

Tribological Assessment of Porous Kozeny Carman Short Bearings Under Viscosity Variation and Elastic Roughness Deformation

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ABSTRACT

This study presents a comprehensive analysis of a porous, rough, elastic short bearing operating under ferrohydrodynamic lubrication with viscosity variation. Rosensweig's viscosity formulation is incorporated to represent magnetoviscous effects, while the Neuringer–Rosensweig model governs magnetic fluid flow in a porous medium. A stochastic averaged modified Reynolds equation is developed to capture the combined influences of porosity, deformable surface roughness, viscosity variation, and elastic deformation. Numerical solutions are obtained using a finite-difference approach to evaluate the pressure profile and corresponding load carrying capacity (LCC).

The results reveal that elastic deformation, porosity, and positively skewed roughness significantly reduce LCC, whereas transverse and negatively skewed roughness textures enhance performance. Viscosity variation and magnetization can partially counteract performance losses when deformation is small and the bearing aspect ratio is optimally chosen. The study highlights the complex interplay between microstructural porosity, surface topology, and magnetoviscous effects, offering design insights for optimizing advanced ferrofluid-lubricated bearing systems.

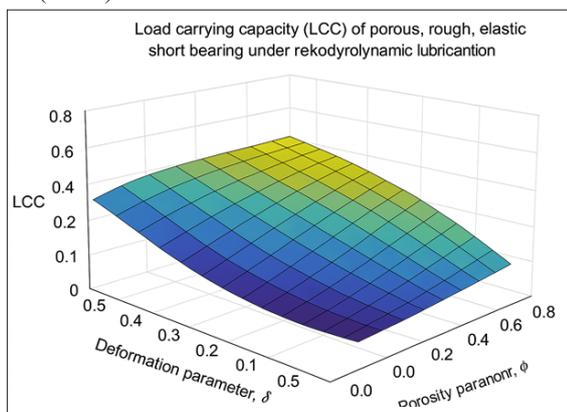
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magnetic fluids), 00A71-Theory of mathematical modeling, 00A72-Simulation and computational methods Nomenclature.



Geometrical Parameters

Symbol	Description
x, z	Coordinate directions along the bearing length and width
L	Length of the bearing in the x -direction
B	Breadth/width of the bearing in the z -direction ($B \ll L$)
h	Film thickness between bearing surfaces
\bar{h}	Mean film thickness
h_s	Random roughness height component (Christensen & Tonder model)
δ	Elastic deformation of the porous facing
u	Sliding velocity of the upper bearing surface

Porous Structure Parameters

Symbol	Description
k_p	Permeability of porous matrix (Kozeny–Carman / Irmay models)
ϕ	Porosity of the porous layer
β	Kozeny–Carman empirical constant
σ	Standard deviation of roughness height distribution

Subject Classification

(Aligned with MSC 2020 standards): 74A55-Theories of friction (tribology), 74M15-Contact problems including roughness, adhesion, and surface effects, 76Dxx-Incompressible viscous fluids, 76Z05-Applications of fluid mechanics to physics (including

S	Skewness parameter (positive/negative roughness)
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Fluid and Ferrofluid Properties

Symbol	Description
ρ	Density of ferrofluid
μ	Dynamic viscosity of ferrofluid
μ_0	Porosity of the porous layer
Magnetic permeability of free space	Kozeny–Carman empirical constant
η	Effective viscosity (Rosensweig’s modified model)
η_0	Base viscosity of carrier liquid
C_1, C_2	Rosensweig viscosity variation constants
χ	Magnetic susceptibility
q	Fluid velocity vector in the film region

Magnetic Field Parameters

Symbol	Description
B	Magnetic field intensity
k	Dimensionless constant controlling magnetic field strength
M	Magnetization of ferrofluid
θ	Magnetic field inclination angle (oblique field)

Lubrication & Governing Equation Parameters

Symbol	Description
p	Film pressure
$\partial p / \partial x$	Pressure gradient in x-direction (neglected for short bearing)
$\langle \cdot \rangle$	Stochastic average (Christensen–Tonder theory)
Eq. (2)	Generalized stochastic averaged modified Reynolds equation
ϵ	Roughness parameter, ratio of roughness amplitude to mean film thickness

Dimensionless Quantities

Symbol	Description
$\bar{x} = x/L$	Film pressure
$\bar{z} = z/B$	Pressure gradient in x-direction (neglected for short bearing)
$\bar{h} = h/h_0$	Stochastic average (Christensen–Tonder theory)
$\bar{p} = p/p_0$	Dimensionless pressure
Λ	Elasticity parameter
Ψ	Permeability or porosity parameter
α	Viscosity variation parameter
β_m	Magnetization parameter

Short Bearing Approximation

Symbol	Description
$\partial p / \partial x \approx 0$	Axial pressure gradient neglected (Prajapati, 1995)
$p = p(z)$	Pressure varies only across the width in short bearing theory

Introduction

Bearings constitute the backbone of numerous engineering systems

ranging from industrial machinery and automotive components to aerospace mechanisms and power generation equipment by supporting rotating shafts and enabling smooth, controlled motion. Their operational reliability, efficiency, and load-handling capacity directly influence the performance and longevity of mechanical systems. Consequently, enhancing bearing performance through advanced lubrication techniques, material innovations, and refined modeling approaches has remained a critical area of tribological research for decades.

A significant advancement in lubrication science has been the incorporation of magnetic fluids (ferrofluids), whose rheological and flow properties can be dynamically modulated through externally applied magnetic fields. Classical studies by established the fundamental physics of ferro hydrodynamics, describing how magnetic polarization and magnetoviscous effects influence lubrication films under varying field intensities [1,2]. Recent investigations continue to demonstrate the advantages of ferrofluid lubrication in enhancing load-carrying capacity, suppressing instabilities, and reducing frictional losses in both slider and journal bearings [3-6]. The ability of ferrofluids to respond instantaneously to magnetic stimuli makes them particularly suitable for high-performance and adaptive bearing systems.

In parallel, porous bearings have gained considerable attention due to their ability to regulate lubricant supply, filter contaminants, and maintain stable pressure distribution. Early works by revealed that porous media significantly influence hydrodynamic and magnetohydrodynamic lubrication characteristics by modifying the flow resistance within the bearing clearance [7, 8]. Subsequent studies demonstrated that porous structures not only enhance lubrication under varying operating conditions but also help mitigate adverse effects of surface irregularities, wear debris, and cavitation [9-11].

These findings established porous-material-based bearings as robust and self-regulating lubrication devices suitable for next-generation tribological applications. Another essential factor influencing bearing performance is the viscosity variation of lubricants, especially magnetic fluids. Viscosity is highly sensitive to temperature gradients, shear rates, and magnetic field strength, making its accurate modeling vital for predicting film thickness and pressure generation. Foundational works by highlighted the significant role of temperature-dependent viscosity variation in hydrodynamic behavior [12,13]. Later research has further explored magnetic-field-induced viscosity variation using Rosensweig’s model, emphasizing the control it offers over lubrication characteristics in both long and short bearing configurations [14-18].

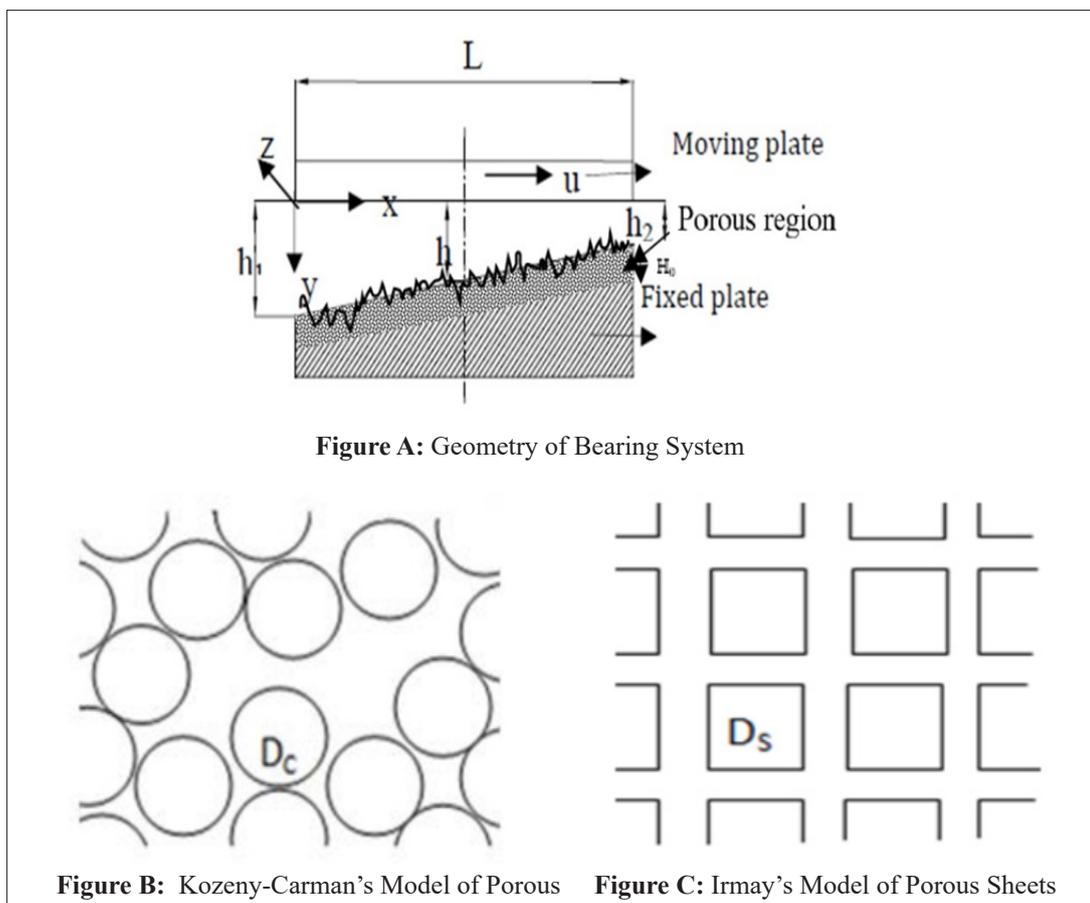
These investigations underline the potential of magneto viscous effects to enhance load-carrying capacity, damping, and operational stability under diverse loading and environmental conditions. Additionally, elastic deformation of bearing surfaces is an unavoidable phenomenon under operational loads. Such deformation alters the hydrodynamic pressure distribution, modifies the lubrication film profile, and significantly affects the overall load-carrying capacity and stiffness of the bearing. Early analysis by showed that deformation must be carefully incorporated into lubrication models to avoid misleading predictions of bearing performance [19]. More recent studies, including those by have demonstrated that when deformation interacts with surface roughness, viscosity variation, and magnetic effects, the resultant tribological behavior becomes highly nonlinear and sensitive to material and operating parameters [20]. This underscores the

importance of advanced coupled modeling approaches that integrate roughness, elasticity, and fluid structure interactions. Given the above developments, the present work seeks to undertake a Kozeny Carman-based comparative analysis of a magnetic-fluid-lubricated grounded short bearing that incorporates viscosity variation, porous medium effects, and deformable roughness.

The Kozeny–Carman formulation is particularly suitable for characterizing porous flow resistance and enables a rigorous study of how porosity interacts with magnetoviscous effects and elastic deformation to determine the bearing’s performance. By combining stochastic roughness modeling, Rosensweig-type viscosity variation, and elastic deformation effects, the proposed analysis aims to provide deeper insights into the complex Multiphysics governing modern ferrofluid based short bearings. The findings of this study are expected to contribute significantly to the optimized design of adaptive, high-efficiency bearings for advanced mechanical systems.

Mathematical Analysis

Figure (A) to (C) determines the slider bearing surfaces of short bearing travels with uniform velocity u in x – direction with the inclusion of different porous structures. L and B are length and breadth of the short bearing in z direction with $B \ll L$ respectively. Following the discussion in Prajapati (1995), the effect of $\partial p/\partial x$ is neglecting [21].



In view of Agrawal (1986), magnetic field remains oblique to the starter [22]. As mentioned in Christensen and Tonder (1970) and the magnitude of the magnetic field is taken as

$$M^2 = kB^2 \left\{ (0.5 + zB^{-1}) \sin(0.5 - zB^{-1}) + (0.5 - zB^{-1}) \sin(0.5 + zB^{-1}) \right\} \quad (1)$$

where k is a suitably chosen constant from dimensionless point of view Bhat (2003), so as to manufacture a magnetic field with desired strength [23]. Under the usual assumptions of hydrodynamic lubrication with the laminar flow (Shimpi et al. (2021)), the following model of Christensen and Tonder (1970) for the film thickness has been considered as

$$h(x) = \bar{h}(x) + h_s + \delta$$

where \bar{h} is the mean film thickness, δ being local deformation of the porous facing and h_s , the part due to surface roughness measured from the mean level $h + \delta$, is hypothetical to be stochastic in nature and governed by the theory of Christensen and Tonder (1970). A model was developed for the simple steady flow of ferrofluid by in the presence of a slowly changing external magnetic field and proposed following equation:

$$-\nabla \left(p - \frac{\mu_0 \bar{\mu} M^2}{2} \right) + \eta \nabla^2 \bar{q} = \rho (\bar{q} \nabla) \bar{q}$$

where ρ , q , μ_0 , μ , η and p represent the fluid density, the fluid velocity in the film region, magnetic susceptibility of the magnetic field, free space permeability, viscosity of fluid and the film pressure respectively. Under hydro-magnetic lubrication theory, stochastically averaging and adopting the properties of magnetic fluid lubrication, the generalized Reynolds equation is comes out as

$$\frac{d^2}{dz^2} \left(p - \frac{\mu_0 \bar{\mu} M^2}{2} \right) = \frac{6\mu u}{g(h)} \frac{d}{dx} (h + p_a p \delta) \quad (2)$$

where,

$$g(h) = (h + p_a p \delta)^3 + 3(h + p_a p \delta)^2 \alpha^* + 3(h + p_a p \delta) (\sigma^{*2} + \alpha^{*2}) + 3\sigma^{*2} \alpha^* + \alpha^{*3} + \epsilon^* + 12\phi H_0$$

As per the information available in the literature, it is possible for the viscosity of the lubricant to vary within the lubricant film [24]. Moreover, the viscosity near the bearing surfaces could differ due to the interactions of additives and surfactants with these surfaces. Rosensweig (1985) made modifications to Einstein's viscosity equation, specifically introducing a quadratic correlation that accurately describes higher concentrations. In such cases, it is possible to consider a two-constant expression, precisely to as:

$$\mu' = \mu (1 + a\phi + b\phi^2) \quad (3)$$

where $a = -\frac{5}{2}$ and $b = \frac{(\frac{5}{2}\phi_c - 1)}{(\phi_c)^2}$ and ϕ_c is the suspension becomes effectively rigid.

The related physical Reynolds boundary conditions of the system, are

$$p \left(\pm \frac{B}{2} \right) = 0$$

and

$$\frac{dp}{dz} (0) = 0 \quad (4)$$

Solving equation (2) under boundary conditions (4), one can obtain the expression for dimensional pressure distribution with variation in viscosity suggested by Rosensweig (1985),

$$p = \frac{\mu_0 \bar{\mu} M^2}{2} - \frac{3\mu_1 u m h_2}{g(h) L J(\phi)} \left(z^2 - \frac{B^2}{4} \right)$$

where

$$J(\phi) = 1 - \frac{5\phi}{2} + \frac{5\phi^2}{2\phi_c} - \frac{\phi^2}{\phi_c^2}$$

Using the following dimensionless quantities,

$$m = \frac{h_1 - h_2}{h_2}, Z = \frac{z}{B}, P = \frac{(h_2)^3}{\mu_1 u B^2} p, \mu^* = \frac{(h_2)^3 k \mu_0 \bar{\mu}}{\mu_1 u}, X = \frac{x}{L}, \bar{L} = \frac{L}{h_2}, \bar{B} = \frac{B}{h_2}$$

$$\alpha = \frac{\alpha^*}{h}, \sigma = \frac{\sigma^*}{h}, \epsilon = \frac{\epsilon^*}{h^3}, \bar{P} = \frac{P_a p'}{h^2}, \delta = \frac{\delta}{h}, \psi = \frac{\phi H_0}{h^3}, W = \frac{h_2^3}{\mu_1 u B^4} w$$

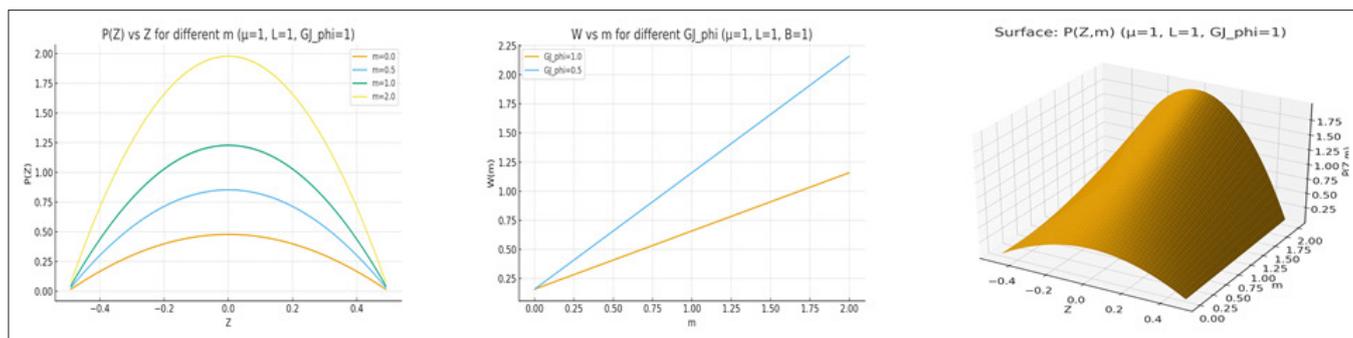
$$G = (1 + \bar{p}\bar{\delta})^3 + 3(1 + \bar{p}\bar{\delta})^2 \alpha + 3(1 + \bar{p}\bar{\delta}) (\sigma^2 + \alpha^2) + 3\sigma^2 \alpha + \alpha^3 + \epsilon + 12\psi \quad (6)$$

one transforms equation (5) to

$$P = \frac{\mu^*}{2} \left[\left(\frac{1}{2} + Z \right) \sin \left(\frac{1}{2} - Z \right) + \left(\frac{1}{2} - Z \right) \sin \left(\frac{1}{2} + Z \right) \right] + \frac{3m}{L G J(\phi)} \left(\frac{1}{4} - Z^2 \right) \quad (7)$$

The LCC in uniform dimension is calculated as

$$W = \frac{\mu^* \bar{L}}{B} (1 - \sin(1)) + \frac{m}{2} \frac{1}{B G J(\phi)} \quad (8)$$



Results and Short Discussions

The present study analyzes the performance of a magnetic fluid-based porous rough short bearing system by incorporating Rosensweig's viscosity variation, elastic deformation, porosity, and surface roughness effects. The pressure distribution derived from Equation (7) and the non-dimensional load-carrying capacity (LCC) from Equation (8) clearly establish that magnetization enhances the bearing performance significantly. This advantage surpasses the outcomes reported by earlier studies involving conventional lubricants and constant-viscosity models. Graphical results confirm that magnetization exhibits a positive correlation with key design parameters such as viscosity variation, porosity, aspect ratio, and elastic deformation.

The enhancement in LCC can be attributed to the increased viscosity of the magnetic fluid under magnetization. While viscosity variation and aspect ratio improve the LCC, increases in deformation and porosity reduce it. Importantly, magnetization and viscosity variation jointly compensate for the adverse effects of porosity and deformation, particularly under negatively skewed roughness and lower deformation levels. The comparative assessment with prior works, including those by validates the significant contribution of this study toward improving the efficiency and robustness of short bearing systems. The findings reinforce the importance of adopting magnetic fluids with variable viscosity for high-performance bearing applications.

Future Scope

Based on the findings and limitations identified, several directions for future research emerge:

Influence of Operating Conditions

Future studies may incorporate varying temperature fields, dynamic loading, and external magnetic field strengths to better assess real-world performance and thermal-magnetic coupling effects.

Optimization of Porous Media and Magnetic Fluids

Further investigation is needed to optimize:

- Pore structure and permeability of the porous matrix,
- Magnetic particle concentration,
- Viscosity magnetization dependency functions, to maximize bearing efficiency.

Advanced Roughness and Deformation Models

The current model can be extended by including:

- Three-dimensional roughness distributions,
- Nonlinear or viscoelastic deformation behavior,
- Anisotropic surface properties, to capture complex tribological interactions more accurately.

Stability And Dynamic Response Analysis

Analyzing the dynamic stability, vibration characteristics, and transient behavior of magnetic fluid-lubricated short bearings will enhance their applicability in high-speed rotating machinery.

Experimental Validation

Constructing prototype short bearings and conducting experiments under controlled magnetic fields would strengthen the practical relevance of the theoretical predictions.

Integration with Smart and Adaptive Systems

Future designs may use:

- Active magnetic control,
- Sensor-integrated porous bearings,
- Adaptive viscosity magnetic fluids, to create intelligent bearing systems capable of self-adjustment under variable operational demands.

Declarations

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