ISSN: 3033-3652

Journal of Physical Mathematics & its Applications

SCIENTIFIC Research and Community

Review Article

Open Access

Exploring Theoretical Fundamentals of a Physical-Mathematical Model for Unsteady Combustion in Solid Rocket Fuel and Developing Methods for Unsteady Velocity Calculation

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ABSTRACT

The article is dedicated to the theoretical study of the non-stationary Combustion Velocity of Solid Rocket Fuel (CVSRF). To calculate the Internal Ballistic Characteristics (IBC) of solid fuel missile engines, combustion velocity laws are used. These laws represent stationary relationships between the Combustion Velocity of Solid Rocket Fuel (CVSRF) and initial pressure and temperature. These relationships are experimentally determined under "stationary" conditions, i.e., by burning fuel samples in stationary conditions.

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Received: November 06, 2023; **Accepted:** November 13, 2023; **Published:** November 29, 2023

Keywords: Solid Fuel, Combustion Velocity, Physical-Mathematical Model, Ballistic Missiles, Intra-Ballistic Characteristic, Mathematical Methods, Numerical Methods, *k*-phase.

MSC: 74A15, 80A25

Relevance of the Work

The article is dedicated to the theoretical investigation of non-stationary combustion velocity of solid rocket propellants (SRPs).

By the present time, a significant number of scientific works by researchers like I.B. Zeldovich, Novozhilov, Prisnyakov, both from Russia and abroad, have been published, focusing on the theoretical description of the processes occur- ring during the combustion of solid propellants in the combustion chambers of rocket engines. However, the results of the existing theory of non-stationary combustion have not found widespread application in the development of solid rocket motors (SRMs).

For the calculation of the internal ballistics characteristics (IBC) of SRMs, burning rate laws are used. These laws describe the burning rate of solid propellants as functions of pressure and initial temperature, which have been experimentally obtained under stationary conditions. This approach is entirely justified when the characteristic time of pressure change $\left(\frac{p}{dp/dt}\right)$ is sufficiently large, on the order of 10^{-1} s or more.

However, the real combustion rate may differ from the value calculated using stationary laws if the pressure change process occurs rapidly, i.e., within a characteristic time on the order of 10^{-3} s or less. Such a process occurs during startup, pressure drop at the end of operation, and throughout the operational cycle of some types of SRMs, especially in the case of impulse SRMs with a firing duration of 5-50 milliseconds used in Crew Escape Systems (CES) for air- craft and spacecraft. When CES is engaged, an error of more than 10 percent in the calculations of acceleration or its rate of increase is unacceptable, and, consequently, an error of more than 5 percent in calculating the burning rate of SRPs is also unacceptable.

Calculations for the IBC of such engines, especially during startup, must take into account all possible factors that can influence the internal chamber characteristics and, therefore, the magnitude of acceleration and its rate of increase, to which the human body is particularly sensitive. In this case, for a more accurate IBC calculation, the stationary burning rate law must be replaced with a relationship that links the non-stationary burning rate and non-stationary pressure.

In the existing theory of non-stationary SRP combustion, there was no question of developing specific methods for calculating the dependence of the nonstationary burning rate on pressure. When theoretically studying the combustion processes of solid propellants, either a time interval is considered in which the influence of initial conditions on the course of the process is negligible, or the problem statement is so general that obtaining a specific solution is not feasible.

Thus, the creation of a theoretical foundation for studying the non-stationary burning rate of solid rocket propellants, which, on one hand, would be based on the accumulated results in this field and, on the other hand, would be suitable for a more precise calculation of the IBC of SRMs, appears highly relevant.

Objective of the Article

The objective of the work is to develop the theoretical foundations of a physicalmathematical model for non-stationary combustion of solid fuel, taking into account characteristic times of the process on the order of 10^{-3} seconds and the influence of initial conditions on the course of non-stationary combustion. This involves deriving an equation for calculating the non-stationary combustion velocity from a closed system of equations and creating methods to calculate the dependence of the solid fuel combustion velocity on pressure under nonstationary conditions.

Notations

In this work, we introduce notations for the key elements of the research. These notations include various symbols and indices that are used throughout the study. Some of the key symbols and indices are defined as follows:

k-phase: Represents the condensed phase of solid fuel, often referred to as the "dark zone."

p: Pressure.

u: Combustion velocity in the condensed solid phase (CVCSP).

t: Time.

x: Distance in a Cartesian coordinate system connected to the combustion surface.

T: Temperature.

f: Temperature gradient.

a: Coefficient of temperature conductivity.

λ: Heat conductivity coefficient.

c: Specific heat capacity.

ρ: Density.

E_s: Activation energy of chemical reactions in the k-phase.

H: Pre-exponential factor.

υ: Gas flow velocity.

R: Gas constant.

Q_s: Heat release of the reaction in the k-phase.

$S = S(p)$: Total surface area of condensed particles per unit volume of gas, related to the probability of recombination of free radicals on the particle surface.

ε: Energy.

The indices are used to specify different conditions or phases within the study. For example, "0" denotes stationary conditions, "s" refers to the combustion surface, s_0 indicates stationary conditions at the combustion surface, "g" represents the gas phase, and "N" is used for initial conditions. The "fr" index stands for the flame front. The main objective of the project is to develop the theoretical foundations of a physical-mathematical model for the non-stationary combustion of solid rocket fuel, taking into account the characteristic time scales of the process on the order of 10^{-3} seconds and the influence of initial conditions on the non-stationary combustion process. This project aims to derive the equation for calculating non-stationary combustion velocity from a closed system of equations and create methods for calculating the dependence of Solid Fuel Combustion Velocity (SFCV) on pressure under non-stationary conditions.

Research Methods

The article applies methods of mathematical analysis. Analytical derivations of formulas for calculating non-stationary combustion velocity were made based on the fundamental laws of classical physics and the physics of gas combustion. Wherever possible, the results of the work were compared with available experimental data. In some cases, experimentally obtained dependencies (e.g., the power law for stationary burning velocity) were derived theoretically.

Scientific Novelty

The results presented in the article are novel and have both theoretical and practical significance. Some of the notable outcomes include

Proposal and justification of a physical-mathematical model for the combustion of solid propellants designed to account for the influence of non-stationary burning velocity of solid ballistic propellants on the Internal Ballistics (IBC) of impulse Solid Rocket Motors (SRMs).

Formulation of the initial-boundary value problem with a moving boundary for a one-dimensional non-stationary heat conduction equation, including the burning velocity as an unknown variable. An exact analytical solution for this problem is derived in the form of a green's function.

First-time application of results from N.N. Semenov's theory of chain-branching chemical reactions to the theory of solid propellant combustion. Using the findings of this theory, a formula for calculating the heat flux from the gas phase to the condensed phase is derived.

Derivation of a nonlinear equation for calculating the non-stationary burning velocity of solid propellants and obtaining its linear approximation in the form of a first kind Volterra integral equation. Approximate analytical methods for solving the nonlinear equation were developed, allowing for the calculation of non-stationary burning velocity.

Development of a model for stationary combustion of solid propellants as a special case of non-stationary combustion. Theoretical derivation of the burning velocity law for fuels containing catalysts.

Specific Aim of the Article

The aim of the article is

- Develop the theoretical foundations of a physical-mathematical model for the non-stationary combustion of solid fuel, taking into account characteristic time scales of the process on the order of 10^{-3} seconds and the effect of initial conditions on the course of the non-stationary combustion process.
- Derive the equation for calculating the non-stationary combustion velocity from a closed system of equations.
- Create methods for calculating the dependence of the solid fuel combustion velocity on pressure under non-stationary conditions.
- The research methods involve various interconnected steps:
- Literature review to understand the current state of research in solid fuel combustion, rocket and missile modeling, and non-stationary velocity calculation methods.
- Data collection and experimentation, including obtaining information about the properties of solid fuel and missile operating conditions.
- Formulation of the physical-mathematical model, including the creation of differential equations that describe the non-stationary combustion process.

- Mathematical simplifications and approximations to make the equations more manageable.
- Selection of numerical methods for solving the equations, such as finite differences, finite elements, or characteristics.
- Computational implementation of the model and simulations.
- Validation and calibration of the model with real experimental data.
- Analysis of the results to extract relevant information.
- Refinement and improvement of the model based on the analysis of the results.
- Communication and documentation of the entire process, methods, results, and findings.

The research requires a multidisciplinary approach that combines knowledge of thermodynamics, combustion chemistry, applied mathematics, and computational methods. Additionally, the project's results include theoretical and practical innovations in modeling solid fuel combustion.

Practical Importance of the Article

Advanced Missile and Rocket Projects: A deep understanding of the combustion process in rockets and missiles is crucial for the development of more efficient, safe, and powerful propulsion systems. This can lead to significantly improvements in the performance and reliability of these systems.

Resource Economy: Accurate physical-mathematical models can help optimize the use of resources, such as fuel and missile components, reducing production and operational costs.

Safety: Understanding combustion behavior is essential to ensure safety during the storage, handling, and operation of missiles and rockets. The knowledge resulting from this project can lead to more effective safety protocols.

Space Missions and National Defense: Missiles are used in various applications, including space exploration, national defense, and satellite launches. A better understanding of solid fuel combustion can improve the accuracy and effectiveness of these missions.

Emissions Reduction and Environmental Impact: The development of advanced combustion models also has environmental implications. Research can contribute to the reduction of pollutant emissions and the optimization of fuel use.

Technological Innovation: The project stimulates innovation in areas such as aerodynamics, thermodynamics, combustion chemistry, and applied mathematics. Advances in these fields have applications beyond missile propulsion.

Human Resource Development: Research projects of this kind provide training and capacity-building opportunities for scientists, engineers, and researchers, contributing to the development of a skilled workforce.

National and International Competitiveness: Countries that invest in research and development in this field can maintain or increase their technological and military competitiveness.

Response to Emerging Threats: In an ever-evolving geopolitical landscape, ongoing research is necessary to address emerging threats and develop effective defense systems.

International Collaboration: Projects of this nature often involve collaborations between researchers, engineers, and institutions from various countries. This promotes knowledge exchange and the building of international relationships.

It is important to emphasize that research into physical-mathematical models of missile solid fuel combustion is highly specialized and requires significant investments in research, computational resources, and experimentation. Furthermore, research must be conducted with ethical considerations, especially regarding the use of missile technology in military and political contexts. Therefore, it is essential to balance practical benefits with ethical responsibility and safety.

The Main Result

The article provides an analysis of the theory of solid fuel combustion (CCS) for both stationary and non-stationary cases. It aims to establish the theoretical foundation for the physical-mathematical model developed in this study. The analysis includes discussions of different methods for describing physicochemical processes in the condensed phase (*k*-phase). The work introduces the concept that combustion velocity at the combustion surface follows an Arrhenius-type equation, which relates the velocity to temperature.

The derivations of equations and dependencies required for the construction of a physical-mathematical model for non-stationary solid fuel combustion. It presents the main equation for calculating non-stationary combustion velocity as the solution to a system of equations, which includes the heat conduction equation in the *k*-phase with boundary and initial conditions, the heat balance equation at the combustion surface, and an additional boundary condition. The project formulates the initial and boundary value problem for the heat conduction equation in a one-dimensional formulation, introducing a coordinate system connected to the combustion surface. The key equations and conditions are derived in this part, and the project elaborates on the physical meaning and significance of these equations. According to works [1], [2], [3], and others, the equation for heat propagation in the solid phase has the following form

$$c\rho\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial x^2} + c\rho u(t)\frac{\partial T}{\partial x} + \rho Q_s W(T,A), \quad x \geq 0. \quad (1)$$

Here, Q_s is the specific heat released during the decomposition of one unit mass of the solid-phase substance, $W(T,A)$ is the rate of the chemical decomposition reaction, and A is the concentration of the decomposing substance.

Equation (1) should be considered together with the equation determining the distribution of the concentration of the decomposing substance in the solid phase. In the cited works, it is noted that the depth of the heated layer in the solid phase is very small, and that the chemical reaction of the solid-phase decomposition practically occurs only on the combustion surface, where the temperature of the solid phase is maximal.

In the paper [1], it is noted that in the *k*–phase, the diffusion coefficient is much smaller than the temperature coefficient of spread. It is also observed that the heating of the *k*–phase occurs due to heat conduction and chemical decomposition reactions, which do not require prior mixing of components through diffusion.

This circumstance allows us to consider the dependence for the function $W(T,A)$,

$$W(T,A) = \begin{cases} u(t) & , \ x = 0, \\ 0 & , \ x > 0, \end{cases}$$

Here, Q_s is the specific heat realease during the decomposition of one unit mass of the solid-phase substance, $W(t,A)$ is the rate of the chemical decomposition reactio, A is the concetrationof the

decomposing substance. In other words, completely abandon the consideration of diffusion processes in the *k*–phase and take into account heat generation by modifying the boundary conditions accordingly. Equation (1) is significantly simplified in this case, but the boundary conditions become somewhat more complex

Overall, the work delves into the complex process of non-stationary solid fuel combustion, providing a theoretical foundation for understanding the interplay of factors such as temperature, pressure, and reaction kinetics in the combustion process. The use of mathematical models and physical principles enables the analysis and prediction of non-stationary combustion behavior:

$$
\begin{array}{lll}
0. & H_2 + O_2 \rightleftarrows 2OH & - \text{ chain branching reactions;} \\
1. & H_2 + OH \rightleftarrows H_2O + H & - \text{ chain propagation reaction;} \\
2. & O_2 + H \rightleftarrows OH + O \\
3. & H_2 + O \rightleftarrows OH + H & \text{chain branching reactions;} \\
4. & H + H + M \rightleftarrows H_2 + M \\
5. & H + OH + M \rightleftarrows H_2O + M & \text{chain breakdown in the volume;} \\
6. & H + \text{parede} - \text{chain de breakdown in the surface.} &
\end{array}
\tag{3}
$$

Here, *M* stands for any molecule or radical. Reaction 6 refers to chain breaking when a free radical collides with the surface of a condensed particle present in the fuel decomposition products. According to the theory of branched chain reactions developed by the academic Semyonov, after reaching the necessary conditions due to the mixture of hydrogen and oxygen that promote chain reactions, the ignition of hydrogen combustion occurs with a delay. This delay, or the induction period, is caused by the need to accumulate a sufficient quantity of free radicals responsible for the continuation and branching of the chains. For the same reason, there is a delay in ignition in the case of solid fuel combustion mixtures (CSM) during the passage of the "dark zone" through the gases [1], [2], are:

1. the presence of a clearly defined pressure threshold below which the reaction practically does not occur, and above which the reaction proceeds at a high speed. The pressure values considered in the theory of combustion are deliberately higher than the limiting ones;

2. the presence of an induction period, i.e., the time from the moment the initial fuel mixture reaches the conditions necessary for spontaneous ignition (specific values of temperature and pressure) until the onset of the rapid reaction. The magnitude of the induction period is inversely proportional to the rate of the chain-branching reaction (reaction 2 of mechanism 3). In other words, the induction period is the time required to accumulate a sufficient number of active centers for the oxidation reaction of hydrogen to proceed;

3. a very high reaction rate after the completion of the induction period - the reaction is perceived as an explosion or flash.

In the works of N.N. Semenov, theoretical dependencies for the rate of branching chain reactions have been obtained. These dependencies do not involve the concepts of "activation energy" and "reaction order", i.e., branching chain reactions are not described by Arrhenius-type equations. A theoretical dependence for the induction period was also derived, which was subsequently experimentally verified [5], [6] and applied to the combustion of hydrocarbons [7]. It is worth noting that the formula for calculating the induction period was known earlier [2], but only in the works of N.N. Semenov and his students did it receive sufficient theoretical and experimental justification. Subsequently, the concept of the induction period has been used in the theory of liquid fuel combustion [4], while in the theory of solid fuel combustion, this concept has not been utilized up to the present.

It is necessary to note that the approximate formula for calculating induction periods was known before. However, it was only in the work of Semenov and his students that it received theoretical and experimental justification. Subsequently, the concept of the induction period was used in the theory of combustion of liquid fuels [3]. In the theory of solid fuel combustion, this concept was not utilized. In this study, the results of the theory of exploratory chain chemical reactions are adopted as the theoretical basis for calculating the heat flux from the gas phase to the *k*–phase.

Methods for calculating the burning rate of solid propellant

The task of calculating the combustion speed of solid fuel was first addressed in the article of Zeldovich [1]. Methods based on flame propagation theory in gases were used to compute the steady-state combustion speed. This involved considering mass conservation equations, where u_g is the flame propagation speed in the gas phase, equations for conserving the number of atoms of each species, energy conservation equations, enthalpy constancy, and diffusion equations. Assuming equal diffusion coefficients and temperature conductivity in established similarity fields of concentration and temperature throughout the gas phase, integration of equations was performed for the "dark zone", anticipating heat release.

Calculating the burning rate of solid propellant is a crucial aspect of designing solid rocket motors and propellant-driven rockets. The burning rate refers to the speed at which the solid propellant burns over time. This measure is essential for understanding engine performance and ensuring safe and efficient operation. There are various approaches to calculating the burning rate of solid propellant, and they can be categorized into analytical, empirical, and numerical methods. Analytical methods BATES-GRUNEISEN Law is based on the Bates-Gruneisen law, which relates the burning rate to gas pressure and propellant density. Internal ballistics Mmdels utilizes mass and energy conservation equations to describe the burning of solid propellant inside the motor. Includes considerations for motor geometry, propellant characteristics, and thermal effects. Empirical Methods proposed by Crawford, it relates the burning rate to gas pressure. Correlation methods from experimental data relies on experimental data, such as laboratory burning tests. Uses statistical techniques to correlate the data and derive empirical equations for the burning rate. Computational fluid dynamics (CFD) modeling uses computational simulations to model fluid dynamics inside the motor. Considers the interaction between gas and solid, including factors like heat transfer and changes in surface area. Finite element models divides the motor into finite elements to analyze the mechanical and thermal behavior of solid propellant during burning. May include thermo-structural coupling to capture thermo-mechanical interactions. Key Considerations: propellant properties, understanding propellant properties such as chemical composition, grain size, and additives is crucial for accurate modeling. Environmental Conditions like temperature and atmospheric pressure influence the burning rate and should be considered in calculations. Experimental validation results from analytical and numerical methods should be validated through practical experiments to ensure accuracy and reliability. The development and application of these methods require advanced knowledge in thermodynamics, fluid mechanics, and computational modeling, reflecting the multidisciplinary nature

of solid propellant rocket motor design.

In the works [1], [2], the calculation of the steady-state burning rate of solid fuel was carried out using the mass conservation equation $\rho u = \rho_g u_g$ where u_g - the flame propagation velocity in the gas phase. Another method for calculating the steady-state burning rate, also proposed by Ya.B. Zeldovich [1], [2], involves the use of the heat balance equation in the k–phase, which is expressed as: $\lambda f_o = \rho c u_o (T_{so} - T_N)$. In this case, to calculate u_o, it is necessary to know the analytical expressions for T_{so} and f_o. In works [1], [2], it was assumed that T_{so} = const, and the heat balance equation was considered valid even for non-stationary combustion regimes.

The calculations for the steady combustion velocity are reduced to solving the heat balance equation in the *k*–phase. To solve this equation, one needs to know the dependence of the steady combustion velocity on pressure and initial temperature, obtained either from experiments or the theory of steady combustion. This approach to calculating the steady combustion velocity is justified because the combustion rate, considered in quasisteady state conditions, behaves similarly to the steady-state conditions, with the same temperature gradient at the surface of the condensed phase. In other words, the quasi-steady combustion velocity is determined solely by the instantaneous values of pressure and temperature gradient and is currently not linked to the temperature distribution throughout the volumes of the condensed phase.

Thus, for the stationary case, the heat balance equation was considered valid, extending to the non-stationary combustion regime. Novozhilov generalized Zeldovich's theory of non-stationary combustion to the case of variable temperature of the hot surface, fundamentally preserving the main idea of accounting for the quasi-stationarity of the process and using it as a link for the stationary dependence of the combustion rate on pressure.

The approach to calculating non-stationary combustion velocity is justified by considering the combustion rate at the moment in a quasi-stationary regime, which equals the rate in a stationary regime with the same temperature gradient at the condensed phase surface. In this way, the quasi-stationary combustion velocity is solely determined by the instantaneous pressure and temperature gradient and is not currently linked to the temperature distribution throughout the condensed phase volume. Therefore, the heat balance equation valid for the stationary case is adopted for non-stationary combustion. Novozhilov extended Zeldovich's theory of non-stationary combustion to cases with changing temperatures on the burning surface. He utilized the stationary combustion law and the dependency of combustion velocity on surface temperature, coupled with vitamin equations for heat flow. Vozhilov introduced an integral equation for calculating non-stationary combustion velocity, applied to processes far from the influence of initial conditions. However, its use in developing a calculation method for non-stationary combustion velocity is significantly challenging, requiring a specific dependency for surface temperature and heat flow.

In Novozhilov's work [8], an integral equation is introduced for calculating the non-stationary combustion velocity, applied to processes occurring far from the influence of initial conditions. The use of this equation in developing methods for calculating non-stationary combustion velocity faces significant challenges, both due to its need for a specific dependency for the surface temperature of combustion and heat flux into the *k*–phase. Romanov's articles at [6]–[8] provide a calculation for non-stationary combustion velocity formulae.

In works [2], [3], [4], equations (1) and (2) were used to calculate the combustion rate in combination with various equations for heat flux. Analytical expressions for some specific cases of pressure variation over time were first obtained in work [2]. However, the practical use of these expressions, even for specific cases, is quite limited primarily because they contain functions of a complex argument, requiring the development of sufficiently complex calculation methods. In the works of B.V. Novozhilov [4], [8], a general integral equation for calculating non-stationary combustion velocity has been derived, which needs to be complemented with specific dependencies for the surface temperature of combustion and heat flux into the combustion phase.

Let's consider various existing descriptions of the physical-mathematical aspects of the combustion process of solid fuel based on the results of theoretical and experimental studies outlined in the works of both domestic and foreign researchers. This review of works dedicated to the study of heat transfer from the gas phase to condensation reveals a significant divergence of opinions among different researchers. Each work essentially employs its own model for calculating heat flux. The fundamental principles of the theory of chain branching combustion reactions, including those introduced by Academician Semenov, are presented. Currently, these principles serve as the theoretical basis for calculating the heat flux from the gas phase to the condensed phase. The review also covers existing methods for calculating the burning rate of solid fuel, demonstrating that these methods may only apply to specific cases of pressure variation over time or possess such a general nature that additional research is required for their application in specific calculation.

Physico-mathematical model of non-stationary combustion of solid fuel

The physico-mathematical model of non-stationary solid fuel combustion adopted in this study is based on the findings of works [2], [3], [4], with certain refinements and additions. The assumptions made during the model construction are primarily formulated in work [3]:

1. the solid fuel is homogeneous and isotropic;

2. the combustion surface is a plane; heat release in the combustion phase occurs only on the combustion surface;

3. the flame front is a plane parallel to the combustion surface; heat release in the "dark" zone does not occur;

4. processes occurring behind the flame front (secondary reactions, combustion
of dispersed particles) do not affect the combustion rate;

5. the processes of solid-phase decomposition and combustion in the gas phase occur much faster than the heating of the combustion phase, i.e., the gas phase is considered non-inertial.

All processes are considered in a one-dimensional setup in a coordinate system attached to the combustion surface. The positive direction of the longitudinal coordinate is considered as the direction into the solid fuel.

The accepted assumptions have been confirmed by experimental publications [1], [3], [4], [5] and [27]. The proposed physical model allows for the mathematical formulation of the problem of non-stationary combustion velocity of solid fuel and the introduction of a linear integral equation. In such a problem formulation, the non-stationary combustion velocity depends not only on the instantaneous temperature gradient, as is the case in a quasi-stationary regime but is determined by the entire history of the combustion process. To convert the calculation of the non-stationary combustion velocity into an integral equation and find the solution to this equation, subsequent paragraphs will address the processes occurring in the gas phase and at the combustion surface. Additionally, the combustion problem will be formulated and solved with an initial condition for the non-stationary equation, involving heat and *k*–phase.

The equation for heat propagation in the *k*–phase

Experimental studies of the solid fuel combustion process indicate that the expanding contribution to the non-stationarity of this process is associated with the condensed phase. In existing physics-mathematical models, the inertia in the *k*–phase is taken into account by the heat conduction equation. According to the work of researchers [1], [2] and [3], and others, the heat propagation equation in the *k*–phase has the following form:

$$\frac{\partial T}{\partial t} = æ\frac{\partial^2 T}{\partial x^2} + u(t)\frac{\partial T}{\partial x}. \tag{4}$$

The boundary conditions for equation (4) in a general form are given by:

$$T(0,t) = T_s(t), \tag{5}$$

$$\frac{dT}{dx}(0,t) = f(t), \tag{6}$$

$$\lim_{x\to\infty} T(x,t) = T_N, \qquad \lim_{x\to\infty}\frac{dT}{dx}(x,t) = 0. \tag{7}$$

The initial condition in a general form is given by:

$$T(x,0) = T_N + \varphi(x). \tag{8}$$

The heat release in the *k*–phase is accounted for by the heat balance equation on the combustion surface.

$$\lambda_g f_g(t) = \lambda f(t) + \rho u(t) Q_s. \tag{9}$$

$$g(p) = \alpha_p S(p)\sqrt{RT/(2\pi)}. \tag{9}$$

In the case where the fuel does not contain combustion catalysts in its composition, the recombination probability α_p is close to zero, so in the first approximation, it is considered that $g(p) = 0$. The general case will be discussed later.

Using the Fourier method, we found the representation in the form of the Green's function solution with initial and boundary conditions for the heat conduction equation (2).

As a result of the solution, a temperature field was obtained containing the combustion velocity as an unknown variable parameter. The representation of the result in the form of a green's equation solution (2) with boundary conditions (4), (5), and initial conditions (6) takes the following form

$$T(x,t) = T_N + \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\varphi(z(x,t,x_1))e^{-x_1^2}\,dx_1 + \cdot\sqrt{\frac{a}{\pi}}\int_0^t F(\tau)e^{-\Psi_*^2(x,t,\tau)}\frac{d\tau}{\sqrt{t-\tau}}, \tag{10}$$

$$\text{onde } z = z(x,t,x_1) = x + \int_0^t u(\tau)\,d\tau - 2x_1\sqrt{at},$$

$$\Psi_*(x,t,\tau) = \frac{x + \int_\tau^{\check{}} u(\tau)\,d\tau}{2\sqrt{a(t-\tau)}},$$

$$F(t) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\left.\frac{d\varphi(z)}{dz}\right|_{x=0} e^{-x_1^2}dx_1 - f(t).$$

The functions $\varphi(x)$, T_s, and $f(t)$, which are involved in the initial and boundary conditions, are specified.

For the function $\varphi(x)$, which is involved in the initial conditions (6), we consider the steady-state temperature distribution, i.e., the Michelson distribution.

$$\varphi(x) = (T_{s_0} - T_N)\exp\left(-\frac{u_0 x}{a}\right), \tag{11}$$

que foi obtido de (2), (6), (7) se $\frac{\partial T}{\partial t} = 0$, $u(t) = u_0$,

$T_s = T_{s_0} = const.$ The expression for $T_s(t)$ is derived from formula (1) in the form of a one-to-one dependence of the surface combustion temperature on the combustion rate of the fuel.

$$T_s(t) = T_N + (T_{s_o} - T_N)\bar{T}_s(U), \tag{12}$$

where $\bar{T}_s = \bar{T}_s(U) = \frac{1+\gamma\theta\ln U}{1-\gamma\ln U}$, $U = U(t) = \frac{u(t)}{u_0}$, $\gamma = \frac{RT_{s_0}}{E_s}$, $\theta = \frac{T_N}{(T_{s_0}-T_N)}$.

The analytical expression for the temperature gradient on the side of the kphase,

$$f(t) = -\frac{u_0}{a}(T_{s_0} - T_N)U(t)Q(t) \tag{13}$$

it is obtained from the definition of the heat flux from the gas phase to the k-phase (8) and the heat balance equation at the combustion surface (3),

$$P = P(t) = \tfrac{p(t)}{p_0},\quad Q(t) = Q(P,U) = (1-\sigma)\tfrac{P^{2\nu}}{U^2}\tfrac{(1-\gamma\eta\ln U)}{(1-\gamma\ln U)} + \sigma,$$

$$\sigma = \frac{Q_s}{c(T_{s_0} - T_N)},$$

$\eta = \frac{T_{fr}}{T_{fr}-T_{s_0}}$, $2\nu = m+1$. It is assumed that the function $P(t)$ is known. Finally, the general solution of equation (2) with boundary conditions (4), (5), and initial conditions (6), where the functions $f(t)$ and $\varphi(x)$ are specified according to formulas (13) and (11), takes the form:

$$T(x,t) = T_N + (T_{s_0} - T_N)\left(e^{-\frac{u_0}{a}\left(\int_0^t u(\tau)\,d\tau - u_0 t + x\right)} + \right.$$

$$+\frac{u_0}{\sqrt{a\pi}}\int_0^t \left(U(\tau)Q(\tau) - e^{-\frac{u_0}{a}\left(\int_0^\tau u(\tau_1)\,d\tau_1 - u_0\tau\right)}\right) \tag{14}$$

$$\left. e^{-\Psi_*^2(x,t,\tau)}\frac{d\tau}{\sqrt{t-\tau}}\right).$$

Now, let's proceed to the second stage of solving the problem regarding the derivation of the main equation for calculating the non-stationary combustion velocity. We will integrate equation (2) term by term with respect to *x* within the limits from 0 to + ∞ Taking into account the boundary conditions, we obtain the energy balance equation in the end of *k*-phase

$$\frac{d\varepsilon}{dt} + af(t) + u(t)[T_s(t) - T_N] = 0, \quad \text{where}$$

$$\varepsilon = \int_0^\infty [T(x,t) - T_N]\,dx. \tag{15}$$

After substituting the temperature values from (14) into (15), and also after substituting the specified functions into (15), $T_s(t)$ and $f(t)$ In accordance with (12) and (13), you obtain the nonlinear equation involving the unknown function U(t) for calculating the non-stationary combustion velocity.

$$\int_0^t \left[U(\tau)(\bar{T}_s(\tau) - E(\tau)) - (U(\tau)Q(\tau) - E(\tau))erf(\Psi(t,\tau))\right]d\tau = 0. \tag{16}$$

Here $E(t) = e^{-n\left(\int_0^t u(\tau)\,d\tau - t\right)}$, $\Psi(t,\tau) = \Psi_*(x,t,\tau)\Big|_{x=0}$,

$$erf(\Psi) = \frac{2}{\sqrt{\pi}}\int_0^\Psi e^{-x_1^2}\,dx_1,$$

$erfc(\Psi) = 1 - erf(\Psi)$, $n = \frac{u_0^2}{a}$. Equation (16) is the energy conservation equation for the k-phase. We will refer to it as the main equation for the non- stationary combustion velocity.

Stationary Combustion Physical-Mathematical Model

Let's consider a physical-mathematical model of stationary combustion of solid fuels as a special case of the non-stationary combustion model presented in the first two sections. The issue of combustion of fuels containing catalysts has also been addressed separately. A stationary model of solid rocket fuel combustion is proposed, which includes the following equations within it

• The energy balance equation for the *k*-phase in stationary conditions (Zel- dovich's equation) is given by:

$$\lambda f(u,p) + c\rho u(T_s - T_N) = 0.$$

• The heat balance equation at the combustion surface. (3).

• Dependency $T_s(t)$ the combustion velocity of the fuel (12).

• Equation for calculating the temperature gradient in the gas phase at the combustion surface.

$$\frac{\lambda_g}{c\rho}f_g(u,p) + u[T_s(u) - T_N - \frac{Q_s}{c}] = 0.$$

• Formula (8) for calculating the heat flux from the flame front to the combustion surface.

• Formula (9) for the number of effective collisions of free radicals with the surface of condensed particles present in the" dark zone."

An equation for calculating the stationary combustion velocity was obtained using the heat flux equation (8) and formula (9). The combustion process of a fuel that does not contain catalysts in its composition was examined in greater detail. In this case

$$\phi(p) = C_5 p - C_2 p^2, \quad \tau(p) = C_0 p^{-m}, \quad h(u,p) = \frac{C_4 u^q}{p^{q+m}},$$

$$f_g(u,p) = -\frac{(T_{fr} - T_s(u))p^{q+m}}{C_4 u^q}.$$

Here, C_0, ... C_5 – these are values that do not depend on pressure, $C_5 \approx 2C_1$. Theoretically, a power law for the combustion velocity is deduced.

$$u = u_1 p^\nu, \text{ where } u_1 = \left[\frac{\lambda_g(T_{fr} - T_s(u))}{c\rho C_4(T_s(u) - T_N - Q_s/c)}\right]^{\frac{1}{1+q}}, \tag{17}$$

$\nu = (m + q)/ (1 + q)$. here u_1 is function T_s and T_N. The process of stationary combustion in the presence of catalysts in the fuel was examined. In this case, as particles pass through the gas flow region, they change in size, i.e., they condense. Some metals present in the catalysts, such as lead, exist in a supersaturated vapor state in the "dark zone." Collisions between free radicals and supersaturated vapor molecules or condensation nuclei lead to radical deactivation, i.e., chain termination, and consequently, an increase in the induction period and the width of the" dark zone."

The proposed mechanism of interaction between free radicals and metals in a supersaturated vapor state allows for refining the heat flux formula for cases where particles condense as they pass through the gas flow region. By using the Gibbs formula for the distribution of condensation nuclei by size, formulas were obtained for the specific surface area of the droplets, corresponding formulas for the rate of formation of free radicals, as well as for the induction period, the width of the "dark zone," and the heat flux from the gas phase to the k-phase.

In this case, the combustion velocity law over the entire range of pressures considered has the following form:

$$u = u_1 p^\nu F(p), \tag{18}$$

where $F(p) = \left(1 - \frac{a_1 e^{-a_2/p^r}}{p^k}\right)^{\frac{1}{q+1}}$, a_1, a_2, k, r constants determined by the physicochemical characteristics of the catalytic substance. The combustion velocity law (18) exhibits a pressure region referred to as the "plateau", characteristic of fuels containing lead compounds. The dependence (18) is also reflected in the results of experiments conducted at the Institute of Scientific Research of Propulsion Engines to measure the combustion rate.

Calculations of the coefficients in the law of stationary combustion velocity in the presence of catalysts in the fuel composition were carried out. A method for calculating the constants that are part of the integral equation for the combustion velocity is proposed.

An Approximate Solution of the Main Equation for Non-Stationary Combustion Velocity

Let's obtain the approximate solution of the main equation for non-stationary combustion velocity.

A linear approximation for the main equation describing the combustion rate of a solid fuel has been obtained. It is assumed that

$$U(t) = 1 + \alpha(t), \tag{19}$$

$$P(t) = 1 + \beta(t), \;\; y(t) = \int_0^t \alpha(\tau)d\tau, \;\; \alpha^2 \ll 1, \;\; \beta^2 \ll 1, \;\; y^2 \ll 1.$$

Under these assumptions, for equation (16), a linear approximation equation was obtained.

$$\int_0^t \left(D\alpha(\tau) + ny(\tau) - (A\beta(\tau) + ny(\tau) - B\alpha(\tau))erf\left(\frac{\sqrt{n(t-\tau)}}{2}\right)\right) d\tau = 0, \tag{20}$$

This corresponds to a first-order Volterra type convolution integral equation, which is linear in terms of $\alpha(t)$. Here, $A = 2\nu(1-\sigma)$, $B = 1 - 2\sigma+\gamma(1-\sigma)(\eta-1)$, $D = \gamma(1 + \theta)$. By using the Laplace integral transform, the general analytical solution of equation (20), which is the linear approximation of the main equation for non-stationary combustion rate, was obtained. It was demonstrated that this solution can be expressed as a linear integral operator of the Volterra convolution type.

$$\alpha(t) = \int_0^t M(z(\tau))\beta(t-\tau)d\tau, \quad where \;\; z = z(\tau) = \sqrt{n\tau}/2, \tag{21}$$

$$M(z) = \frac{An}{4D}\left(\frac{e^{-z^2}}{\sqrt{\pi}z} - \bar{a}\frac{(1-\bar{a})}{\bar{b}-\bar{a}}e^{-(1-\bar{a}^2)z^2}erfc(\bar{a}z) + \bar{b}\frac{(1-\bar{b})}{\bar{b}-\bar{a}}e^{-(1-\bar{b}^2)z^2}erfc(\bar{b}z)\right).$$

n.They determine the form of the solution, that is, the presence or absence of an oscillatory process. So, if $\bar{a}$ and $\bar{b}$ are real numbers, the calculation of the combustion rate is performed using the formulas (19) and (21). If $\bar{a}$ and $\bar{b}$ are complex numberssão números complexos, then the formula for calculating the combustion rate is written as

$$U(t) = 1 + \frac{An}{4D}\int_0^t \beta(t-\tau)\left(\frac{e^{-z^2}}{\sqrt{\pi}z} + F_1(z)e^{Nz^2}\cos(\omega z^2 + \Phi(z))\right) d\tau,$$

where $F_1(z)$, $\Phi(z)$, ω and N are determined by the constants A, B, D, and n. Solutions for the linear approximation equation of the main equation for the combustion rate were obtained under different pressure variation laws. Analytical expressions for the reduced combustion velocity were derived by substituting specific pressure variation laws into the formula (21) in cases of step pressure variations, smooth transitions from one regime to another, and oscillatory pressure variations.

Conclusion

At the same time, the characteristics of the Missel Motor of Solid Propellant were calculated using the stationary combustion velocity law obtained for fuel containing combustion catalyst in its composition. Based on the calculation results, the following conclusions can be drawn: In the range of small pressure gradients, the calculation results, both in stationary and non-stationary theory, satisfactorily coincide with experimental results. In the initial regime, the calculation results in non-stationary theory differ from experimental results by no more than 1 to 2 percent, while in the case of calculations in stationary theory, this discrepancy is around 10 percent.

Funding

Funding: This work received financial support from UNISCTEM - University of Sciences and Technology of Mozambique

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