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Mathematical Modeling of the Dynamics between the Ozone Layer, Infrared Rays and Holes in the Ozone Layer

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ABSTRACT

This paper analyzes the environmental situation of our planet, especially in relation to global warming. It discusses the dynamics between the ozone layer, infrared rays and holes in the ozone layer. It also highlights the dangers that the decrease in the density of the ozone layer represents due to the unconscious activity of the planet's biggest polluters. It also highlights the problems that global warming poses to humanity, especially the melting of the polar ice caps. A model was created that simulates this dynamic using a system of differential equations. A qualitative analysis of the system's trajectories was performed, and conclusions were drawn about the future situation of this process.

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Introduction

Almost all of society is concerned about the future consequences for our planet due to climate change. These are caused by a set of changes in a short period, and in almost all directions. Such as temperature, air pollution, and other aspects that affect the quality of life on the planet. All of this is caused by natural and human factors, and can affect the environment, the economy and human health [A].

There are many examples of natural disasters that can be given as a result of climate change, such as floods, landslides and other calamities caused by this situation; all of this has taken the toll of countless lives, natural disasters and affected the economies of many countries.

The melting of ice is a phenomenon caused by global warming, which is a result of the increase in the greenhouse effect; the melting of glaciers is another consequence of climate change and poses serious threats to the planet; many countries are at risk of losing part of their territories due to being taken over by ocean waters. All of these extreme events are increasingly intense and frequent climate phenomena due to global warming, which can lead to the increase in many diseases due to the release of viruses and bacteria [B, 1].

The ozone layer filters the infrared and ultraviolet rays from the sun, as well as type B ultraviolet rays (UV-B), which are harmful to living beings; this is why this dynamic between these two concentrations is interesting. The ozone layer and the greenhouse effect are two natural phenomena that protect the Earth. The greenhouse effect maintains a temperature suitable for life. However, the intensification of the greenhouse effect by man threatens the climate [C]. The greenhouse effect guarantees the necessary heat; without it, life on our planet would be impossible, as the cold would prevent survival [2-5].

The hole in the ozone layer is a region where there is a low concentration of this gas in the stratosphere, below the level that would be considered normal. The ozone layer is extremely important for planet Earth, since it filters ultraviolet (UV) rays emitted by the Sun. The problem of the hole in the ozone layer and the climate crisis are similar: both are global and vital. Just as we act to protect the ozone layer, we can face the climate crisis. The international community has recognized the need to act together to ensure the quality of life on our planet [D].

Global oblivion involves the process of pollution, because without pollution control there is no possibility of a healthy and comfortable life for the whole society. Cachapuz A expresses the need to contribute positively: "It was necessary for the avalanche of information of the most diverse types and by the most diverse means with which we are confronted to better understand that information is only a necessary condition of knowledge [6]. Perhaps the most perverse thing is that the construction of knowledge is as easy as the current access to information" [7].

Footnote

[A] https://www.google.com/search?q=mudan%C3%A7as+clim%C3%A1ticas+resumo&sca_esv=8a5f3c529fc99551&sxsrf=AHTn8zrZ7qQpeHM- YIfOmiJhlf7NXR3s5Apercentage3A1740068027060&ei=u1S3Z. Lit on 02/2025

[B] tps://www.google.com/search?q=o+descongelamento+devido+as+mudan%C3%A7 as+clim%C3%A1ticas&oq=o+descongelamento+devido+as+mudan%C3%A7as+clim%C3%A1ticas&aqs=chrome.69i57j33i160l2.37607j0j7&sourceid=chrome&ie=UTF-8. Lit on 02/2025

[C] https://www.google.com/search?q=camada+de+ox%C3%B4nio+e+raios+infraverm elhos.&oq=camada+de+ox%C3%B4nio+e+raios+infravermelhos.&aqs=chrome..69i57j0i512i546j0i751j0i512i546l3.135. Lit on 02/2025

[D].https://www.google.com/search?q=os+buracos+e+a+camada+de+ozonio&oq=os+b uracos+e+a+camada+de+ozonio&aqs=chrome..69i57j0i22i30j0i512i546j0i546i649j0i7. Lit on 02/2025

In the work entitled Ecological Literacy: a discussion on the philosophical and sociological aspects of environmental education, the following considerations are made: "A human civilization and its consumer culture, driven in recent years by the advent of technology, has led to a devastating process of its fundamental ecosystems and, consequently, to a crisis in society, such as economic, social, educational and why not philosophical" [8,9].

The severe depletion of the ozone layer will lead to an increase in diseases, taking away comfort and quality of life, such as melanomas, skin cancer, ocular cataracts and suppression of the immune system in humans and other species. Thanks to some products generated by man and called halocarbons; the destruction of the ozone layer has accelerated compared to its natural rate. This causes the thinning of the layer and the generation of the well-known ozone holes, with which the Earth loses protection against solar radiation [10,11].

The problem of reducing holes in the ozone layer involves the problem of eliminating pollution and reducing polluting agents such as excess vehicles, large factories in cities, among others [12,13].

The process of eliminating liquid pollution in oxidation ponds uses a system of differential equations with constant coefficients, but in general the additions of pollution occur periodically, making the model non-autonomous and mainly periodic in relation to time; Batista E, et al. and Valiente A, et al. he simulation is done through non-autonomous and particularly periodic systems; this is a specific example of a compartmental model [14,15].

Different real-life problems are treated using equations and systems of ordinary differential equations, not only environmental problems, but also others related to life, comfort, diseases and functions of the human body systems [3,4,16-18].

Research into environmental problems, particularly those related to global warming, which involves the structural behavior of the atmosphere surrounding the Earth. The objective of this work is to develop a model that simulates the dynamics between the ozone layer and infrared rays and the holes in the ozone layer [19].

Formulation of the Model

To create a model that simulates the environmental situation that occurs on our planet, it is necessary to introduce certain functions that represent the values of the magnitudes used at a given time; in addition, certain principles are needed to obtain the variations of each of them.

The variation in the density of the ozone layer is inversely proportional to its density and decreases with the action of infrared rays and increases proportionally to the density of the holes; in the same way, with the increase in the density of the ozone layer, the infrared rays decrease and increase in relation to their own density; However, the density of the holes decreases in relation to the density of the ozone layer, according to infrared rays, and decreases proportionally to its own concentration.

The functions are also introduced,

$\tilde{x}_1$ Is the total density of the ozone layer at the moment t.

$\tilde{x}_2$ Is the total density of infrared rays at the moment t.

$\tilde{x}_3$ Is the total density of holes in the ozone layer at the moment t.

The region will also be considered.

$$V\alpha=\{(\bar{x}_1,\bar{x}_2,\bar{x}_3)\in x_1^2+x_2^2+x_3^2<\alpha^2\}$$

Which represents the open set of admissible values of the oxone layer, infrared rays and holes in the oxone layer.

Unknown functions are further defined x_i (i=1,2,3) by the following expression $x_i=\tilde{x}_i-\bar{x}_i$ (i=1,2,3); here it is verified that if $x_i\to 0$, then $\tilde{x}_i\to\bar{x}_i$ (i=1,2,3), which would constitute the main objective of this work. In this way the model will be given by the following system of equations,

$$\begin{cases} x_1' = -a_1x_1 - a_2x_2 + a_3x_3 + X_1(x_1,x_2,x_3) \\ x_2' = -b_1x_1 + b_2x_2 - b_3x_2^2 + X_2(x_1,x_2,x_3) \\ x_3' = -c_1x_1 + c_1x_2 - c_2x_3 + X_3(x_1,x_2,x_3) \end{cases} \quad (1)$$

Where the coefficients have the following meaning.

a_1 represents the decrease in the density of the oxone layer in relation to its own concentration

a_2 represents the decrease in the density of the ozone layer in relation to the density of infrared rays.

a_3 represents the increase in the density of the oxone layer in relation to the density of the holes in the oxone layer.

b_1 represents the decrease in the density of infrared rays in relation to the density of the ozone layer.

b_2 represents the increase in the density of infrared rays in relation to its own density

b_3 represents the increase in the density of infrared rays in relation to the density of holes in the ozone layer.

c_1 represents the reduction in the density of holes in the oxone layer in relation to the density of the oxone layer.

c_2 represents the increase in the density of holes in the ozone layer in relation to the density of infrared rays.

c_3 represents the decrease in the density of holes in the ozone layer in relation to its own density

The functions $X_i(x_1,x_2,x_3)$, (i=1,2,3) are disturbances not inherent to the process, which may at a given moment produce certain alterations that could change the final state of the dynamics presented; from a mathematical point of view they represent higher order infinitesimals, in general they admit the following development in series of potentials

$$X_i(x_1,x_2,x_3,) = \sum_{|p|\geq 2} x_i^p x_1^{p_1} x_2^{p_2} x_3^{p_3} \ (i=1,2), |p| = p_1+p_2+p_3$$

The characteristic matrix equation of the linear part of system (1) has the following form,

$$\begin{vmatrix} a_1-\lambda & -a_2 & -a_3 \\ -b_1 & b_2-\lambda & b_3 \\ -c_1 & c_2 & -c_3-\lambda \end{vmatrix} = 0$$

This expression is equivalent to,

$$\lambda^3+n_1\lambda^2+n^2\lambda+n_3=0 \quad (2)$$

Where,

$n_1=a_1+c_3-b_2$

$n_2=a_1c_3+a_3c_1+a_1b_2-b_2c_3-a_2b_1$

$n_3=a_3b_1c_2-a_2b_1c_3-a^3b_2c^1$

The Hurwitz matrix associated with the polynomial on the first member of equation (2) has the form,

$$H=\begin{bmatrix} n_1 & 1 & 0 \\ n_3 & n_2 & n_1 \\ 0 & 0 & n_3 \end{bmatrix}$$

The conditions of Hurwitz's theorem allow us to arrive at concrete expressions with respect to n_1, n_2 and n_3, because it would have to be fulfilled that all the smaller diagonals of H are positive, that is to say,

$n_1>0$, $n_1\, n_2-n_3>0$ and $n_3\,(n^1\, n_2-n_3)>0$

It follows that,

$n_1>0$, $n_2>0$ and $n_3>0$, furthermore that $n_1\, n_2>n_3$.

In this case, applying the first approximation method, the following result is obtained.

Theorem 1

The null solution of system (1) is asymptotically stable if the following conditions are satisfied: $n_1>0$, $n_2>0$, $n_3>0$, and $n^1\, n_2>n_3$. otherwise, it is unstable.

The proof is a direct consequence of the conditions of Hurwitz's theorem, since Hurwitz's conditions guarantee that all solutions of equation (2) have negative real parts; applying the method of first approximation, the stability of the null solution of system (1) can be guaranteed.

Note 1: If the following conditions are satisfied $n_1>0$, $n_2>0$, $n_3>0$, and $n_1\, n_2>n_3$, then the densities of the ozone layer, infrared rays and holes in the oxone layer converge to ideal values; that is to say, there will be no significant effects on the environment, otherwise appropriate measures must be taken to control pollution or any other contaminating element.

Quasi-Normal Form

It may be the case that when identifying the coefficients, the Hurwitz conditions are not satisfied, and in this way theorem 1 could not be applied to reach conclusions regarding the environmental situation that is presented. In this case, two situations could arise in which it can be concluded directly because it is unstable, or that it is necessary to continue the research to reach conclusions regarding the situation that is presented.

It may be the case that, $n_1>0$, $n_2>0$, $n_3=0$, thus the eigenvalues of the matrix of the linear part of the system (1) present a zero value and the other two with a negative real part; this makes it a critical case, that is to say, the first approximation method cannot be applied here; thus, to conclude regarding the behavior of the trajectories of the system (1) it is necessary to use the second Lyapunov method, but for this it is first necessary to simplify this system by applying the Analytical Theory of Differential Equations. By means of a non-degenerate transformation of the form, X=QY, system (1) is reduced to the form,

$$\begin{cases} y_1' = Y_1(y_1,y_2,y_3) \\ y_2' = \lambda_2 y_2 + Y_2(y_1,y_2,y_3) \\ y_3' = \lambda_3 y_3 + Y_3(y_1,y_2,y_3) \end{cases} \quad (3)$$

Where $Re\lambda_i<0$, (i=2,3), and the series, $Y_i\,(y_1,y_2,y_3)$, (i=1,2,3), admit a development similar to the series $X_i\,(x_1,x_2,x_3)$, (i=1,2,3

Theorem 2: The change of variables,

$$\begin{cases} y_1 = z_1 + h_1(z_1) + h^0(z_1,z_2,z_3) \\ y_2 = z_2 + h_2(z_1) \\ y_3 = z_3 + h_3(z_1) \end{cases} \quad (4)$$

transforms system (3) into the following quasi-normal form,

$$\begin{cases} z_1' = Z_1(z_1) \\ z_2' = \lambda_2 z_2 + Z_2(z_1,z_2,z_3) \\ z_3' = \lambda_3 z_3 + Z_3(z_1,z_2,z_3) \end{cases} \quad (5)$$

Where h_0, Z_2, Z_3 cancel each other out

$$z_2=z_3=0.$$

Proof: By deriving transformation (4) along the trajectories of systems (3) and (5), we obtain the system of equations,

$$\begin{cases} (p_2\lambda_2+p_3\lambda_3)h^0 + Z_1(z_1) = Y_1 - \frac{dh_1}{dz_1}Z_1 - \sum_{i=1}^{3}\frac{\partial h^0}{\partial z_i}Z_i \\ \lambda_2 h_2 - Z_2 = \frac{dh_2}{dz_1}Z_1 - Y_2 \\ \lambda_3 h_3 - Z_3 = \frac{dh_3}{dz_1}Z_1 - Y_3 \end{cases} \quad (6)$$

To determine the series that intervene in the systems and in the transformation, we will separate the coefficients of the power of degree $p=(p_1,p_2,p_3)$ in the following two cases:

Case I: Doing it in the system (6) $z_2=z_3=0$, is to say when the vector vector of exponents has the form $p=(p_1,^{0,0})$ results in the system

$$\begin{cases} Z_1(z_1) = Y_1(z_1+h_1,h_2,h_3) - \frac{dh_1}{dz_1}Z_1 \\ \lambda_2 h_2 - Z_2 = \frac{dh_2}{dz_1}Z_1 - Y_2(z_1+h_1,h_2,h_3) \\ \lambda_3 h_3 - Z_3 = \frac{dh_3}{dz_1}Z_1 - Y_3(z_1+h_1,h_2,h_3) \end{cases} \quad (7)$$

System (7) allows determining the coefficients of the series Z_1, h_1, h_2 and h_3, where because it is the resonant case $h_1=0$ and the other series are determined uniquely

Case II: For the case when $z_2\neq 0$ and $z_3\neq 0$ from system (6) it follows that,

$$\begin{cases} (p_2\lambda_2+p_3\lambda_3)h^0 = Y_1(z_1+h_1+h^0,z_2+h_2,z_3+h_3) - \sum_{i=1}^{3}\frac{\partial h^0}{\partial z_i}Z_i \\ Z_2 = Y_2(z_1+h_1+h^0,z_2+h_2,z_3+h_3) \\ Z_3 = Y_3(z_1+h_1+h^0,z_2+h_2,z_3+h_3) \end{cases} \quad (8)$$

Since the series of system (3) are known expressions, system (8) allows the calculation of the series h_0,Z_2 and Z_3. This proves the existence of the change of variables indicated in theorem 2.

To carry out the qualitative study, it is considered that in system (5) the function
Z_1 admits the following development in power series:

$$Z_1\,(z_1) = \alpha z_1^{s} + \cdots$$

Where α the first nonzero coeficiente and s is the corresponding power. This is the algebraic case for which it is not necessary to demonstrate the convergence of the series; a truncation could be done, constituting these polynomials, and therefore they would always be convergent.

Theorem 3
If $n_1>0$, $n_2>0$, $n_3=0$, $\alpha<0$ and s odd, then the trajectories of system (5) are asymptotically stable, otherwise they are unstable.
Proof: Consider the positive-definite Lyapunov function,

$$V(z_1, z_2, z_3) = \frac{1}{2}(z_1^2 + z_2^2 + z_3^2)$$

This function is such that its derivative along the trajectories of system (5) has the following expression

$$V'(z_1,z_2,z_3)=\alpha z_1^{s+1}+\lambda_2 z_2^2+\lambda 3\, z_3^2+R(z_1,z_2,z_3)$$

The derivative $V'(z_1,z_2,z_3)$ is negative definite, because in R only has powers of degrees greater than s+1 in relation z_1 and superior to the second with respect to z_2 and z_3 and how the function V is positive definite this allows us to state that the equilibrium position (0,0,0) is asymptotically stable.

Note 2
If the conditions are met, $a_1+c_2>b_2$, $a_1 c_3+a_3 c_1+a_1 b_2>a_2 b_1+b_2 c3$, $a_3 b_1 c_2=a_2 b_1 c_3+a_3 b_2 c_1$, $\alpha<0$ and s odd, then we have a critical case but even so the densities of the ozone layer, infrared rays and holes in the ozone layer converge to acceptable values; and therefore, there will be no significant effects on the environment, and the situation in which the three coexist at a given time is still in place.

Conclusion
1. Environmental problems will always be transcendental for all scientists, both for environmentalists and for anyone, especially for mathematicians who, through modeling, can reach conclusions.
2. Under the conditions $n_1>0$, $n_2>0$, $n_3>0$ and $n_1 n_2>n_3$, The convergence of the admissible values of the density of the ozone layer, infrared rays and holes in the ozone layer is guaranteed; this ensures that there will be no significant impact on the environment. Otherwise, appropriate measures must be taken to control pollution or any other contaminating element.
3. The Analytical Theory of Differential Equations is a significant tool in simplifying models expressed by systems of differential equations in order to draw conclusions regarding the behavior of the process.
4. If $a_1+c_2>b_2$, $a_1 c_3+a_3 c_1+a_1 b_2>a_2 b_1+b_2 c_3$, $a_3 b_1 c_2=a_2 b_1 c_3+a_3 b_2 c_1$, $\alpha<0$ and s odd; even though it is a critical case, it can be guaranteed that environmental problems in particular, the case of global warming, will not be a significant situation, as the effects will be controllable.

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