0.2 °C	0.37 ± 0.2 °C	0.34 ± 0.2 °C	0.32 ± 0.2 °C	0.31 ± 0.2 °C	0.27 ± 0.2 °C
Cover cloud cl_n	0.43 ± 0.1 %	0.40 ± 0.1 %	0.38 ± 0.1 %	0.36 ± 0.1 %	0.34 ± 0.1 %	0.30 ± 0.1 %
Strat. s_n	1.96 ± 0.1 %	2.14 ± 0.1 %	1.73 ± 0.1 %	1.62 ± 0.1 %	1.55 ± 0.1 %	1.35 ± 0.1 %
Sinking S_n	39.37 ± 8 m	36.89 ± 8 m	34.54 ± 8 m	32.42 ± 8 m	31.00 ± 8 m	27.02 ± 8 m

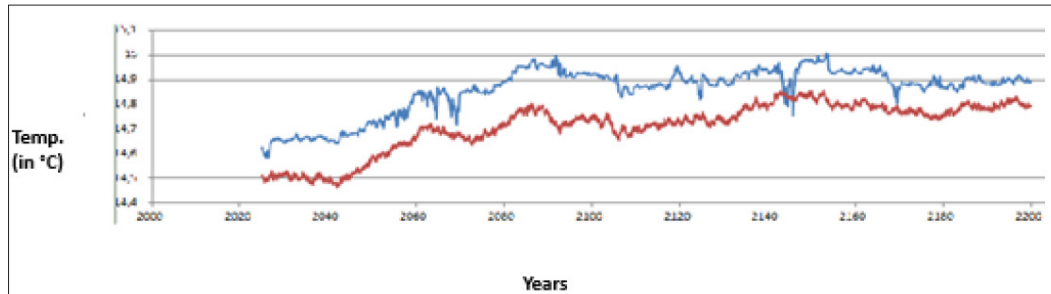


Figure 6: Simulation of t_n (Blue Curve) and θ_n (Red Curve) Over the Period 2026-2200 and within the Framework of Scenario S₄

Tests on Model Constants

To determine their relative importance in the evolution of the five simulated climate parameters, we conducted four series of tests on the constants used in the model's construction. For this purpose, we used scenario S₅.

Tables 6, 7, 8, and 9 show the variations (denoted Δt^i , $\Delta \theta^i$, ΔS^i , Δs^i and Δcl^i) of the five theoretical parameters t_n^i , θ_n^i , S_n^i , s_n^i and cl_n^i , based on these tests, for the entire period 1870–2200.

Test 1: Here we test λ , the coefficient that essentially determines the sinking of the mixing layer S_n using the relationship $S_{n+3} = S_{n+2} + \lambda (\theta_n - \theta_{n-1}) + L_{Th}$ from row 18 of Table 1, with two values:
- $\lambda = 0$ (no sinking regardless of the temperature increase in the UOS).
- $\lambda = 200$, twice the value used in the model.

We observe that the depth of the mixing layer interacts quite strongly with the other parameters. If it could not sink, temperatures and other parameters would increase more rapidly; conversely, if it could sink more rapidly than the calibration on the data indicated, this would reduce the variations of the other parameters. In other words, the mixing layer has a fairly strong buffering and reducing capacity on variations in the major global climate parameters.

Table 6: Tests on λ Values

λ	Δt^i (°C)	$\Delta \theta^i$ (°C)	ΔS^i (m)	Δs^i (%)	Δcl^i (%)
$\lambda = 100$ (model)	1.37	0.31	31	1.55	0.34
$\lambda = 0$	1.42	0.36	0	1.81	0.40
$\lambda = 200$	1.34	0.28	56	1.39	0.31

Test 2: This time, we test the coefficient γ , which governs the evolution of atmospheric albedo and is strongly correlated with cloud cover by the relationship $a_n = \gamma \times (68 + cl_n) \times 0.01$ in row 5 of Table 1:
 $\gamma = 0$ (no atmospheric albedo effect ; solar radiation reaches the ground entirely).
 $\gamma = 1$ (the atmospheric albedo effect corresponds exactly to cloud cover).

The influence of γ on the variations of the other parameters is similar to that of the constant λ : according to our model, atmospheric albedo has a real regulating effect on the climate, although slightly less significant than that of the depth of the mixing layer.

Table 7: Tests on γ Values

γ	Δt^i (°C)	$\Delta \theta^i$ (°C)	ΔS^i (m)	Δs^i (%)	Δcl^i (%)
$\gamma = 0.44$ (model)	1.37	0.31	31	1.55	0.34
$\gamma = 0$	1.39	0.36	36	1.82	0.40
$\gamma = 1$	1.35	0.23	24	1.20	0.26

Test 3 : We test the coefficient β , which governs the evolution of cloud cover (cl_n), using the relationship $cl_{n+1} = cl_n + \beta (\theta_{n+1} - \theta_n)$ from row 7 of Table 1:

- $\beta = 0$ (cl_n does not vary, regardless of atmospheric temperature).

- $\beta = 10$ (the variation of cl_n is then very sensitive to temperature increases).

This time, we observe no apparent influence on the other parameters. Therefore, according to our model, a 3% increase or decrease in global cloud cover does not significantly change the climate. It seems that the cooling "*parasol effect*" of clouds is almost exactly offset by the warming greenhouse effect, which is linked to air humidity and therefore partly to clouds.

Table 8: Tests on β Values

β	Δt^t (°C)	$\Delta \theta^t$ (°C)	ΔS^t (m)	Δs^t (%)	Δcl^t (%)
$\beta = 1,1$ (model)	1.37	0.31	31	1.55	0.34
$\beta = 0$	1.37	0.31	31	1.55	0
$\beta = 10$	1.37	0.31	31	1.55	3.10

Test 4 : Finally, we test the coefficient μ that governs the evolution of stratification s_n using the relationship $s_{n+1} = s_n + \mu (\theta_{n+1} - \theta_n)$ from row 10 of Table 1:

$\mu = 0$ (s_n does not vary, regardless of the UOS temperature).

$\mu = 10$ (the variation of s_n is then very sensitive to increases in UOS temperature).

As with β , we observe no significant impact of μ on the other parameters. The UOS temperature θ_n interacts directly with stratification, but stratification itself has no apparent influence on the other major global climate parameters. This is despite the relationship

$K'_n = \frac{\theta_n K'_1}{\theta_1 s_n}$ (row 9 of Table 1) where it is involved in the calculation by the model of the transfer to the deep ocean layers.

Table 9: Tests on μ Values

μ	Δt^t (°C)	$\Delta \theta^t$ (°C)	ΔS^t (m)	Δs^t (%)	Δcl^t (%)
$\mu = 5$ (model)	1.37	0.31	31	1.55	0.34
$\mu = 0$	1.37	0.31	31	0	0.34
$\mu = 10$	1.37	0.31	31	3.11	0.34

Discussion

Comparison with the results described in [22]

As clearly shown in Table 4, the simulations provided by the model for the periods 2026–2100 and 2026–2200 demonstrate little sensitivity to the six scenarios S1, S2, S3, S4, S5 and 280 ppm in which they are placed. These scenarios, however, differ significantly with regard to the projected CO₂ changes (see Figure 4). In contrast, and as Figure 7 clearly indicates, the atmospheric temperature changes presented in by the first working group of the IPCC are much more sensitive to the GHG emission scenarios considered [16].

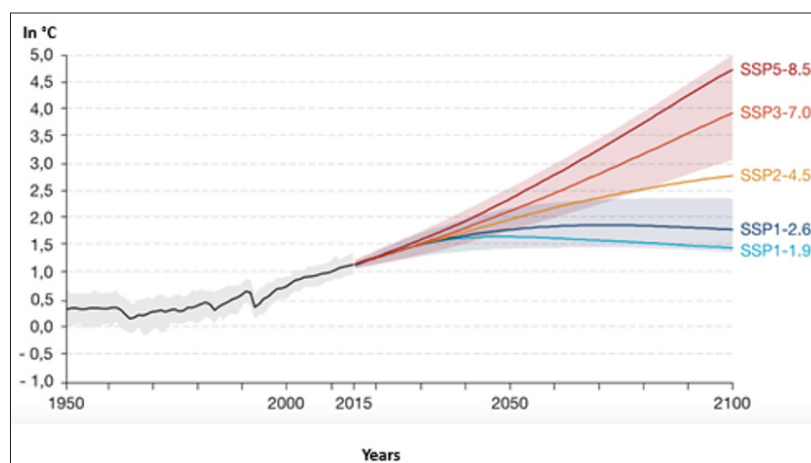


Figure 7: Temperature Changes According to the IPCC Scenarios. Source: [16].

This is indeed what the Technical Summary by indicates: "In the 21st century, a 2°C increase in global surface temperature compared to 1850–1900 will be exceeded in the case of SSP5–8.5 and SSP3–7.0; it is extremely likely that this level will be exceeded in the case of SSP2–4.5, but it is unlikely in the case of SSP1–2.6 and extremely unlikely in the case of SSP1–1.9" [21]. For the medium term (2041–2060), exceeding the 2°C warming threshold is very likely in the case of SSP5–8.5, likely in the case of SSP3–7.0, and more likely than unlikely in the case of SSP2–4.5. Table 10 directly illustrates this difference between the results obtained by the two approaches:

Table 10: For Scenarios that are Quite Similar in Terms of CO₂ Emissions, Comparison of Atmospheric Temperature Changes Predicted by [22] and those Simulated by Our Model

Scenarios used for simulations by the model	S1	S2	S3	S4	S5	280 ppm
IC 95% for the evolution over the period 1870-2100	1.44 °C [1.1 - 1.8]	1.40 °C [1.06 - 1.76]	1.38 °C [1.03 - 1.73]	1.35 °C [1.00 - 1.70]	1.33 °C [0.98 - 1.68]	1.25 °C [0.90 - 1.60]
SSP scenarios	SSP5-8.5	SSP3-7.0	SSP2-4.5	SSP1-2.6	SSP1-1.9	
IC 90% for the evolution of atmospheric temperature to the horizon 2081-2100 based on the average over the period 1850-1900.	4.4 °C [3.3 - 5.7]	3.6 °C [2.8 - 4.6]	2.7 °C [2.1 - 3.5]	1.8 °C [1.3- 2.4]	1.4 °C [1.0 - 1.8]	

For the most demanding scenarios (SSP1-2.6 and 1.9 on the one hand, and S4, S5, and 280 ppm on the other), the results obtained by the two approaches are quite similar. However, we observe much greater sensitivity for the SSP5-8.5, SSP3-7.0, and even SSP2-4.5 scenarios than for the S1, S2, and S scenarios. For the former, the projected increase ranges from 3°C to over 4°C by 2100, with significant variations between each scenario. For the latter, the projected increase is always less than 2°C, and the differences between them are on the order of a tenth of a degree °C.

This difference is also reflected in the estimation of the Transient Climate Response (TCR). Specifically, the TCR is the short-term sensitivity to a doubling of carbon. To calculate it, a conventional assumption is generally made of a steady increase of 1% in CO₂ per year, resulting in a total doubling time of 70 years. The IPCC's predicted TCR is 1.8 ± 0.6 °C, which falls within the 90% confidence interval [1.2 °C; 2.4 °C] ([16]). Scenario S1 achieves a doubling, as it starts in January 1870 with a CO₂ concentration of 282 ppm and reaches double that (564 ppm) in January 2059. The model then predicts a net temperature increase of 1.3 ± 0.3 °C, corresponding to a 95% confidence interval of [1.0 °C; 1.6 °C]. However, this doubling would then take 189 years, which is significantly longer than the required 70 years. If we recalculate starting from the historical concentration of 314 ppm in 1971 and stopping at its doubling according to scenario S1 in 2111, the model then predicts a temperature increase of 1 ± 0.3 °C, therefore a 95% confidence interval (CI) of [0.7 °C; 1.3 °C]. This time, for a period of 140 years. Finally, if we theoretically test the model for a doubling based on an annual increase of 1% and therefore a total duration of 70 years, we find that the 90% CI of the TCR falls within [0.7 °C - 1.1 °C]. Therefore, this is significantly lower than the TCR specified in whose 90% CI is [1.2 °C; 2.4 °C] [10].

Such differences in results must be analyzed. This first requires understanding the meaning of the words used. When the Technical Report uses the terms *probable*, *improbable*, *extremely probable*, etc., it does so within this specific context: “*Probability is a quantitative measure of confidence in an outcome, expressed in probabilistic terms (e.g., according to the statistical analysis of observations or model results, or both, according to the expert opinion formulated by the team of authors or from a systematic quantitative review of expert opinions, or both)*” [21]. This is not a purely mathematical definition (the one used by that science), since a significant degree of subjectivity (expert opinions) is involved. Furthermore, the fact that a methodical quantitative study—in other words, statistical analysis—is conducted on these subjective opinions to conclude that a given event is probable, very probable, improbable, or highly improbable, etc., does not, however, confer upon the use of these terms the meaning one would expect in the mathematical theory of probability.

The reality is this: although implemented using powerful computers, GCMs are incapable of providing, within a reasonable timeframe, a sufficient number of simulations to, for example, obtain confidence intervals as we were able to do with our model. Therefore, the simulations obtained by the GCMs of major climate institutes are collected and then analyzed by experts. For example, the French models (IPSL and CNRM-Cerfacs) that have participated or are participating in the international Climate Simulation Comparison Exercises (CMIP) are independently examined for months at a time by teams from around the world who analyze all available CMIP simulations [58].

Based on this data collection and these opinions, each of the resulting forecasts is qualified using probabilistic terminology, which doesn't precisely correspond to the probabilistic framework as defined in mathematics. Of course, this doesn't call into question the professionalism of this practice, which is carried out with the necessary care and expertise. But even if the conclusions reached have a certain value, a more or less significant degree of subjectivity cannot be ruled out, whether at the intermediate level of the assessment or at its final stage, when it comes to drafting the reports for the IPCC.

The ability to easily obtain the 95% confidence intervals for the simulated parameters is therefore a powerful advantage of our model compared to general cyclic models (GCMs), as it eliminates any subjectivity in the probabilistic results. Furthermore, section 4 clearly demonstrated the model's excellent ability to accurately represent historical climate trends. Finally, the tests performed on the relevant constants, as presented in section 5 did not reveal any glaring anomalies that would suggest this model suffers from a major flaw. Therefore, even if they originate from leading international climatological experts, external results that partially contradict our own are not sufficient in themselves to call into question the robustness and reliability of this EBM, nor the accuracy of the results it has produced. Until proven otherwise, these results are well-established, and we therefore take them fully into account. certainly, has shortcomings, notably:

- Other GHGs (CH₄, N₂O, etc.) are simply treated as equivalent to CO₂. This approximation likely leads to a slight underestimation of the total radiative forcing caused by GHGs.
- The albedo calculation does not account for potential ice melt.
- Methane releases from permafrost exposed by warming near the poles are not taken into account [59].
- The same applies to the gradual, long-term release of heat accumulated in the deep ocean [16].

Therefore, although the simulations covering the 1870–2025 period are validated by historical observations, our forecasts for the year 2200 likely underestimate the actual outcome, particularly

regarding scenarios S1, S2, and S3. Further work will thus be required to refine the model. However, in our view, even with an improved model, the temperature forecasts we obtain will remain significantly lower than those presented in the report for scenarios SSP5-8.5, 3-7.0, and 2-4.5 [16].

Which Scenario should we Aim for?

Over the period 1870-2200, the model indicates that between the least severe scenario (S1) and the most severe (280 ppm), there is less than four-tenths of a degree Celsius difference in the evolution of t_n and less than two-tenths for that of θ_n . This might suggest that global efforts to reduce CO₂ emissions would not be truly effective. This hasty conclusion is undoubtedly a serious error, particularly for the following reasons:

- As noted in the previous section, the model in its current state does not account for certain relatively significant climatic phenomena. This likely leads to an underestimation of predicted temperatures compared to future reality, particularly in scenarios requiring less stringent GHG emission reductions. Consequently, the actual sensitivity—depending on the scenario—is likely greater than what our current model predicts. An average deviation of a few tenths of a degree Celsius from the global average can translate into a much larger deviation at the regional level. Studies show that land is warming faster than the oceans, with a warming rate two to three times greater [21]. Polar and mountainous regions are experiencing even more pronounced warming, which accelerates ice melt and the release of greenhouse gases trapped in permafrost [60]. These mechanisms create feedback loops that amplify warming far beyond the global average. Our model does not account for the possibility of such phenomena, and this is a weakness that must be kept in mind for its proper use.
- A few tenths of a degree more or less significantly alter the energy and moisture present in the atmosphere. According to the Clausius-Clapeyron law, a 1°C increase allows the atmosphere to retain approximately 7% more moisture, which intensifies extreme precipitation and droughts [61]. This translates into an increase in the violence of climatic events, such as hurricanes, whose intensity can increase by 10 to 20% with even moderate warming [62,63]. And as is well known, water vapor has a strong greenhouse effect.
- The oceans, which absorb approximately 30% of the CO₂ emitted by human activities, are already suffering from the resulting acidification. This leads to the death of coral reefs, a decline in fish populations, and major disruptions to marine ecosystems [64,65]. Reducing CO₂ emissions is therefore essential to limit these impacts and preserve marine biodiversity.
- Finally, every reduction in emissions, however small, has tangible benefits: improved air quality, reduced economic costs associated with natural disasters, and preservation of living conditions for future generations [66,16].

All these reasons combine to recommend a scenario that is truly demanding in terms of greenhouse gas emissions. But at the same time, the constraints its implementation would entail must not be too excessive. Of the six proposed in this study, scenario S5 undoubtedly offers the best compromise. Considering, in particular, the four points specified above, scenarios S1, S2, S3, and S4 are indeed insufficient in terms of CO₂ reduction. As for the 280-ppm scenario, it is undoubtedly far too difficult to implement, especially due to the overly severe economic, industrial, and therefore human impact it would impose.

Conclusion

This work makes original and structuring contributions to climate modeling. It proposes a theoretical and numerical validation of the Principle of Radiative Invariance, enabling a simplified analytical estimation of CO₂ emission levels, whereas classical approaches rely on complex numerical calculations. It introduces a precise definition of Dynamic Heat Capacity comprehensively integrating the effects of turbulence, currents, and energy exchanges in the atmosphere and oceans, without resorting to empirical parameterizations. Using a new method for studying climate time series (the MAL) and implicitly the mean field method, the model built with these elements is validated by comparison with historical data from 1870 to 2025. Combining MAL with CTD and PIR offers a unique approach to modeling the climate in a deterministic and accessible way, while still capturing complex dynamics. By remaining consistent with fundamental physical principles, it enables policymakers and non-specialists to obtain readily understandable projections.

The model thus possesses significant strengths regarding the reliability of global forecasts—strengths that make it far superior in this crucial respect to earlier EBMs such as the one presented and to GCMs [30]. However, unlike the latter, it cannot predict regional climate differences, seasonal variations, monsoons, extreme events (e.g., heatwaves, hurricanes), and so on. Clearly, GCMs will remain irreplaceable in many respects. Yet, the predictive power of the model presented here—combined with the simplicity of its underlying principles and numerical implementation—makes it a highly valuable tool for understanding and forecasting the global climate future. It thus provides long-term forecasts (to 2200) in a much more deterministic than stochastic manner. The model presented here has the inherent limitations of EBMs, and it is clear that in many respects, GCMs will remain irreplaceable. But its predictive power, combined with its simplicity in terms of its fundamentals and numerical programming, makes it a highly relevant tool for understanding and forecasting the future of the global climate. By offering a deterministic and lightweight alternative to current complex models, this work contributes to informing debates on climate mitigation strategies, while also paving the way for collaborations to refine the results already obtained.

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