

Revival Policy of a Ruined Firm

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ABSTRACT

This article examines the problem of a manager who plans to combat with the disaster of war and epidemic with reducing the social costs simultaneously. If we move forward to slow policies then it may regulate the effective spread rate of the disease with randomness. We present a comprehensive theoretical analysis demonstrating the formation of the most favourable policy. In all our tests the latter is demonstrated by three different stages: the disaster is first freely affect the units of a firm and damaged it vigorously and then the manager of the firm tries to put up a fight against the disaster and finally a regeneration policy is adopted to revive the damaged units. Stochastic process indicates the premature position of the disease in its first stage with the positions of the boost growing rate of the disaster which is controlled after a long time.

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Introduction

The effect of disaster on firms has received widespread attention in the recent past. Most of it was affected in the units of a firm due to Pandemic virus and different types of battle in most part of the globe. Only recently has there been more interest in how armed conflicts affects physical infrastructure and human capital both of which can have a shocking blow on both persons and firms. There are also some investigations of how battle affects firm level productivity which is influenced by technology, capital, organisational structure, management practices and the number of workers as well as their skills [1,2]. Then, during the last two years, both economy and production growth retarded significantly, and economy began to jettison its growth dramatically. For the first time in this century, a significantly distributional crisis is observed. Pandemic and armed conflicts have created a tension among the working age people as they have begun to lose their work due to lockdown and destruction of industries. We have entered an entirely new phase in our economic activities.

The retreat of the growth of economy due to closure did completely break the economic structure of small business setup. The main difficulties of the firms were that the owners had little cash at the starting off after disaster. These uneven and unprecedented circumstances compelled the firm owners to cut the expenses, take additional loan and ultimately declare bankruptcy. They can, therefore, be tried to seek funding through relief aid with having laid the true example of welfare economy.

Battles, it is true, did start its impact on economy amid fanfare and create weak units of a firm. But reparations for the damage done to financial position of a country must soon to follow. It is

seen that weak firms matter for aggregate productivity by dragging down average (unweighted) productivity and also by consuming scarce resources they congest markets, and also deter entry of potentially innovative young units. The typically healthy firm's investment is less as capital is deepening, particularly more capital is sunk in firms, as it affects on young and more productive units. The economists paid their attention for remedy and recommended different measures such as cutting interest rates of central banks and application of monetary, fiscal and health. If deep recovery of economy is a firm's aspiration, the essence of public policy; rehabilitation of weak units of a firm thus implying lower social costs to job churn than if only exist was envisaged; and also reallocation of capital to weak units of firm and productivity growth of laggard units via more efficient technology diffusion are the needed reforms. The function of the armed conflicts were to kill people of any category of socio-economic group without differing rich and poor and strong and weak units of a firm. So it is necessary to invest more money on insolvency reform to reduce disordered congestion. But at the first time, we have to study the effects of armed conflicts on economies of a country.

Impact of Armed Conflicts on Economy

The empirical evidence of five days armed conflicts in Georgia in August 2008 showed that over 1,00,000 people were displaced and there was a significant harm to roads estimating US \$150 million, destruction of civilian infrastructure and property calculating approximately US \$ 350 million and lost fiscal revenues measuring US \$ 300 million in the second half of 2008 [3]. This amounts to about 6.3 percent of Georgia's GDP in 2008, - a considerable amount for such a small period.

Afghanistan and Syria rank as the least peaceful countries globally and suffer the highest economic cost of violence as measured against their GDP. It is seen that Afghan Government did not provide the basic amenities to their people due to unable to collect

the fiscal revenues, resulted a significant economic losses and steady state depression [4]. Adding to this, the war in Afghanistan also lowered national income by 50% in 2016 that is approximately US \$ 1billion [5]. Also in the case of Syria, the country fell from the lowest 14-th rank to the 2-nd lowest in the year 2017 due to armed conflict [5]. In the current discourse, pandemic also plays an infamous role.

Impact of Pandemic on Economy

Zhu is credited with having given the first information of pneumonia case on December 8, 2019 in a wet market in Wuhan, the capital city of Hubei Province of China [6]. They studied the economic and social impacts of corona virus and endeavoured to understand the socioeconomic effects of lockdown with the role of government to the pandemic. The researchers set the ball rolling. Corona virus inaugurated a new age in economics [7].

In the virus disaster period 2019-20, corona virus affected five key economic indicators namely GDP, Unemployment rate, Inflation rate, Interest rate and Industry output. All these factors sounded amusing; but they were the scenarios of Indian economy. GDP growth rate had recourse to - 2 %, - 4 % and - 6 % respectively in the year 2019 to 2021 [8]. Yet the highly significant was unemployment rate. It took 2 %, 6 % and 12 % increase. Inflation rate also drew the attention of economists as it was confined to growth amid 5 % to 6 % [9]. RBI (Reserve Bank of India) made a significant study of marginal standing facility rate and cut it by 75 basis points taking it down to 4.65 % [10].

From the above discussion it is observed that the effect of pandemic on the economy had been fast and deep such as the U. S. unemployment rate soared to 14.7 percent in April 2020, up from 4.4 percent about a month earlier [11]. The role of the non-linear formation of the primary strong set-up of the maximal investigation policy constituting of lockdown and quarantines plans is generally difficult but it is reviewed systematically by the investigators [12]. The researchers find out a relationship between human capital loss and unemployment due to pandemic by studying labour market carefully [13]. They appropriately caution that pandemic may lead to a considerable diminishing of total factor productivity and sick the firm at last.

On Sick Units of a Firm

Now it is critical to ask whether all the units of a firm become sick or not, in front of the attack of pandemic and armed conflicts? This is a related and conceptually important question and from that point, we will name the duo pandemic and armed conflicts as a disaster. Disaster does affect a firm of course, but there is no strong reason to expect that it would sick all the units of a firm at a time.

Now, the question is what is the definition of a sick unit of a firm? A unit is sick when it is failed to support itself through the operation of internal resources. Furthermore, its snowballing cash losses are equal to fifty percent or more of its peak net worth during the last five years is also an added property of sick unit. One more condition of sick is that it has defaulted in meeting four consecutive instalments of interest. In India, if a unit is remained closed for a period of more than six months, then Development Commissioner may announce it as a sick.

The causes of sickness are both internal and external and also often a combination of both. External factors are government policies on pricing, duties, taxes, high interest rates, taxes on profit, slackness in demand, sluggishness in export markets, high labour cost, inadequate availability of inputs, lack of infrastructure

etc. The internal factors are wrong planning, wrong location, faulty technology, capital cost, and technological obsolescence, management deficiencies and industrial unrest.

In this paper we propose three types of units of a firm such as units approaching to sickness, disordered and redeemed due to disaster in which both time and sickness rate are stochastic. We study the spreading rate of disaster in a stochastic process with limited in time as the duration of disaster is limited. The manager seeks a process to minimize the anticipated expenditures with a time limit concerning a relief from any source, may be government or non-government, and is received. We have taken time as arbitrary and free from Weiner operation.

We have considered a second order continuous and differentiable lowest expenditure function applied on HJB equation. Spreading rate as a circumlocutory chance variable with an absolute study of the control case is the premier investigation so far concerned. It is known that spreading rate tends to fall after hitting an optimum. The movement of the spreading rate is between 0 and $\gamma > 0$. Here the manager applies the sickness policy by studying the average mean of the sickness rate which ultimately converges exponentially. Furthermore, a separate cost function is introduced which is quadratic in nature both in the manager's attempt and in the sick units also.

The sickness effect of units of a firm, focussed in our study, is characterized by three distinct types. In the first case, the disaster evolves in different units, then the manager imposes different restrictions on costs and cost related parameters and in the last case the restrictions may be relaxed by the manager. We also inspect the consequence of the maximal intensity M measuring sickness rate (the revival policy from sickness fluctuates between 0 and M) of the recovered which is less than M. The difference between the utmost sickness of the units and the last recovered would be considered as maximal between tariffs and recovered units happens when $M = 1$ is considered. Research aimed at ascertaining the consequence the starting of the closing plans of units and weakening the worse effect of economy of the firms in the longer period. Note, however, that an unpredictability of the sickness rate persuades manager to operate promptly and carry the process in the long run with a view to avoid heavy blow of α .

The outline of the paper is as follows. We begin with a conceptual overview of the issues relating to unpredictability of sickness rate and cost related parameters. The literary review is discussed in section 2. Section 3 develops a model that describes a revival policy of the sick unit of a firm by a manager. The concluding remarks belong to section 4.

Literary Review

As discussed earlier, we approach the series of issues of health and economic policy by constructing a framework which appears at the transaction between estimates capable of containing infection and the firms able to keep away from economic fall down. The questions briefly outline such a framework – developed by and from it extracts key problems around the characteristic's traits of the current epidemic which we consider in the economic case of the firms in our model [12,14-15]. The impact of pandemic fell heavily on firms due to closures and mass layoffs and uncertain length of the disaster. Many units of a firm are financially weak and the weakness augments as length of the crisis time increases. For those reason firm owners especially small businessmen always try to seek financial help from different governmental aids established at the time of pandemic [16].

Think of a unit of a firm that is affected by pandemic or armed conflicts. Hence it affects the financial condition of a firm that is comprised of interest rate, inflation rate and industry output. It also includes GDP of a country and unemployment rate [10]. Taken together, these economic indicators show that there should be an interconnection between war and economy. There are so many papers regarding war and its effect on socioeconomic indicators in different countries, such as on Libya, Syria, Croatia, Sri Lanka, Rwanda, and Ukraine [17-23]. A detailed and extensive study of the long run effects of war on the economic performance in Afghanistan is studied by Hameed; giving an unbalanced increase in prices and the increase of cost of living in the war-affected countries [24].

Armed conflicts with related economy are associated with risk and uncertainty. Risk is considered as an incident that has negative impacts on the outcome of the firm's operation and also do impact on the sustainability of the supply chain [25-27]. As the supply chain is associated with uncertainties because a firm is itself dynamic in nature and carries a risk propensity then the effect in disaster led firm, is driven by a probability space comprised of Autonomous Planning, Installed Capital, Measurable Space of Sickness of a Unit and Internal Planning [28].

The high intensive innovative companies always try to invest on autonomous planning to get profit as planning aimed at ascertaining the circumstances under which policies are conducive to get rid from the risk propensity and at scrutinizing the paths through which policies influence the economic performance is likely to get more productive [29].

Alongside this lessened policy, there is an interpretation on the investment good and stock market prices behave differently as reflecting the preserve of costs for installing new capital [30]. The discussion in the study of the above papers tries an answer to an obvious question: What type of capital affect the economic performances most? The simple answer is nothing but installed capital.

Barring the installed capital, the continuing financial pressures on firms lower investment and employment; and finally create zombie firms that are unable to cover debt servicing costs from current profits by misallocating resources and productivity growth as discussed by Banerjee & Hoffman [31].

Jeseviciute-Ufartiene undertakes an analysis using Spearman's correlation method between development and planning in Lithuanians firms. The study reveals that managers are far from having been related directly with the planning of management and development but they do not deny its importance [32]. The issue of planning and management is studied by Ghani with the help of SWOT analysis in Malaysia. This method studies the critical internal and external factors of an organization and also indicates the firm's potencies and faults so as to be getting ready for the risks and chances outside [33].

By studying the papers cited above we have tried to establish a relationship between the economic performances and the probability space, comprises of autonomous planning, installed capital, sickness of a firm and internal planning. Research aimed at ascertaining the factors under which probability space are conducive to economic performances and at scrutinizing the channels through which factors influence economic performances is likely to prove more productive.

Model

Let G be a group comprised of five economic indicators such as GDP, Unemployment rate, Inflation rate, Interest rate and Industry output. Let k be a factor belongs to G . We define $\alpha(t)$ to be the proportion of k at the time of sickness of a unit of a firm and k at the time of normal situation. So, $\alpha(t) = 1$ normally apart from sickness situation. As soon as the smoothness of unit deteriorates, $\alpha(t)$ is dropped below 1 which damages economy. There have been characteristics to be studied in the case of $\alpha(t)$. The first characteristic is that the policy makers did get little chances to recover it when unit started to sick. Second, it needs time to restart the business after sickness. The real fact is that some industries especially small-scale industries have gone insolvent due to lack of capital at that crucial time.

We consider $A_t + D_t + R_t = 1$ for all time $t \geq 0$, where A_t is the percentage of units of a firm that are approaching to sickness, D_t is the percent of disordered units and R_t is the fragment of redeemed units. Big farms are reliant on small firms and ancillary those are also dependent on labour power. Letting α_t be the rate of sickness of units of a firm at small span of time dt . Let the fraction of units of a small firm that are closed due to unexpected situation be $\alpha_t A_t dt$ in time dt . The percentage of disordered firms that get disturbed due to shut down of few units of a firm within dt time is $D_t \alpha_t A_t dt$. The fraction of disordered firms is opened by γD_t since the units are recovered from unexpected situation and open the units within time dt .

The movement of A_t and D_t is written as,

$$dA_t = -D_t \alpha_t A_t dt, t > 0, A_0 = a, (1)$$

and

$$dD_t = (D_t \alpha_t A_t - \gamma D_t) dt, t > 0, D_0 = b, (2)$$

where a and b belongs to zero and one, such that $a + b$ also belongs to zero and one.

Note that for any $t \geq 0$, and for any choice of $(\alpha_t)_t$ we can deduce

$$A_t = a e^{-\int_0^t \alpha_u D_u du} \text{ and } D_t = b e^{-\gamma t + \int_0^t \alpha_u A_u du}, (3)$$

and therefore $A_t > 0$ and $D_t > 0$ for all $t \geq 0$.

Moreover, $\frac{d(A_t + D_t)}{dt} = -\gamma D_t < 0$ for all $t > 0$, which means that

$$A_t + D_t < 1 \text{ for all } t > 0.$$

We assume that the sickness rate α_t of a unit of a firm is dependent on time, random but likely regulated. We define a probability space $\{\Lambda, K, P = (K_t)_t, I\}$, where Λ = Autonomous Planning, K = Installed Capital, P = Measurable space of sickness of a unit and I = Internal Planning. We also define a linear Brownian motion W_t . For a specified and stable M greater than zero, we take $(\chi_t)_t$ in $\mathfrak{B} = [\chi : \Lambda \times [0, \infty) \rightarrow [0, M], (\chi_t)_t]$. (4)

We also suppose that the sickness rate is influenced by random events rather than deterministic pathways,

$$d\alpha_t = a(\alpha_t, \chi_t) dt + \varphi(\alpha_t) dW_t, t > 0, \alpha_0 = c > 0. (5)$$

Turning to the flow of sickness, it is observed that the procedure $(\chi_t)_t$ has a significant impact on it, and therefore it is the sole duty of a manager to lower the flow. Now, $\chi = 0$ indicates that no measure is taken to put an end to the flow of sickness, and in the contrast $\chi = M$ shows that the necessary step taken is optimal. The point to be noted that the movement $(W_t)_t$ obeys a seemingly random motion of impulses influencing the flow of sickness emerging out

of the random collisions of those impulses and it is beyond the command of the manager.

Now to study the activities of $(\alpha_t)_t$ the ensuing presumptions are taken.

Presumption 1.

i) For all $\chi \in \mathbb{B}$, an exceptional result comes from equation (4) and it takes the values from $I \subseteq (0, \infty)$.

ii) a : $I \times [0, M] \rightarrow \mathbb{R}$ posses a definite derivatives, so that $\kappa_a > 0$ gives,

$$\sup_{n \in \mathbb{N}} \sup_{(c, \chi) \in I \times [0, M]} \left| \frac{\partial^n}{\partial c^n} a(c, \chi) \right| \leq \kappa_a .$$

iii) μ represents the values of I which is taken taken from all the real numbers from zero to infinity excluding zero and infinity and possesses an upper bound and lower bound also. It is a continuous function and takes the infinite number of derivatives on the values of I with respect to change in its input values and the rate of change of function is confined within a certain range irrespective of operations and hence $\kappa_\psi > 0$ exists so that

$$\sup_{n \in \mathbb{N}} \sup_{z \in I} |\mu^{(n)}(z)| \leq \kappa_\psi .$$

$(\alpha_t)_t$ measures a rational and effective per unit of sickness and eventually return to their long term average so that,

$$d\alpha_t = \lambda (\tilde{\alpha} (M - \chi_t) - \alpha_t) dt + \mu \alpha_t (\zeta - \alpha_t) dW_t, t > 0, \alpha_0 = y \in (0, \zeta), (6)$$

for some $\lambda, \zeta, \mu > 0, \tilde{\alpha} \in (0, \zeta)$. Here α and ζ are not able to achieve the spreading by the sickness rate of $(\alpha_t)_t$, that picks up from $I = (0, \zeta)$ due to time t greater than zero. Here $\tilde{\alpha}$ is considered as an innate and decaying sickness process and eventually returns with λ in the case of $\alpha = 0$. It concludes that ζ shows the optimal effect of sickness measured per unit and μ takes the measurements of $(\alpha_t)_t$ all over $\tilde{\alpha}$.

The Firm Manager Problem

The sickness of a unit generates firm costs, that increases with respect to the fraction of the ancillaries are hampered. The revival process of the unit leads to raise the cost due to former unused production at the time of no working. The firm manager employs policies $(\chi_t)_t$ to adjust the sickness rate α to reduce the sickness. Suppose that a possible revival is possible against the sickness at a certain period of flow of time τ . Here τ moves as a function of exponential and probability distribution with a fixed positive limit v_0 free from $(W_t)_t$. Now the manager tries to solve the problem

$$\inf_{\chi \in \mathbb{B}} E \left[\int_0^\tau e^{-\delta t} C(D_t, \chi_t) dt \right] . (7)$$

Here, $\delta \geq 0$ estimates the options of the firm manager and also calculates the continuous tariffs computing the anti-effect of the sickness of the unit of the firm by F which maps $[0, 1] \times [0, M]$ to $[0, \infty]$.

The ensuing requisites are fulfilled by F .

Presumption 2:

a) $(y_1, \chi) \rightarrow F(y_1, \chi)$ satisfies the law of convexity and continuity which is acted on the product of closed interval between zero and one and also of closed interval between zero and M .

b) If y_1 belongs to the closed interval of zero and one $y_1 \in [0, 1]$, it is taken that $\chi \rightarrow F(y_1, \chi)$ is non decreasing.

c) If χ belongs to the closed interval between zero and M , it is taken that $y_1 \rightarrow F(y_1, \chi)$ is non decreasing.

d) For positive number K_1 and for χ belong to the closed interval of 0 and M for $S = [0, 1] \times [0, 1]$, $F(y_1, \chi)$ satisfies a Lipchitz condition that is

$$|F(y_1, \chi) - F(y'_1, \chi)| \leq K_1 |y_1 - y'_1|, \text{ for all } (y_1, y'_1) \text{ belongs to } S.$$

e) Now if y_1 maps to $F(y_1, \chi)$ and the property of semi concavity holds on the closed set of zero and one with the property of uniformity in regard to $\chi \in [0, M]$, such as, for a positive K_1 and for $\chi \in [0, M]$ and $\xi_1 \in [0, 1]$ obeys

$$\begin{aligned} & \xi_1 F(y_1, \chi) + (1 - \xi_1) F(y'_1, \chi) - F(\xi_1 y_1 + (1 - \xi_1) y'_1, \chi) \\ & \leq K_1 \xi_1 (1 - \xi_1) |y_1 - y'_1|^2, \text{ for all } (y_1, y'_1) \in [0, 1] \times [0, 1]. \end{aligned}$$

We can assume $F(0,0) = 0$ and $y_1 \rightarrow F(y_1, \chi)$ indicates the increasing costs of the firm when huge number of ancillaries are disturbed from various reasons.

Without loss of generality, we take $F(0, 0) = 0$. Convexity of $y \rightarrow F(y, \chi)$ captures the fact that the firm costs might be higher if large number ancillaries are disturbed from various reasons.

We have applied Fubini's theorem by stating τ and $(W_t)_t$ and rearranging the problem of (7) as

$$\inf_{\chi \in \mathbb{B}} E \left[\int_0^\tau e^{-\delta t} F(D_t, \chi_t) dt \right] = \inf_{\chi \in \mathbb{B}} E \left[\int_0^\infty e^{-\rho t} F(D_t, \chi_t) dt \right] (8)$$

where $\rho = \rho_0 + \delta$.

Here it is suggested that the firm manager is fully aware of the degree of development of unit Q_t which will be achieved soon and also assume this develops as a random equation as stated below,

$$dQ_t = \xi(Q_t) dt + \Theta(Q_t) dA_t, Q_0 = q \in \mathbb{R}_+,$$

for appropriate ξ and Θ , and $(A_t)_t$ which is unconstrained of $(W_t)_t$ obeys the usual Brownian motion. There exist always blows from external to disturb the system $(A_t)_t$ at the time of the reviving achievement of the unit while ξ computes the instant movement of the revival condition. We suppose a notation $(M_t)_t$ for Markov process which is continuous and indexed by time bearing two positions zero and one, where zero indicates that the problem is unsolvable and one means that a ray of hope has appeared to solve the problem. It is also accepted that the position one is a state that once entered cannot be left and the process of Markov bears the changing estimate from position zero to position one under the equation $(\psi(t, Q_t))_t$. Here, $\mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ gives

$$\Delta_t = \int_0^t (\psi(s, Q_s)) ds < \infty \text{ for each and every positive time } t. \Delta_t$$

(s_1, s_2) satisfies the condition $\Delta_t(s_1, s_2) \leq \Delta_t(s_1, s_3)$ whenever $s_2 < s_3 < s_3$. This indicates that the critical the problem is, the quicker the problem is handled by the manager.

The problem can be rewritten as,

$$\inf_{\chi \in \mathbb{B}} E \left[\int_0^\tau e^{-\delta t} F(D_t, \chi_t) dt \right]$$

where $(D_t)_t$ and $(\chi_t)_t$ are stated earlier. Again,

$$\tau_1 = \text{infimum} \{ M_t = 1, \text{ for positive time} \}.$$

Q is unchangeable when W is considered and it gives a similar expression,

$$J(a, b, c, q) = \inf_{\chi \in \mathcal{B}} E \left[\int_0^\infty e^{-\delta t - \Delta t} F(D_t, \chi_t) dt \right],$$

which represents a four-dimensional random function. This question is left to the future researchers.

In order to tackle the problem from dynamic programming, it is convenient to keep track of the initial values of (A_t, D_t, α_t) . It is considered,

$$G = \{(a, b, c) \in R^3 : (a, b) \in (0, 1) \times (0, 1), a + b < 1, c \in I\}.$$

Now, one can take (A_t, D_t, α_t) for $(a, b, c) \in G$ with $\chi \in \mathcal{B}$ and rewrite as $(A_t^{a,b,c;\chi}, D_t^{a,b,c;\chi}, \alpha_t^{c;\chi})$. Here the equations (3) and (4) show that A_t and D_t rely on (a, b, c) and also on χ by α_t , so that α_t relies on c and χ . Then one can place $(A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c) = (A_t^{a,b,c;0}, D_t^{a,b,c;0}, \alpha_t^{c;0})$ to get the results of (1), (2), and (4) at the time of $\chi = 0$.

Now each and every $(a, b, c) \in G$, one can obtain the viscosity solution to the HJB equation performed by a manager

$$J_1(a, b, c) = \inf_{\chi \in \mathcal{B}} E \left[\int_0^\infty e^{-\rho t} C(D^{a,b,c;\chi}_t, \chi_t) dt \right] \quad (9)$$

Here F is nonnegative and J_1 is well defined and solves the dynamic problem.

Dilemma of a manager and its findings

By taking L_1 as a differential operator reacted on $F^{1,1,2}(R^3)$ and gives the equation,

$$(L_1 \varphi)(a, b, c) = a b c (\varphi_b - \varphi_a)(a, b, c) - \gamma b \varphi_b(a, b, c) + \frac{1}{2} \mu^2(c) \varphi_{cc}(a, b, c). \quad (10)$$

Further, for some $(b, c, h) \in (0, 1) \times I \times R$, set

$$F(b, c, p) = \inf_{\chi \in [0, M]} (F(b, \chi) - a(c, \chi)h). \quad (11)$$

It obeys the law of continuity which is acted on $(0, 1) \times I \times R$. Now, considering the presumptions 1 (b) and 3 (d), the existence of an unchangeable positive factor H satisfies the following inequality

$$\begin{aligned} & |F(b_1, c_1, h_1) - F(b, c, h)| \\ & \leq \sup_{\chi \in [0, M]} (|F(b_1, \chi) - F(b, \chi)| + |a(c_1, \chi) - a(c, \chi)| \\ & + |a(c, \chi)| |h_1 - h|) \\ & \leq H(|b_1 - b| + |c_1 - c| + |h_1 - h|). \end{aligned}$$

It is our expectation that J can be solved from the HJB equation

$$\rho j(a, b, c) = (L j)(a, b, c) + F(b, c, j_c(a, b, c)), \quad (a, b, c) \in G. \quad (12)$$

Now, we want to get the primary outcomes such as to solve the equation (12) by J.

Proposition 1:

Here, we find a positive H so that for all $m = (a, b, c)$, $m_1 = (a_1, b_1, c_1) \in G$
 (a) $0 \leq J(m) \leq H$ and $(|J(m) - J(m_1)| \leq H |m - m_1|)$ shows that J satisfies the bounded property and the continuity of Lipschitz property on G;

b) take $\xi \in [0, 1]$ and positive H
 $\xi J(m) + (1 - \xi) J(m_1) - J(\xi m + (1 - \xi) m_1) = H \xi (1 - \xi) |m - m_1|^2$;
 shows the continuity of J on G.
 Further, J comes as a viscous solution to the HJB equation (12).

Proof

The nonnegative and bounded property of F on $[0, 1] \times [0, 1]$ gives the proof of (a). We have considered our problem in the context of time, place and social environment which gives the proof of (b) according to Proposition (1) in Yong and Zhou (1999). Again, the semi-concavity characteristic of (b) is also deduced from Yong and Zhou (1999) in Proposition (4.5). Lastly, the property of viscosity is obtained from Theorem (5.2) of Yong and Zhou (1999).

J resolves Hamiltonian-Jacobi-Bellman equation (12) as a viscous solution and it also satisfies the property of semi-concavity. Hence it gives a desired result.

Proposition 2. J_c bears a nonempty solution set and satisfies the property of continuity on G.

Proof

Suppose $(a, b, c) \in G$. With the property of semi-concavity of J, it is easy to find out the derivatives of J along with the directed path of c at (a, b, c) that is indicated, correspondingly, by

$J_c^-(a, b, c), J_c^+(a, b, c)$. Consequently, it is easy to find out a chronological successive mappings $(\bar{\theta}^n)_n$ which is a subset of $F^2(G)$ from the auxiliary theorem A in Conjecture so that

$$\begin{cases} \bar{\theta}^n(\bar{a}, \bar{b}, \bar{c}) = J(\bar{a}, \bar{b}, \bar{c}) \\ \bar{\theta}^n \geq V_j \text{ in a neighbourhood of } (\bar{a}, \bar{b}, \bar{c}) \\ |D \bar{\theta}^n(\bar{a}, \bar{b}, \bar{c})| \leq \bar{M} < \infty \\ \bar{\theta}_{cc}^n(\bar{a}, \bar{b}, \bar{c}) \xrightarrow{n \rightarrow \infty} -\infty. \end{cases} \quad (13)$$

Now from the Proposition 1, we get the inequality and property of viscosity

$$\rho J(\bar{a}, \bar{b}, \bar{c}) \leq L(\bar{\theta}^n)(\bar{a}, \bar{b}, \bar{c}) + F^*(\bar{b}, c, \bar{\theta}_c^-(\bar{a}, \bar{b}, \bar{c})).$$

By applying (13) and the limiting value of n tending to infinity, it is easy to reach a disagreement. Hence J_c exists for all $(a, b, c) \in G$. Here we are in a position to prove the continuity of J_c . Get a chronological successive mappings $(m^n)_n \subset G$ so that $m^n \rightarrow m \in G$ and take $\gamma^n = (\gamma_a^n, \gamma_b^n, \gamma_c^n) \in D^+ J(m^n)$. The semi-concavity of J indicates that the set is not void. We get $\gamma_c^n = V_c(m^n)$ as J_c exists at all points of G. J maintains the property of semi-concavity on every compact subset of G. The super gradient $D^+ J$ is bounded around its neighbourhood so that it is easy to find out a subsequence $(m^{nk})_k$ that maintains $\gamma^{nk} \rightarrow \gamma = (\gamma_a^n, \gamma_b^n, \gamma_c^n)$. As V_z exists and $\gamma \in D^+ J(m)$ then we get $\gamma_c = J_c(m)$. Therefore it is established that if we take a chronological successive mappings $(m^n)_n \subset G$ which tends to m, it is also possible to draw out a subsequence $(m^{nk})_k \subset G$ so that $J_c(m^{nk}) \rightarrow J_c(m)$. The proof is established from the line of reasoning regarding subsequence.

Hence the above results give us a position to establish that J solves the Hamiltonian-Jacobi-Bellman equation.

Theorem

The ensuing consequences contain:

a) $J \in F^2(G)$ and bears a solution to the Hamiltonian-Jacobi-Bellman equation traditionally.

b) Suppose

$$\hat{\chi}(a, b, c) = \arg \min_{\chi \in [0, M]} (F(b, \chi) - a(c, \chi)J_c(a, b, c)), (a, b, c) \in G. \quad (14)$$

As long as the system of equations

$$\begin{cases} dA_t = -D_t \alpha_t A_t dt, A_0 = a, \\ dD_t = (D_t \alpha_t A_t - \gamma D_t) dt, D_0 = b, \\ d\alpha_t = a(\alpha_t, \hat{\chi}(A_t, D_t, \alpha_t))dt + \varphi(\alpha_t)dW_t, \alpha_0 = z. \end{cases} \quad (15)$$

admits a unique solution $(A_t^*, D_t^*, \alpha_t^*)$, then the control

$$\chi_t^* = \hat{\chi}(A_t^*, D_t^*, \alpha_t^*), \quad (16)$$

is optimal for (9) and $(\alpha_t^*)_t$ bears a maximally restrained estimate of the firm; namely,

$$J(a, b, c) = \text{Mean Value of } \left[\int_0^\infty e^{-\rho t} F(D_t^*, \chi_t^*) dt \right].$$

Proof: Derivation of (a). First Move: Recall (11) and represent $S(a, b, c) = F(b, c, J_c(a, b, c))$. According to equation (11) and with the help of continuous property of F on $(0, 1) \times I \times R$, it is founded that S also maintains the property of continuity on G . Again, hence the range of F is included in a closed interval $[0, 1] \times [0, M]$, J_c is also included in a closed interval of G with the help of Proposition 1 (a), and a (\cdot, χ) is also bounded, that suggests a positive K so that

$$|S(a, b, c)| \leq K, \forall (a, b, c) \in G \quad (17)$$

Set now,

$$j(a, b, c) = E \left[\int_0^\infty e^{-\rho t} S(A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c) \right], (a, b, c) \in G. \quad (18)$$

The differential operator \mathcal{L} satisfies the Hörmander's condition. In fact, for any $m = (a, b, c) \in G$ the process $(M_t^m)_t = (A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c)$ produces a continuous and infinitely differentiable function $p(t, m, \cdot)$, $t > 0$ in R^3 . According to Fubini's theorem it can be rewritten as

$$j(a, b, c) = \int_0^\infty e^{-\rho t} \left(\int_0^\infty S(a', b', c_1) \times p(t, a, b, c; a', b', c_1) da' db' dc_1 \right) dt.$$

Now, as $(a, b, c) \in G$, suppose

$$\tau_n = \inf \{t \geq 0 : | (A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c) | \geq n\}, n \in N, \text{ and observe that the}$$

decisions and values are dependent only on current state that is the property associated with Markov gives

$$\begin{aligned} & e^{-\rho(t \wedge \tau_n)} j(A_{(t \wedge \tau_n)}^{a,b,c}, D_{(t \wedge \tau_n)}^{a,b,c}, \alpha_{(t \wedge \tau_n)}^c) + \int_0^{t \wedge \tau_n} S(A_u^{a,b,c}, D_u^{a,b,c}, \alpha_u^c) du \\ &= E \left[\int_0^\infty e^{-\rho t} F(A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c) dt \mid F_{t \wedge \tau_n} \right]. \end{aligned}$$

It is seen that $j \in F^2(G)$. Then we must try to seek an identity used in the calculus of Ito to find out the differential of a time depended function of a stochastic process by adding other terms. Now, we attain the first moment and notice that the integral of the random possess the null expectation. The result comes from the interpretation of τ_n and the behaviour of the continuity of v_a . It gives

$$\begin{aligned} & E \left[\int_0^{t \wedge \tau_n} e^{-\rho u} (\mathcal{L}j + S - vj)(A_u^{a,b,c}, D_u^{a,b,c}, \alpha_u^c) du \right] + j(a, b, c) \\ &= E \left[\int_0^\infty e^{-\rho t} S(A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c) dt \right]; \quad (19) \end{aligned}$$

that is, by (17),

$$E \left[\int_0^{t \wedge \tau_n} e^{-\rho u} (\mathcal{L}j + S - vj)(A_u^{a,b,c}, D_u^{a,b,c}, \alpha_u^c) du \right] = 0.$$

Dividing both sides of (18) by the divisor t , applying the Lagrange's mean value theorem on the integration, allowing t moves towards zero, and taking the property of continuity of t that tends to $(A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c)$, it is easy to get j as a conventional answer to the problem

$$v\Theta = \mathcal{L}\Theta + S \text{ on } G. \quad (20)$$

Derivation of (a). Second Move: Suppose $(a, b, c) \in G$ together with $(K_n)_n$ which is arranged in an ascending order of disclosed and bounded subsets of G in a manner so that $\bigcup_{n \in N} K_n = G$. Finishing time and its explanation

$$\rho_n = \inf \{t \geq 0 : (A_t^{a,b,c}, D_t^{a,b,c}, \alpha_t^c) \in K_n\}, n \in N,$$

we set,

$$\hat{j}_n(a, b, c) = E \left[\int_0^{\rho_n} F(A_u^{a,b,c}, D_u^{a,b,c}, \alpha_u^c) du + e^{-\rho \rho_n} V(A_{\rho_n}^{a,b,c}, D_{\rho_n}^{a,b,c}, \alpha_{\rho_n}^c) \right]. \quad (21)$$

If $(a, b, c) \notin K_n$, then $\hat{j}_n(a, b, c) = j(a, b, c)$ as $\rho_n = 0$ almost surely. Take then $(a, b, c) \in K_n$. From the derivation of first move together with the property of continuity of J operating on K_n , \hat{j}_n solves the equation

$$v\Theta = \mathcal{L}\Theta + S, \text{ on } K_n, \Theta = J \text{ on } \partial K_n. \quad (22)$$

It is observed here that J obeys the solution of partial differential equation towards the equivalent mathematical problem and also solves the problem uniquely, we get $\hat{j}_n = J$ on $\overline{K_n}$. The limiting value of G is not solvable with regard to $(A_t^{a,b,c}, D_t^{a,b,c}, c)$ due to

$\rho_n \uparrow \infty$ and by selecting $n \uparrow \infty$ in (19), it is observed that

$$J(a, b, c) = \lim_{n \uparrow \infty} \hat{j}_n(a, b, c) = j(a, b, c), (a, b, c) \in G.$$

Since the range of J is included in its closed interval, $j(a, b, c)$ obeys the property of Lebesgue integration theory. As $J = j$ on G , and so it leads to $J \in F^2(G)$ and gives a solution (19) with the help of first move. Hence J is a viscous solution to the Hamiltonian-Jacobi-Bellman equation (12).

Derivation of (b). Pham (2009) shows in chapter 3.5 that in the traditional case to dynamic programming corresponds in proving that given a smooth solution to the HJB equation, this claimant coincides with the value function which is called a verification theorem and also formulates it as a general version and is also based on Ito's lemma which is an identity used in Ito's calculus to find the differential of a time dependent function of a stochastic process. As J is analytic and single valued function, the most desirable function of (16) comes after by applying Ito's lemma-based verification theorem.

Conclusion

Here, the problem is studied from the angle of firm's manager which during a blow to the firm due to armed conflicts and disaster is called into question to best possible stability the protection of organization together with its obstructive financial effect of acute damages from hostilities. Randomness produces the adversities of the firm as time proceeds. The manager can execute strategies to lessen the tendency of the hardships of the organization. According to our design, the above-mentioned investigation seeks to find a derivation to determine the smallest possible amount of firm's expenditure function related with the equation of motion and also administers a most favourable command in the structure of reaction [34-37].

Conjecture

Auxiliary Theorem A

Let Q be an open neighbourhood of $\Theta = (0, 0, 0) \in R^3$. Let $P: Q \rightarrow R$ be a semi concave function with such that $P_c^-(\Theta) > P_c^+(\Theta)$.

Now we can find out a real-valued functions defined on $(\Theta^n)_n \subset F^2(Q)$ in a manner that

$$\left\{ \begin{array}{l} \Theta^n(\Theta) = P(\Theta) = 0 \\ \Theta^n \geq P \text{ in a neighbourhood of } \Theta \\ |D \Theta^n(\Theta)| \leq M < \infty \\ \Theta_{cc}^n(\Theta) \xrightarrow{n \rightarrow \infty} -\infty. \end{array} \right. \quad (A.1)$$

Proof: As P admits the property of semi concavity, it is easy to find out a non-negative number F_0 so that

$$\hat{P}: Q \rightarrow R, \hat{P}(a, b, c) = P(a, b, c) - C_0(a^2 + b^2 + c^2),$$

maintains concavity. Choose and set F_0 . As $P_c^-(\Theta) > P_c^+(\Theta)$, also $\hat{P}_c^-(\Theta) > \hat{P}_c^+(\Theta)$, it equals the assertion of \hat{P} . Now according to the theorem of Rockafellar (1970, Theorem 23.4), one can find out,

$$\gamma = (\gamma_a, \gamma_b, \gamma_c), \varsigma = (\varsigma_a, \varsigma_b, \varsigma_c) \in D^+P(\Theta) \text{ such that } \gamma_c > \varsigma_c.$$

Set $h(e) = \langle \gamma, \varsigma \rangle \wedge \langle \varsigma, e \rangle$

and notice that $\hat{P}(\Theta) = 0 = h(\Theta)$ and that by concavity,

$$\hat{P}(e) \leq h(e), \forall e \in Q.$$

Define S as the set of vectors defined as $\text{Span}(\gamma - \varsigma)^+$ as well as represents it by $\Omega: R^3 \rightarrow A$.

Set $e \in R^3$ and we the decomposition is

$$e = \Pi e + \frac{\gamma - \varsigma}{|\gamma - \varsigma|} r, r = \frac{\langle e, \gamma - \varsigma \rangle}{|\gamma - \varsigma|}.$$

Take, as $e \in Q$,

$$\bar{\Theta}^n(e) = h(\Omega e) + \Psi^n(r),$$

where

$$\Psi^n: R \rightarrow R, \Psi^n(r) = -\frac{n}{2} r^2 + \frac{1}{2} \frac{\langle \gamma + \varsigma, \gamma - \varsigma \rangle}{|\gamma - \varsigma|} r.$$

The above chronological mappings produce the equations (A.1) of auxiliary theorem A which make the factors of initial two equations. Specifically, the second equation comes from the following derivation,

$$h(e) = h(\Omega e) + \begin{cases} \frac{\langle \varsigma, \gamma - \varsigma \rangle}{|\gamma - \varsigma|} r \text{ if } r \geq 0, \\ \frac{\langle \gamma, \gamma - \varsigma \rangle}{|\gamma - \varsigma|} r \text{ if } r < 0 \end{cases} \quad (A.2)$$

It is also seen by considering A.2 that

$$D \bar{\Theta}^n(e) = \Omega \gamma (= \Omega \varsigma) + \frac{\gamma - \varsigma}{|\gamma - \varsigma|} \frac{d \Psi^n(r)}{dr}.$$

So

$$\bar{\Theta}^n(e) = \langle \Omega \gamma, (0,0,1) \rangle + \langle \frac{\gamma - \varsigma}{|\gamma - \varsigma|}, (0,0,1) \rangle \frac{d \Psi^n(r)}{dr}$$

$$= \langle \gamma, (0,0,1) \rangle + \frac{\gamma_c - \varsigma_c}{|\gamma - \varsigma|} \frac{d \Psi^n(r)}{dr}.$$

$$\bar{\Theta}_{cc}^n(e) = \frac{\gamma_c - \varsigma_c}{|\gamma - \varsigma|} \frac{d^2 \Psi^n}{d r^2}(s)$$

which then imply them.

Denote by $e = (e_1, e_2, e_3) = (a, b, c)$ belongs to Q. Taking

$\alpha = (\alpha_1, \alpha_2, \alpha_3) \in N^3$ it is indicated as

$$|\alpha| = \sum_{i=1}^3 \alpha_i \text{ and } D_q^\alpha = \frac{\partial |\alpha|}{\partial e_1^{\alpha_1} + \partial e_2^{\alpha_2} + \partial e_3^{\alpha_3}}, \text{ with the convention}$$

that ∂^0 is the identity.

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