

On the Dynamic Characteristics of Orthotropic Rectangular Plates under the Influence of Moving Distributed Masses and Resting on a Variable Elastic Pasternak Foundation

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ABSTRACT

This work studies the dynamic characteristics of orthotropic rectangular plates under the influence of moving distributed masses and resting on a variable elastic Pasternak foundation. The governing equation is a fourth order partial differential equation with variable and singular coefficients. The solutions to the problem are obtained by transforming the fourth order partial differential equation for the problem to a set of coupled second order ordinary differential equations using the technique of Shadnam et al, then simplified using asymptotic method of Struble [11]. The closed form solution is analyzed, resonance conditions are obtained and the results are depicted graphically for both cases of moving distributed mass and moving distributed force.

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Introduction

Vibrations of anisotropic plates have been studied extensively by many authors. As the three-dimensional equations of motion result in solutions that generally are of a considerably higher complexity than in the isotropic case [1,2]. The main interest has concerned approximate plate theories. Here the bulk of work deals with the special case of orthotropic plates. This is partly due to the fact that the involved constitutive relations, in general, an isotropic material results in quite complicated plate equations even for an approximate theory. Moreover, in many engineering applications using composites the material configuration is orthotropic. In many engineering applications, structures may be simplified to rectangular plates with or without inter-mediate supports in one or two directions. Examples of moving materials include cables, belts and chains in power transmission lines; conveyor belts; paper and plastic sheets in different processes; metal strips in a thin sheet production line; band saw blade or rotating blades in turbo machineries.

Relatively few researchers have focused on the dynamic response of the axially moving isotropic and laminated composite plate. Wang presented numerical analysis of moving orthotropic thin plates [3]. Using a finite strip method (FSM), free vibration analysis of axially moving laminated composite and viscoelastic plates was presented by Hatami et al. [4,5]. Banichuk et al. examined the instability of an axially moving elastic plate using 2D formulation [6]. Luo and

Hamidzadeh studied equilibrium and buckling stability of axially traveling plates using an analytical method [7]. However, to the best of the authors knowledge, vibration analysis of axially moving FG plates with or without line supports is missing in the existing literature. Approximate numerical methods have been broadly used to solve partial differential equations that arise in almost all engineering disciplines. The most commonly used numerical methods for such applications are the finite difference, finite element and Ritz methods. Most engineering problems can be solved by these methods to achieve suitable precision if a proper and sufficient number of grid points are used. Later, Shu and Du [8] studied free vibration of the beams and plates with clamped and simply supported boundary conditions. Free vibration analysis of moderately thick trapezoidal symmetrically laminated plates with various combinations of boundary conditions was investigated by Zamani et al. using the GDQ method [9].

The problems related to thin structural bodies (rods, beams, plates, and shells) with other bodies have widespread application in various fields of science and technology (ie engineering, physics, mathematics etc). The physical phenomena involved in the impact event include structural responses, contact effects and wave propagation. The problems associated with these are always topical issues in the field of applied mechanics. Since these problems belong to the problems related to dynamic contact interaction, their solution is connected with cumbersome mathematical tasks. To this moment, several researchers had worked and some are still working on the dynamic behavior of orthotropic rectangular plates. Awodola and Adeoye investigated the behavior of simply supported orthotropic rectangular plate by applying the technique of variable separable [10]. Adeoye and

Awodola studied the dynamic behavior of orthotropic rectangular plate with clamped-clamped boundary conditions by making use of the technique of Shadnam [11].

Due to inability of researchers to solve orthotropic plates problems by analytical methods, this research aims at tackling and solving the governing equation by analytical m and also considers the effect of some pertinent parameters on the deflection of the plate (ie. on the solutions obtained).

Governing Equation

The dynamic transverse displacement $W(x, y, t)$ of orthotropic rectangular plates when it is resting on a bi-parametric elastic foundation and traversed by distributed mass M_r moving with constant velocity c_r along a straight line parallel to the x -axis issuing from point $y=s$ on the y -axis with flexural rigidities D_x and D_y is governed by the fourth order partial differential equation given as

$$\begin{aligned}
 & D_x \frac{\partial^4}{\partial x^4} W(x, y, t) + 2B \frac{\partial^4}{\partial x^2 \partial y^2} W(x, y, t) + D_y \frac{\partial^4}{\partial y^4} W(x, y, t) + \mu \frac{\partial^2}{\partial t^2} W(x, y, t) - \rho h R_0 \\
 & \left[\frac{\partial^4}{\partial x^2 \partial t^2} W(x, y, t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x, y, t) \right] + K_0(4x - 3x^2 + x^3)W(x, y, t) + S_0(-13 + \\
 & 12x + 3x^2) \frac{\partial}{\partial x} W(x, y, t) - S_0(12 - 13x + 6x^2 + x^3) \left[\frac{\partial^2}{\partial x^2} W(x, y, t) + \frac{\partial^2}{\partial y^2} W(x, y, t) \right] \\
 & - \sum_{r=1}^N \left[M_r g H(x - ct) H(y - s) - M_r \left(\frac{\partial^2}{\partial t^2} W(x, y, t) + 2c_r \frac{\partial^2}{\partial x \partial t} W(x, y, t) + c_r^2 \right. \right. \\
 & \left. \left. \frac{\partial^2}{\partial x^2} W(x, y, t) \right) H(x - c_r t) H(y - s) \right] = 0
 \end{aligned} \tag{2.1}$$

where D_x and D_y are the flexural rigidities of the plate along x and y axes respectively

$$D_x = \frac{E_x h^3}{12(1 - \nu_x \nu_y)}, \quad D_y = \frac{E_y h^3}{12(1 - \nu_x \nu_y)}, \quad B = D_x D_y + \frac{G_o h^3}{6} \tag{2.2}$$

E_x and E_y are the Young's moduli along x and y axes respectively, G_o is the rigidity modulus, ν_x and ν_y are Poisson's ratios for the material such that $E_x \nu_y = E_y \nu_x$, ρ is the mass density per unit volume of the plate, h is the plate thickness, t is the time, x and y are the spatial coordinates in x and y directions respectively, R_o is the rotatory inertia correction factor, K_o is the foundation constant, S_o shear modulus and g is the acceleration due to gravity, $H(\cdot)$ is the Heaviside function.

Rewriting equation (2.1), one obtains

$$\begin{aligned}
 & \mu \frac{\partial^2}{\partial t^2} W(x, y, t) + \mu \omega_n^2 W(x, y, t) = \rho h R_0 \left[\frac{\partial^4}{\partial x^2 \partial t^2} W(x, y, t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x, y, t) \right] - 2B \\
 & \frac{\partial^4}{\partial x^2 \partial y^2} W(x, y, t) - D_y \frac{\partial^4}{\partial y^4} W(x, y, t) - D_x \frac{\partial^4}{\partial x^4} W(x, y, t) + \mu \omega_n^2 W(x, y, t) - K_0(4x - \\
 & 3x^2 + x^3)W(x, y, t) + G_o(-13 + 12x + 3x^2) \frac{\partial}{\partial x} W(x, y, t) - G_o(12 - 13x + 6x^2 + x^3) \left[\right. \\
 & \left. \frac{\partial^2}{\partial x^2} W(x, y, t) + \frac{\partial^2}{\partial y^2} W(x, y, t) \right] + \sum_{r=1}^N \left[M_r g H(x - c_r t) H(y - s) - M_r \left(\frac{\partial^2}{\partial t^2} W(x, y, t) \right. \right. \\
 & \left. \left. + 2c_r \frac{\partial^2}{\partial x \partial t} W(x, y, t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x, y, t) \right) H(x - c_r t) H(y - s) \right]
 \end{aligned} \tag{2.3}$$

Simplifying equation (2.3) further, one obtains

$$\begin{aligned} \frac{\partial^2}{\partial t^2} W(x, y, t) + \omega_n^2 W(x, y, t) = \sum_{r=1}^N \left[R_0 \left(\frac{\partial^4}{\partial x^2 \partial t^2} W(x, y, t) + \frac{\partial^4}{\partial y^2 \partial t^2} W(x, y, t) \right) - \frac{2B}{\mu} \right. \\ \left. \frac{\partial^4}{\partial x^2 \partial y^2} W(x, y, t) - \frac{D_y}{\mu} \frac{\partial^4}{\partial y^4} W(x, y, t) - \frac{D_x}{\mu} \frac{\partial^4}{\partial x^4} W(x, y, t) + \omega_n^2 W(x, y, t) - \frac{K_0}{\mu} (4x - 3x^2 \right. \\ \left. + x^3) W(x, y, t) + \frac{G_o}{\mu} (-13 + 12x + 3x^2) \frac{\partial}{\partial x} W(x, y, t) - \frac{G_0}{\mu} (12 - 13x + 6x^2 + x^3) \left(\frac{\partial^2}{\partial x^2} \right. \right. \\ \left. \left. W(x, y, t) + \frac{\partial^2}{\partial y^2} W(x, y, t) \right) + \sum_{r=1}^N \left(\frac{M_r}{\mu} g H(x - c_r t) H(y - s) - \frac{M_r}{\mu} \left(\frac{\partial^2}{\partial t^2} W(x, y, t) + 2c_r \right. \right. \right. \\ \left. \left. \left. \frac{\partial^2}{\partial x \partial t} W(x, y, t) + c_r^2 \frac{\partial^2}{\partial x^2} W(x, y, t) \right) H(x - c_r t) H(y - s) \right) \right] \end{aligned} \quad (2.4)$$

where ω_n^2 is the natural frequencies, $n = 1, 2, 3, \dots$

The initial conditions, without any loss of generality, is taken as

$$W(x, y, t) = 0 = \frac{\partial}{\partial t} W(x, y, t) \quad (2.5)$$

Analytical Approximate Solution

In order to solve equation (2.4), one makes use of the technique of Shadnam et al which requires that the deflection of the plates be in series form as

$$W(x, y, t) = \sum_{n=1}^N \Psi_n(x, y) Q_n(t) \quad (3.1)$$

where

$$\Psi_n(x, y) = \Psi_m(x) \Psi_m(y)$$

$$\Psi_m(x) = \sin \varphi_m x + A_m \cos \varphi_m x + B_m \sinh \varphi_m x + C_m \cosh \varphi_m x$$

$$\Psi_m(y) = \sin \varphi_m y + A_m \cos \varphi_m y + B_m \sinh \varphi_m y + C_m \cosh \varphi_m y$$

$$\varphi_m = \frac{\xi_m}{L_x}, \quad \varphi_m = \frac{\xi_m}{L_y}$$

The right hand side of equation (2.4), taken into account equation (3.1), written in the form of series takes the form

$$\sum_{n=1}^{\infty} \left[R_0 \left(\frac{\partial^2}{\partial x^2} \Psi_n(x, y) \ddot{Q}_n(t) + \frac{\partial^4}{\partial y^2} \Psi_n(x, y) \ddot{Q}_n(t) \right) - \frac{2B}{\mu} \frac{\partial^4}{\partial x^2 \partial y^2} \Psi_n(x, y) Q_n(t) - \frac{D_y}{\mu} \frac{\partial^4}{\partial y^4} \Psi_n(x, y) Q_n(t) - \frac{D_x}{\mu} \frac{\partial^4}{\partial x^4} \Psi_n(x, y) Q_n(t) + \omega_n^2 \Psi_n(x, y) Q_n(t) - \frac{K_0}{\mu} (4x - 3x^2 + x^3) \Psi_n(x, y) Q_n(t) + \frac{G_o}{\mu} (-13 + 12x + 3x^2) \frac{\partial}{\partial x} \Psi_n(x, y) Q_n(t) - \frac{G_0}{\mu} (12 - 13x + 6x^2 + x^3) \left(\frac{\partial^2}{\partial x^2} \Psi_n(x, y) Q_n(t) + \frac{\partial^2}{\partial y^2} \Psi_n(x, y) Q_n(t) \right) + \sum_{r=1}^N \left(\frac{M_r}{\mu} g H(x - c_r t) H(y - s) - \frac{M_r}{\mu} \left(\Psi_n(x, y) \ddot{Q}_n(t) + 2c \frac{\partial}{\partial x} \Psi_n(x, y) \dot{Q}_n(t) + c_r^2 \frac{\partial^2}{\partial x^2} \Psi_n(x, y) Q_n(t) \right) H(x - c_r t) H(y - s) \right) \right] = \sum_{n=1}^N \Psi_n(x, y) \Theta_n(t) \quad (3.2)$$

Multiplying both sides of equation (3.2) by $\Psi_m(x, y)$ and integrating on area A of the plate and considering the orthogonality of $\Psi_n(x, y)$, one obtains

$$\Theta_n(t) = \frac{1}{\theta^*} \sum_{n=1}^{\infty} \int_A \left[R_0 \left(\frac{\partial^2}{\partial x^2} \Psi_n(x, y) \ddot{Q}_n(t) + \frac{\partial^4}{\partial y^2} \Psi_n(x, y) \ddot{Q}_n(t) \right) - \frac{2B}{\mu} \frac{\partial^4}{\partial x^2 \partial y^2} \Psi_n(x, y) Q_n(t) - \frac{D_y}{\mu} \frac{\partial^4}{\partial y^4} \Psi_n(x, y) Q_n(t) - \frac{D_x}{\mu} \frac{\partial^4}{\partial x^4} \Psi_n(x, y) Q_n(t) + \omega_n^2 \Psi_n(x, y) Q_n(t) - \frac{K_0}{\mu} (4x - 3x^2 + x^3) \Psi_n(x, y) Q_n(t) + \frac{G_o}{\mu} (-13 + 12x + 3x^2) \frac{\partial}{\partial x} \Psi_n(x, y) Q_n(t) - \frac{G_0}{\mu} (12 - 13x + 6x^2 + x^3) \left(\frac{\partial^2}{\partial x^2} \Psi_n(x, y) Q_n(t) + \frac{\partial^2}{\partial y^2} \Psi_n(x, y) Q_n(t) \right) + \sum_{r=1}^N \left(\frac{M_r}{\mu} g H(x - c_r t) H(y - s) - \frac{M_r}{\mu} \left(\Psi_n(x, y) \ddot{Q}_n(t) + 2c \frac{\partial}{\partial x} \Psi_n(x, y) \dot{Q}_n(t) + c_r^2 \frac{\partial^2}{\partial x^2} \Psi_n(x, y) Q_n(t) \right) H(x - c_r t) H(y - s) \right) \right] \Psi_m(x, y) dA \quad (3.3)$$

and zero when $n \neq m$ where

where

$$\theta^* = \int_A \Psi_n^2(x, y) dA \quad (3.4)$$

Making use of equation (3.3) and taking into account equation (3.2), equation (2.4) can be written as

$$\Psi_n(x, y) \left[\omega_n^2 Q_n(t) + \ddot{Q}_n(t) \right] = \frac{\Psi_n(x, y)}{\theta^*} \sum_{q=1}^{\infty} \int_A \left[R_0 \left(\frac{\partial^2 \Psi_q(x, y)}{\partial x^2} \Psi_m(x, y) \ddot{Q}_q(t) + \frac{\partial^2 \Psi_q(x, y)}{\partial y^2} \Psi_m(x, y) \ddot{Q}_q(t) \right) - \frac{2B}{\mu} \frac{\partial^4 \Psi_q(x, y)}{\partial x^2 \partial y^2} \Psi_m(x, y) Q_q(t) - \frac{D_y}{\mu} \frac{\partial^4 \Psi_q(x, y)}{\partial y^4} \Psi_m(x, y) Q_q(t) + \frac{D_x}{\mu} \frac{\partial^4 \Psi_q(x, y)}{\partial x^4} \Psi_m(x, y) Q_q(t) - \frac{K_0}{\mu} (4x - 3x^2 + x^3) \Psi_q(x, y) \Psi_m(x, y) Q_q(t) + \frac{G_o}{\mu} (-13 + 12x + 3x^2) \frac{\partial \Psi_q(x, y)}{\partial x} \Psi_m(x, y) Q_q(t) \right]$$

$$\begin{aligned} & \Psi_m(x, y)Q_q(t) - \frac{G_o}{\mu}(12 - 13x + 6x^2 + x^3) \left(\frac{\partial^2 \Psi_q(x, y)}{\partial x^2} \Psi_m(x, y)Q_q(t) + \frac{\partial^2 \Psi_q(x, y)}{\partial y^2} \Psi_m(x, y) \right. \\ & \left. Q_q(t) \right) + \sum_{r=1}^N \left(\frac{M_r}{\mu} g \Psi_m(x, y)H(x - c_r t)H(y - s) - \frac{M_r}{\mu} \left(\Psi_q(x, y)\Psi_m(x, y)\ddot{Q}_q(t) + 2c_r \right. \right. \\ & \left. \left. \frac{\partial \Phi_q(x, y)}{\partial x} \Phi_m(x, y)\dot{Q}_q(t) + c_r^2 \frac{\partial^2 \Phi_q(x, y)}{\partial x^2} \Phi_m(x, y)Q_q(t) \right) H(x - c_r t)H(y - s) \right) \Big] dA \end{aligned} \quad (3.5)$$

On further simplification of equation (3.5), one obtains

$$\begin{aligned} \ddot{Q}_n(t) + \omega_n^2 Q_n(t) = & \frac{1}{\theta^*} \sum_{q=1}^{\infty} \int_A \left[R_0 \left(\frac{\partial^2 \Psi_q(x, y)}{\partial x^2} \Psi_m(x, y)\ddot{Q}_q(t) + \frac{\partial^2 \Psi_q(x, y)}{\partial y^2} \Psi_m(x, y)\ddot{Q}_q(t) \right) \right. \\ & - \frac{2B}{\mu} \frac{\partial^2 \Psi_q(x, y)}{\partial x^2 \partial y^2} \Psi_m(x, y)Q_q(t) - \frac{D_y}{\mu} \frac{\partial^4 \Psi_q(x, y)}{\partial y^4} \Psi_m(x, y)Q_q(t) - \frac{D_x}{\mu} \frac{\partial^4 \Psi_q(x, y)}{\partial x^4} \Psi_m(x, y) \\ & Q_q(t) + \omega_q^2 \Psi_q(x, y)\Psi_m(x, y)Q_n(t) - \frac{K_0}{\mu} (4x - 3x^2 + x^3) \Psi_q(x, y)\Psi_m(x, y)Q_q(t) + \frac{G_o}{\mu} (-13 \\ & + 12x + 3x^2) \frac{\partial \Psi_q(x, y)}{\partial x} \Psi_m(x, y)Q_q(t) - \frac{G_o}{\mu} (12 - 13x + 6x^2 + x^3) \left(\frac{\partial^2 \Psi_q(x, y)}{\partial x^2} \Psi_m(x, y)Q_q(t) \right. \\ & \left. + \frac{\partial^2 \Psi_q(x, y)}{\partial y^2} \Psi_m(x, y)Q_q(t) \right) + \sum_{r=1}^N \left(\frac{M_r}{\mu} g \Psi_m(x, y)H(x - c_r t)H(y - s) - \frac{M_r}{\mu} \left(\Psi_q(x, y)\Psi_m(x, y) \right. \right. \\ & \left. \left. \ddot{Q}_q(t) + 2c_r \frac{\partial \Phi_q(x, y)}{\partial x} \Phi_m(x, y)\dot{Q}_q(t) + c_r^2 \frac{\partial^2 \Phi_q(x, y)}{\partial x^2} \Phi_m(x, y)Q_q(t) \right) H(x - c_r t)H(y - s) \right) \Big] dA \end{aligned} \quad (3.6)$$

The system of equations in equation (3.6) is a set of coupled ordinary differential equations where $H(x - c_r t)$ and $H(y - s)$ are the Heaviside functions which are defined as

$$H(x - c_r t) = \begin{cases} 1, & \text{for } x \geq c_r t \\ 0, & \text{for } x < c_r t \end{cases}, \quad H(y - s) = \begin{cases} 1, & \text{for } y \geq s \\ 0, & \text{for } y < s \end{cases} \quad (3.7)$$

With the properties

$$(i) \frac{d}{dx} [H(x - c_r t)] = \delta(x - c_r t), \quad \frac{d}{dy} [H(y - s)] = \delta(y - s) \quad (3.8)$$

$$(ii) f(x)H(x - c_r t) = \begin{cases} f(x), & \text{for } x \geq c_r t \\ 0, & \text{for } x < c_r t \end{cases}, \quad f(y)H(y - s) = \begin{cases} f(y), & \text{for } y \geq s \\ 0, & \text{for } y < s \end{cases} \quad (3.9)$$

Using the Fourier series representation, the Heaviside functions take the form

$$H(x - c_r t) = \frac{1}{4} + \frac{1}{\pi} \sum_{r=1}^N \frac{\sin(2n + 1)\pi(x - c_r t)}{2n + 1}, \quad 0 < x < 1 \quad (3.10)$$

$$H(y - s) = \frac{1}{4} + \frac{1}{\pi} \sum_{r=1}^N \frac{\sin(2n + 1)\pi(y - s)}{2n + 1}, \quad 0 < y < 1 \quad (3.11)$$

On putting equations (3.7) to (3.11) into equation (3.6) and simplifying, one obtains

$$\begin{aligned}
 \ddot{Q}_n(t) + \omega_n^2 Q_n(t) - \frac{1}{\theta^*} \sum_{q=1}^{\infty} \left[R_0 T_1 \ddot{Q}_q(t) - \frac{2B}{\mu} T_2 Q_q(t) - \frac{D_y}{\mu} T_3 Q_q(t) - \frac{D_x}{\mu} T_4 Q_q(t) + (\omega_q^2 F_4^* - \right. \\
 \left. \frac{K_0}{\mu} F_5^*) Q_q(t) + \frac{G_0}{\mu} (T_6 + T_7) Q_q(t) - \sum_{r=1}^N \frac{M_r}{\mu} \left(\left(T_8 + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi c_r t}{2j+1} \right) \right) \right. \\
 \left. \left(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi c_r t}{2j+1} \right) \right. \\
 \left. + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) \ddot{Q}_q(t) + \\
 2c_r \left(T_9 + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi c_r t}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right. \\
 \left. + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi c_r t}{2j+1} \right) \right. \\
 \left. + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) \dot{Q}_q(t) + c_r^2 \left(T_{10} + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi c_r t}{2j+1} \right) \right. \\
 \left. \left(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right. \\
 \left. + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi c_r t}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi c_r t}{2j+1} \right) + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) Q_q(t) \right] = \sum_{q=1}^{\infty} \sum_{r=1}^N \frac{M_r g}{\mu \theta^*} \Psi_m(ct) \Psi_m(s) \tag{3.12}
 \end{aligned}$$

which is the transformed equation governing the problem of an orthotropic rectangular plate resting on bi-parametric elastic foundation.

Where

$$T_1 = \int_A \left[\frac{\partial^2}{\partial x^2} \Psi_q(x, y) \Psi_m(x, y) + \frac{\partial^2}{\partial y^2} \Psi_q(x, y) \Psi_m(x, y) \right] dA \tag{3.13}$$

$$T_2 = \int_A \frac{\partial^2}{\partial x^2} \left[\frac{\partial^2}{\partial x^2} \Psi_q(x, y) \right] \Psi_m(x, y) dA \tag{3.14}$$

$$T_3 = \int_A \frac{\partial^4}{\partial y^4} \left[\Psi_q(x, y) \right] \Psi_m(x, y) dA \tag{3.15}$$

$$T_4 = \int_A \frac{\partial^4}{\partial x^4} \left[\Psi_q(x, y) \right] \Psi_m(x, y) dA \tag{3.16}$$

$$F_4^* = \int_A \Psi_q(x, y) \Psi_m(x, y) dA \quad (3.17)$$

$$T_5 = 4U_1 - 3U_2 + U_3, \quad T_6 = -13A_1 + 12A_2 + 3A_3 \quad (3.18)$$

$$T_7 = 12f_1 - 13f_2 + 6f_3 + f_4 + 12f_5 - 13f_6 + 6f_7 + f_8 \quad (3.19)$$

$$T_8 = \frac{1}{16} \int_A \Psi_q(x, y) \Psi_m(x, y) dA \quad (3.20)$$

$$E_1^* = \int_A \Psi_q(x, y) \Psi_m(x, y) \sin(2j + 1)\pi x dA \quad (3.21)$$

$$E_2^* = \int_A \Psi_q(x, y) \Psi_m(x, y) \cos(2j + 1)\pi x dA \quad (3.22)$$

$$E_3^* = \int_A \Psi_q(x, y) \Psi_m(x, y) \sin(2k + 1)\pi y dA \quad (3.23)$$

$$E_4^* = \int_A \Psi_q(x, y) \Psi_m(x, y) \cos(2k + 1)\pi y dA \quad (3.24)$$

$$E_5^* = E_1^*, \quad E_6^* = E_2^*, \quad E_7^* = E_3^*, \quad E_8^* = E_4^* \quad (3.25)$$

$$T_9 = \frac{1}{16} \int_A \frac{\partial}{\partial x} \Psi_q(x, y) \Psi_m(x, y) dA \quad (3.26)$$

$$E_9^* = \int_A \frac{\partial}{\partial x} \left(\Psi_q(x, y) \right) \Psi_m(x, y) \sin(2j + 1)\pi x dA \quad (3.27)$$

$$E_{10}^* = \int_A \frac{\partial}{\partial x} \left(\Psi_q(x, y) \right) \Psi_m(x, y) \cos(2j + 1)\pi x dA \quad (3.28)$$

$$E_{11}^* = \int_A \frac{\partial}{\partial x} \left(\Psi_q(x, y) \right) \Psi_m(x, y) \sin(2k + 1)\pi y dA \quad (3.29)$$

$$E_{12}^* = \int_A \frac{\partial}{\partial x} \Psi_q(x, y) \Psi_m(x, y) \cos(2k + 1)\pi y dA \quad (3.30)$$

$$E_{13}^* = E_9^*, \quad E_{14}^* = E_{10}^*, \quad E_{15}^* = E_{11}^*, \quad E_{16}^* = E_{12}^* \quad (3.31)$$

$$T_{10} = \frac{1}{16} \int_A \frac{\partial^2}{\partial x^2} \left(\Psi_q(x, y) \right) \Psi_m(x, y) dA \quad (3.32)$$

$$E_{17}^* = \int_A \frac{\partial^2}{\partial x^2} \left(\Psi_q(x, y) \right) \Psi_m(x, y) \sin(2j + 1)\pi x dA \quad (3.33)$$

$$E_{18}^* = \int_A \frac{\partial^2}{\partial x^2} \left(\Psi_q(x, y) \right) \Psi_m(x, y) \cos(2j + 1)\pi x dA \quad (3.34)$$

$$E_{19}^* = \int_A \Psi_q(x, y) \Psi_m(x, y) \sin(2k + 1)\pi y dA \quad (3.35)$$

$$E_{20}^* = \int_A \frac{\partial^2}{\partial x^2} \left(\Psi_q(x, y) \right) \Psi_m(x, y) \cos(2k + 1)\pi y dA \quad (3.36)$$

$$E_{21}^* = E_{17}^*, \quad E_{22}^* = E_{18}^*, \quad E_{23}^* = E_{19}^*, \quad E_{24}^* = E_{20}^* \quad (3.37)$$

$\Psi_m(x, y)$ is assumed to be the products of functions $\Psi_{pm}(x)\Psi_{bm}(y)$ which are the beam functions in the directions of x and y axes respectively. That is

$$\Psi_m(x, y) = \Psi_{pm}(x)\Psi_{bm}(y) \quad (3.38)$$

where

$$\Psi_{pm}(x) = \sin \lambda_{pm}x + A_{pm} \cos \lambda_{pm}x + B_{pm} \sinh \lambda_{pm}x + C_{pm} \cosh \lambda_{pm}x \quad (3.39)$$

$$\Psi_{bm}(y) = \sin \lambda_{bm}y + A_{bm} \cos \lambda_{bm}y + B_{bm} \sinh \lambda_{bm}y + C_{bm} \cosh \lambda_{bm}y \quad (3.40)$$

where $A_{pm}, B_{pm}, C_{pm}, A_{bm}, B_{bm}$ and C_{bm} are constants determined by the boundary conditions. And Ψ_{pm} and Ψ_{bm} are called the mode frequencies

where

$$\lambda_{pm} = \frac{\xi_{pm}}{L_x}, \quad \lambda_{bm} = \frac{\xi_{bm}}{L_y} \quad (3.41)$$

Considering a unit mass, equation (3.12) can be re-written as

$$\begin{aligned} \ddot{Q}_n(t) + \omega_n^2 Q_n(t) - \frac{1}{\theta^*} \sum_{q=1}^{\infty} \left[R_0 T_1 \ddot{Q}_q(t) - \frac{2B}{\mu} T_2 Q_q(t) - \frac{D_y}{\mu} T_3 Q_q(t) - \frac{D_x}{\mu} T_4 Q_q(t) + \right. \\ \left. (\omega_q^2 F_4^* - \frac{K_0}{\mu} T_5) Q_q(t) + \frac{G_0}{\mu} (T_6 + T_7) Q_q(t) - \alpha \rho \left(\left(T_8 + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} \right. \right. \right. \right. \\ \left. \left. \left. - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1} \right) + \frac{1}{4\pi} \left(\right. \right. \\ \left. \left. \sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \right. \right. \\ \left. \left. \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right] \ddot{Q}_q(t) + 2c \left(T_9 + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \right. \right. \\ \left. \left. \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{13}^* \right. \right. \end{aligned}$$

$$\begin{aligned} & \left. \left(\frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) \dot{Q}_q(t) + c^2 \left(T_{10} + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \right) \\ & \left(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} B_{20}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \\ & \left. + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) Q_q(t) \Bigg] = \sum_{q=1}^{\infty} \sum_{r=1}^N \frac{Mg}{\mu\theta^*} \Psi_m(ct) \Psi_m(s) \end{aligned} \quad (3.42)$$

equation (3.42) is the fundamental equation of the problem. where

$$\alpha = \frac{M}{\mu\varrho}, \quad \varrho = L_x L_y \quad (3.43)$$

$$\Psi_m(ct) = \sin \chi_m t + A_m \cos \chi_m t + B_m \sinh \chi_m t + C_m \cosh \chi_m t \quad (3.44)$$

$$\Psi_m(s) = \sin \nu_m s + A_m \cos \nu_m s + B_m \sinh \nu_m s + C_m \cosh \nu_m s \quad (3.45)$$

$$\chi_m = \frac{\xi_m c}{L_x}, \quad \nu_m = \frac{\xi_m s}{L_y} \quad (3.46)$$

Orthotropic Rectangular Plate Traversed by a Moving Force

In moving force, it is assumed that only the load being transferred to the structure and that no external force acts on the load. In this case, the inertia effect is negligible. Setting $\ddot{w} = 0$ in the fundamental equation (3.42), one obtains

$$\begin{aligned} & \ddot{Q}_n(t) + \left(1 - \frac{F_4^*}{\theta^*} \right) \omega_n^2 Q_n(t) - \frac{1}{\mu\theta^*} \left(\mu R_0 T_1 \ddot{Q}_n(t) - 2BT_2 Q_n(t) - D_y T_3 Q_n(t) - D_x T_4 Q_n(t) \right. \\ & \left. - K_o T_5 Q_n(t) + G_0(T_6 + T_7) Q_n(t) \right) - \frac{1}{\mu\theta^*} \sum_{q=1}^{\infty} \left[\mu R_0 T_1 \ddot{Q}_q(t) - 2BT_2 Q_q(t) - D_y T_3 Q_q(t) \right. \\ & \left. - D_x T_4 Q_q(t) + (\mu\omega_q^2 F_4^* - K_o T_5) Q_q(t) + G_0(T_6 + T_7) Q_q(t) \right] = \sum_{q=1}^{\infty} \sum_{r=1}^N \frac{Mg}{\mu\theta^*} \Psi_m(ct) \Psi_m(s) \end{aligned} \quad (3.47)$$

which can further be simplified as

$$\begin{aligned} & \ddot{Q}_n(t) + \chi_n^2 Q_n(t) - \tau^* \left(\mu R_0 T_1 \ddot{Q}_n(t) - 2BT_2 Q_n(t) - D_y T_3 Q_n(t) - D_x T_4 Q_n(t) - K_o T_5 \right. \\ & \left. Q_n(t) + G_0(T_6 + T_7) Q_n(t) \right) - \tau^* \sum_{q=1}^{\infty} \left[\mu R_0 T_1 \ddot{Q}_q(t) - 2BT_2 Q_q(t) - D_y T_3 Q_q(t) - D_x T_4 \right. \\ & \left. Q_q(t) + (\mu\omega_q^2 F_4^* - K_o T_5) Q_q(t) + G_0(T_6 + T_7) Q_q(t) \right] = \sum_{q=1}^{\infty} \sum_{r=1}^N Mg\tau^* \Psi_m(ct) \Psi_m(s) \end{aligned} \quad (3.48)$$

On expanding, re-arranging and simplifying equation (3.48), one obtains

$$\ddot{Q}_n(t) + \frac{(\chi_n^2 - \tau^* J_6^*)}{[1 - \tau^* \mu R_0 T_1]} Q_n(t) + \frac{\tau}{[1 - \tau^* \mu R_0 T_1]} \sum_{q=1, q \neq n}^{\infty} \left(\mu R_0 T_1 \ddot{Q}_q(t) - 2BT_2 Q_q(t) - D_y T_3 Q_q(t) - D_x T_4 Q_q(t) + (\mu \omega_q^2 F_4^* - K_o T_5) Q_q(t) + G_0 (T_6 + T_7) Q_q(t) \right) = \frac{\tau^* Mg}{[1 - \tau^* \mu R_0 F_1^*]} \Psi_m(ct) \Psi_m(s) \quad (3.49)$$

where

$$\tau^* = \frac{1}{\mu \theta^*}, \quad \chi_n^2 = \left(1 - \frac{F_4^*}{\theta^*}\right) \omega_n^2, \quad J_6^* = -2BT_2 - D_y T_3 - D_x T_4 - K_o T_5 + G_0 (T_6 + T_7) \quad (3.50)$$

For any arbitrary ratio Υ , dened as $\tau^* = \frac{\Upsilon}{1+\Upsilon}$, one obtains

$$\Upsilon = \frac{\tau^*}{1 - \tau^*} = \tau^* + o(\tau^{*2}) + \dots \quad (3.51)$$

For only $o(\tau^*)$, one obtains

$$\tau^* = \Upsilon \quad (3.52)$$

Applying binomial expansion,

$$\frac{1}{1 - \Upsilon \mu R_0 F_1^*} = 1 + \Upsilon \mu R_0 F_1^* + o(\Upsilon^2) + \dots \quad (3.53)$$

On putting equation (3.53) into equation (3.49), one obtains

$$\ddot{Q}_n(t) + (\chi_n^2 - \Upsilon J_6^*) (1 + \Upsilon \mu R_0 T_1 + o(\Upsilon^2) + \dots) Q_n(t) + \Upsilon (1 + \tau^* \mu R_0 T_1 + o(\Upsilon^2) + \dots) \sum_{q=1, q \neq n}^{\infty} \left(\mu R_0 F_1^* \ddot{Q}_q(t) - 2BF_2^* Q_q(t) - D_y F_3^* Q_q(t) - D_x F_4^* Q_q(t) + (\mu \omega_q^2 - K_0) F_5^* Q_q(t) + G_0 (F_6^* + F_7^*) Q_q(t) \right) = (1 + \Upsilon \mu R_0 T_1 + o(\Upsilon^2) + \dots) Mg \Psi_m(ct) \Psi_m(s) \quad (3.54)$$

Retaining only $o(\Upsilon)$, equation (3.54) becomes

$$\ddot{Q}_n(t) + (\chi_n^2 (1 + \Upsilon \mu R_0 T_1) - \Upsilon J_6^*) Q_n(t) + \Upsilon \sum_{q=1, q \neq n}^{\infty} \left(\mu R_0 F_1^* \ddot{Q}_q(t) - 2BF_2^* Q_q(t) - D_y F_3^* Q_q(t) - D_x F_4^* Q_q(t) + (\mu \omega_q^2 - K_0) F_5^* Q_q(t) + G_0 (F_6^* + F_7^*) Q_q(t) \right) = \Upsilon Mg \Psi_m(ct) \Psi_m(s) \quad (3.55)$$

which is simplified further as

$$\ddot{Q}_n(t) + J_7^* Q_n(t) + \Upsilon \sum_{q=1, q \neq n}^{\infty} \left(\mu R_0 F_1^* \ddot{Q}_q(t) - 2BF_2^* Q_q(t) - D_y F_3^* Q_q(t) - D_x F_4^* Q_q(t) + (\mu \omega_q^2 - K_0) F_5^* Q_q(t) + G_0 (F_6^* + F_7^*) Q_q(t) \right) = \Upsilon Mg \Psi_m(ct) \Psi_m(s) \quad (3.56)$$

where

$$J_7^* = \chi_n^2 (1 + \Upsilon \mu R_0 T_1) - \Upsilon J_6^* \quad (3.57)$$

Using Struble's technique, the solution to the homogeneous part of part of equation (3.56) is assumed to take the form

$$Q_n(t) = \varphi(n, t) \cos(\chi_n t - \rho(n, t)) + \dots \quad (3.58)$$

Where

$$\varphi(n, t) = \epsilon_n \quad (3.59)$$

and

$$\phi(n, t) = \left(\frac{\chi_n^2 - J_7^*}{2\chi_n} \right) t + \iota_n \quad (3.60)$$

On putting equations (3.59) and (3.60) into equation (3.58), one obtains

$$Q_n(t) = \epsilon_n \cos \left(\chi_n t - \left(\frac{\chi_n^2 - J_7^* \varphi(n, t)}{2\chi_n} \right) t - \iota_n \right) \quad (3.61)$$

On further simplification, one obtains

$$Q_n(t) = \epsilon_n \cos(v_n t - \iota_n) \quad (3.62)$$

Where

$$v_n = \chi_n - \left(\frac{\chi_n^2 - J_7^* \varphi(n, t)}{2\chi_n} \right) \quad (3.63)$$

is the modified frequency for moving force problem for orthotropic rectangular plate resting on variable elastic bi-parametric foundation.

Using equation (3.63), the homogeneous part of equation (3.56)

$$\ddot{Q}_n(t) + v_n^2 Q_n(t) = 0 \quad (3.64)$$

Hence, the entire equation (3.56) gives

$$\ddot{Q}_n(t) + v_n^2 Q_n(t) = \Upsilon M g \Psi_m(ct) \Psi_m(s) \quad (3.65)$$

Re-writing equation (3.65), one obtains

$$\begin{aligned} \ddot{Q}_n(t) + v_n^2 Q_n(t) = \Upsilon M g \Psi_m(s) [\sin \chi_m t + A_m \cos \chi_m t + B_m \sinh \chi_m t \\ + C_m \cosh \chi_m t] \end{aligned} \quad (3.66)$$

To obtain the solution to equation (3.66), one makes use of Laplace transformation techniques to obtain

$$\begin{aligned} Q_n(t) = \frac{Mg\Upsilon\Phi_m(s)}{v_n(\chi_m^4 - v_n^4)} \left[(\chi_m^2 + v_n^2)(\chi_m \sin v_n t - v_n \sin \chi_m t) - A_m v_n (\chi_m^2 + v_n^2) \right. \\ (\cos \chi_m t - \cos v_n t) - B_m (\chi_m^2 - v_n^2) (\alpha_m \sin v_n t - v_n \sinh \chi_m t) + C_m v_n (\chi_m^2 - v_n^2) \\ \left. (\cosh \chi_m t - \cos v_n t) \right] \end{aligned} \quad (3.67)$$

which on inversion yields

$$\begin{aligned}
 W(x, y, t) = & \sum_{jm=1}^{\infty} \sum_{hm=1}^{\infty} \frac{Mg\Upsilon\Phi_m(s)}{v_n(\chi_m^4 - v_n^4)} \left[(\chi_m^2 + v_n^2)(\chi_m \sin v_n t - v_n \sin \chi_m t) - A_m v_n \right. \\
 & (\chi_m^2 + v_n^2)(\cos \chi_m t - \cos v_n t) - B_m(\chi_m^2 - v_n^2)(\chi_m \sin v_n t - v_n \sinh \chi_m t) + C_m v_n \\
 & \left. (\chi_m^2 - v_n^2)(\cosh \chi_m t - \cos v_n t) \right] \left(\sin \frac{\phi_{jm}}{L_x} x + A_{jm} \cos \frac{\phi_{jm}}{L_x} x + B_{jm} \sinh \frac{\phi_{jm}}{L_x} x + \right. \\
 & \left. C_{jm} \cosh \frac{\phi_{jm}}{L_x} x \right) \left(\sin \frac{\phi_{hm}}{L_y} y + A_{hm} \cos \frac{\phi_{hm}}{L_y} y + B_{hm} \sinh \frac{\phi_{hm}}{L_y} y + C_{hm} \cosh \frac{\phi_{hm}}{L_y} y \right) \tag{3.68}
 \end{aligned}$$

which is the transverse displacement response to a moving force of orthotropic rectangular plate resting on variable elastic bi-parametric foundation.

Orthotropic Rectangular Plate Traversed by a Moving Mass

In moving mass problem, the moving load is assumed rigid, and the weight and as well as inertia forces are transferred to the moving load. That is the inertia effect is not negligible. Thus $\varpi = 0$ and so it is required to solve the entire equation (3.42). To solve the equation, one employs analytical approximate method. This method is known as an approximate analytical method of Struble. The homogeneous part of equation (3.42) shall be replaced by a free system operator defined by the modified frequency v_n . Thus, the entire equation becomes

$$\begin{aligned}
 \ddot{Q}_n(t) + v_n^2 Q_n(t) + \alpha \varrho^* \sum_{q=1}^{\infty} & \left[\left(F_8^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \right) \right. \\
 & \left. \left(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi s}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \right. \right. \\
 & \left. \left. \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) \\
 \ddot{Q}_q(t) + 2c & \left(F_9^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \right) \left(\sum_{k=1}^{\infty} E_{11}^* \right. \\
 & \left. \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{j=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \right. \\
 & \left. \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \Big) \dot{Q}_q(t) \\
 + c^2 & \left(F_{10}^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \right) \left(\sum_{k=1}^{\infty} E_{19}^* \right. \\
 & \left. \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \right. \\
 & \left. \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \Big) Q_q(t) \tag{3.69} \\
 = & \sum_{q=1}^{\infty} \frac{g\alpha}{\theta^*} \Phi_m(ct) \Phi_m(s)
 \end{aligned}$$

where $\varrho^* = \frac{\rho}{\theta^*}$

On expanding, simplifying and rearranging equation (3.69), one obtains

$$\begin{aligned}
 & \ddot{Q}_n(t) + 2c\alpha\varrho^* \left(F_9^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \left(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) \dot{Q}_n(t) + \\
 & \left(v_n^2 \left(1 - \alpha\varrho^* \left(F_8^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) \right) \right) \\
 & + c^2\alpha\varrho^* \left(F_{10}^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) Q_n(t) \\
 & + \alpha\varrho^* \sum_{q=1, q \neq n}^{\infty} \left[\left(F_8^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_1^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_2^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_3^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_4^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_5^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_6^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) \right] \ddot{Q}_q(t) + \\
 & 2c \left(F_9^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_9^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{10}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_{11}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{12}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{13}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{14}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \left(\sum_{k=1}^{\infty} E_{15}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{16}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) \dot{Q}_q(t) + c^2 \left(F_{10}^* + \frac{1}{\pi^2} \left(\sum_{j=1}^{\infty} E_{17}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) Q_q(t)
 \end{aligned}$$

$$\begin{aligned} & \left. \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{18}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) \left(\sum_{k=1}^{\infty} E_{19}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{20}^* \right. \\ & \left. \frac{\sin(2k+1)\pi ct}{2k+1} \right) + \frac{1}{4\pi} \left(\sum_{j=1}^{\infty} E_{21}^* \frac{\cos(2j+1)\pi ct}{2j+1} - \sum_{j=1}^{\infty} E_{22}^* \frac{\sin(2j+1)\pi ct}{2j+1} \right) + \frac{1}{4\pi} \\ & \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \Big] Q_q(t) = \frac{g\alpha}{\theta^*} \Phi_m(ct) \Phi_m(s) \end{aligned} \quad (3.70)$$

Applying modied asymptotic method of Struble, the solution to equation (3.70) takes the form

$$Q_n(t) = \psi(n, t) \cos(v_n t - G(n, t)) + v_n Q_1(t) + \dots \quad (3.71)$$

where

$$\psi(n, t) = \beta e^{-\alpha \varrho^* J_s t} \quad (3.72)$$

and

$$\begin{aligned} F(n, t) = & \frac{1}{2v_n} \left[v_n^2 \varpi \varphi^* \left(F_9^* + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) \right. \\ & \left. - c^2 \varpi \varphi^* \left(F_{10}^* + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) \right] t + \tau_n \end{aligned} \quad (3.73)$$

On putting equations (3.72) and (3.73) into equation (3.71), one obtains

$$\begin{aligned} Q_n(t) = & Z e^{-\alpha \varrho^* J_s t} \cos \left[v_n t - \frac{1}{2v_n} \left(v_n^2 \alpha \varrho^* \left(F_8^* + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \right. \right. \right. \right. \\ & \left. \left. \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) - c^2 \alpha \varrho^* \left(F_{10}^* + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) \right. \\ & \left. \left. \left. \right) \right] t - \tau_n \end{aligned} \quad (3.74)$$

On further simplifications, one obtains

$$Q_n(t) = Z e^{-\alpha \varrho^* J_s t} \cos[\beta_n t - \varepsilon_n] \quad (3.75)$$

where

$$\begin{aligned} \beta_n = & v_n - \frac{1}{2v_n} \left(v_n^2 \alpha \varrho^* \left(F_8^* + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_7^* \frac{\cos(2k+1)\pi s}{2k+1} - \sum_{k=1}^{\infty} E_8^* \frac{\sin(2k+1)\pi s}{2k+1} \right) \right) \right. \\ & \left. - c^2 \alpha \varrho^* \left(F_{10}^* + \frac{1}{4\pi} \left(\sum_{k=1}^{\infty} E_{23}^* \frac{\cos(2k+1)\pi ct}{2k+1} - \sum_{k=1}^{\infty} E_{24}^* \frac{\sin(2k+1)\pi ct}{2k+1} \right) \right) \right) \end{aligned} \quad (3.76)$$

is the modified frequency representing the frequency of the free system. Using the same argument used to solve equation (3.56), the homogeneous part of equation (3.70) becomes

$$\ddot{Q}_n(t) + \beta_n^2 Q_n(t) = 0 \tag{3.77}$$

Hence, the entire equation becomes

$$\ddot{Q}_n(t) + \beta_n^2 Q_n(t) = \frac{g\alpha}{\theta^*} \Phi_m(ct) \Phi_m(s) \tag{3.78}$$

Rewriting equation (3.78), one obtains

$$\ddot{Q}_n(t) + \beta_n^2 Q_n(t) = \frac{g\alpha}{\theta^*} \Phi_m(s) [\sin \chi_m t + A_m \cos \chi_m t + B_m \sinh \chi_m t + C_m \cosh \chi_m t] \tag{3.79}$$

Following the procedures applied to solve equation (3.66), one obtains

$$Q_n(t) = \frac{g\alpha \Phi_m(s)}{\theta^* \beta_n (\chi_m^4 - \beta_n^4)} \left[(\chi_m^2 + \beta_n^2) (\chi_m \sin \beta_n t - \beta_n \sin \chi_m t) - A_m \beta_n (\chi_m^2 + \beta_n^2) (\cos \chi_m t - \cos \beta_n t) - B_m (\chi_m^2 - \beta_n^2) (\chi_m \sin \beta_n t - \beta_n \sinh \chi_m t) + C_m \beta_n (\chi_m^2 - \beta_n^2) (\cosh \chi_m t - \cos \beta_n t) \right] \tag{3.80}$$

which on inversion yields

$$W(x, y, t) = \sum_{jm=1}^{\infty} \sum_{hm=1}^{\infty} \frac{g\alpha \Phi_m(s)}{\theta^* \beta_n (\chi_m^4 - \beta_n^4)} \left[(\chi_m^2 + \beta_n^2) (\chi_m \sin \beta_n t - \beta_n \sin \chi_m t) - A_m \beta_n (\chi_m^2 + \beta_n^2) (\cos \chi_m t - \cos \beta_n t) - B_m (\chi_m^2 - \beta_n^2) (\chi_m \sin \beta_n t - \beta_n \sinh \chi_m t) + C_m \beta_n (\chi_m^2 - \beta_n^2) (\cosh \chi_m t - \cos \beta_n t) \right] \left(\sin \frac{\phi_{jm}}{L_x} x + A_{jm} \cos \frac{\phi_{jm}}{L_x} x + B_{jm} \sinh \frac{\phi_{jm}}{L_x} x + C_{jm} \cosh \frac{\phi_{jm}}{L_x} x \right) \left(\sin \frac{\phi_{hm}}{L_y} y + A_{hm} \cos \frac{\phi_{hm}}{L_y} y + B_{hm} \sinh \frac{\phi_{hm}}{L_y} y + C_{hm} \cosh \frac{\phi_{hm}}{L_y} y \right) \tag{3.81}$$

which is the transverse displacement response to a moving mass of an orthotropic rectangular plate resting on variable elastic bi-parametric foundation.

Illustrative Examples

Orthotropic Rectangular Plate with Free Right-Hand End and Clamped Left-Hand End (The Cantilever)

For an orthotropic rectangular plate with free right hand and clamped left hand ends considered, the boundary conditions are given by

$$W(0, y, t) = 0 = \frac{\partial W(L_x, y, t)}{\partial x} \qquad \frac{\partial^2 W(0, y, t)}{\partial x^2} = 0 = \frac{\partial^3 W(L_x, y, t)}{\partial x^3} \tag{3.82}$$

$$W(x, 0, t) = 0 = \frac{\partial W(x, L_y, t)}{\partial y} \qquad \frac{\partial^2 W(x, 0, t)}{\partial y^2} = 0 = \frac{\partial^3 W(x, L_y, t)}{\partial y^3} \tag{3.83}$$

Thus, for the normal modes

$$\Phi_{pm}(0) = 0 = \frac{\partial \Phi_{pm}(0)}{\partial x} \quad \frac{\partial^2 \Phi_{pm}(0)}{\partial x^2} = 0 = \frac{\partial^3 \xi_{pm}(L_x)}{\partial x^3} = 0 \quad (3.84)$$

$$\Phi_{qm}(0) = 0 = \frac{\partial \Phi_{qm}(0)}{\partial y} \quad \frac{\partial^2 \Phi_{qm}(0)}{\partial y^2} = 0 = \frac{\partial^3 \xi_{qm}(L_y)}{\partial y^3} = 0 \quad (3.85)$$

Using equations (3.39) and (3.40) in equations (3.84) and (3.85), it can be shown that

$$A_{pm} = \frac{\sin \xi_{pm} - \sinh \xi_{pm}}{\cos \xi_{pm} - \cosh \xi_{pm}} = \frac{\cos \xi_{pm} - \cosh \xi_{pm}}{\sinh \xi_{pm} + \sin \xi_{pm}} \quad (3.86)$$

$$A_{qm} = \frac{\sin \xi_{qm} - \sinh \xi_{qm}}{\cos \xi_{qm} - \cosh \xi_{qm}} = \frac{\cos \xi_{qm} - \cosh \xi_{qm}}{\sinh \xi_{qm} + \sin \xi_{qm}} \quad (3.87)$$

In the same vein, we have

$$A_m = \frac{\sin \xi_m - \sinh \xi_m}{\cos \xi_m - \cosh \xi_m} = \frac{\cos \xi_m - \cosh \xi_m}{\sinh \xi_m + \sin \xi_m} \quad (3.88)$$

$$B_{pm} = -1, \quad B_{qm} = -1, \quad \Rightarrow B_m = -1 \quad (3.89)$$

$$C_{pm} = -A_{pm}, \quad C_{qm} = -A_{qm}, \quad \Rightarrow C_m = -A_m \quad (3.90)$$

and from equation (3.88), one obtains

$$\cos \xi_m \cosh \xi_m = -1 \quad (3.91)$$

which is termed the frequency equation for the dynamical problem, such that

$$\xi_1 = 1.875, \quad \xi_2 = 4.694, \quad \xi_3 = 7.855 \quad (3.92)$$

On using equations (3.86), (3.87), (3.88), (3.89), (3.90) and (3.292) in equations (3.68) and (3.81), one obtains the displacement response to a moving force and a moving mass of cantilever orthotropic rectangular plate resting on bi-parametric condition respectively.

Graphs for the Cantilever End Conditions

Figures 1 and 2 display the effect of foundation modulus (K_o) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of foundation modulus (K_o) increases.

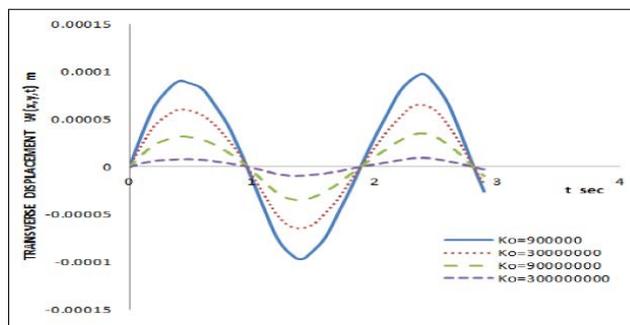


Figure 1: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Force

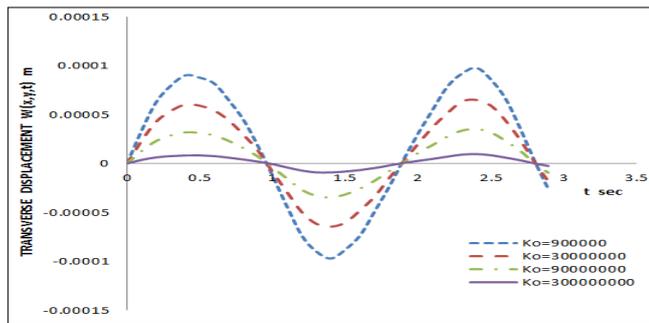


Figure 2: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying K_o and Traversed by Moving Mass

Figures 3 and 4 display the effect of shear modulus (G_o) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of shear modulus (G_o) increases.

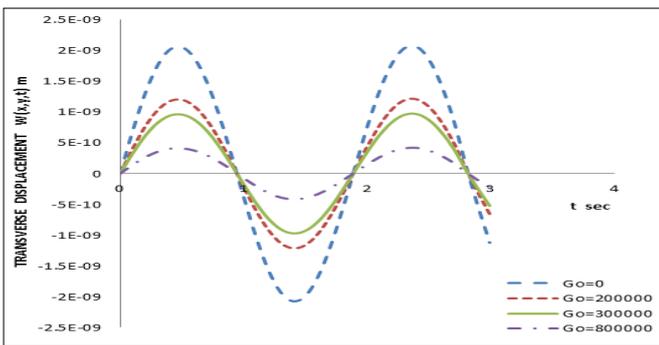


Figure 3: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Force

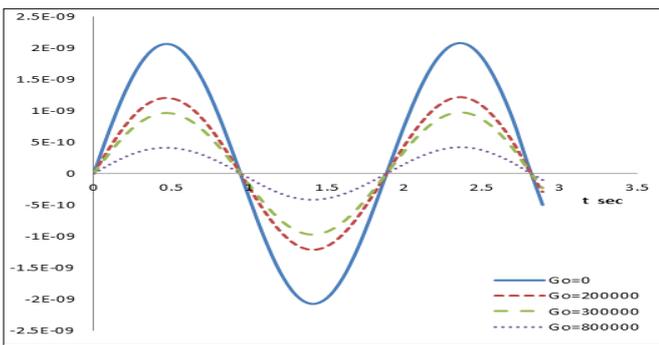


Figure 4: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying G_o and Traversed by Moving Mass

Figures 5 and 6 display the effect of flexural rigidity of the plate along x-axis (D_x) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity (D_x) increases.

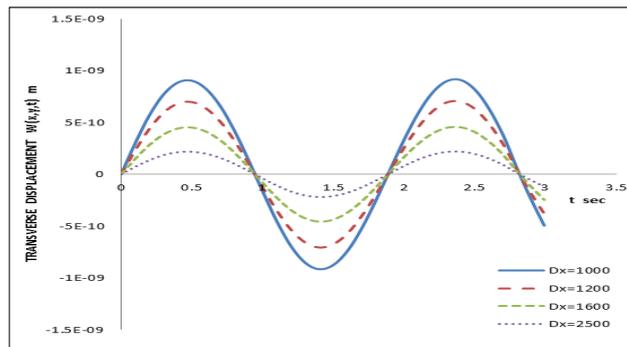


Figure 5: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_x and Traversed by Moving Force

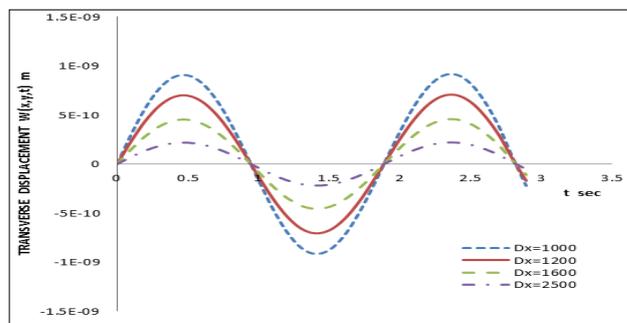


Figure 6: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_x and Traversed by Moving Mass

Figures 7 and 8 display the effect of flexural rigidity of the plate along y-axis (D_y) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the value of flexural rigidity (D_y) increases.

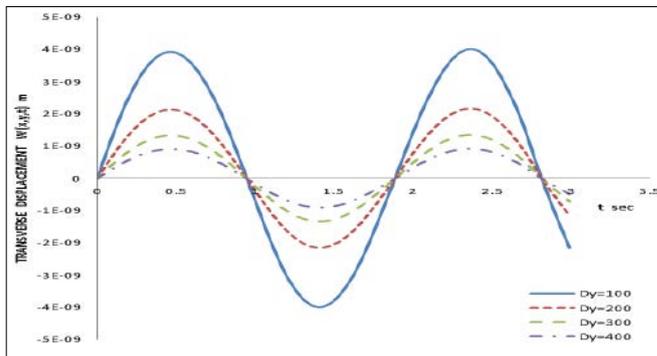


Figure 7: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_y and Traversed by Moving Force

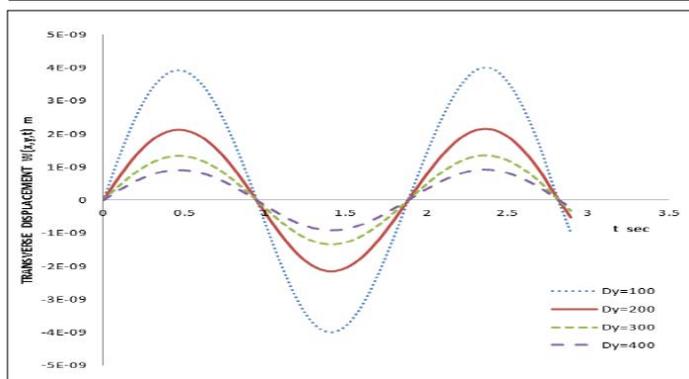


Figure 8: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying D_y and Traversed by Moving Force

Figures 9 and 10 display the effect of rotatory inertia (R_o) on the deflection profile of cantilever orthotropic rectangular plate under the action of load moving at constant velocity in both cases of moving distributed forces and moving distributed masses respectively. The graphs show that the response amplitude decreases as the R_o increases.

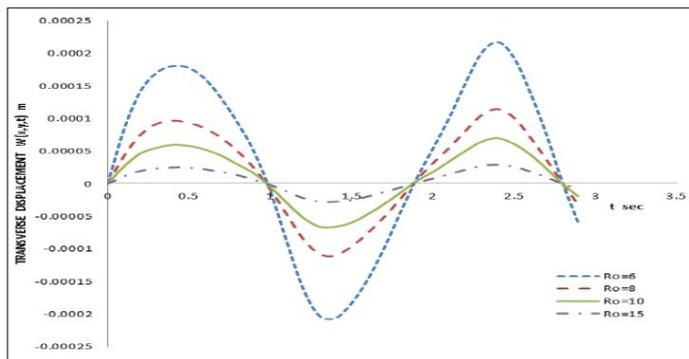


Figure 9: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying R_o and Traversed by Moving Force

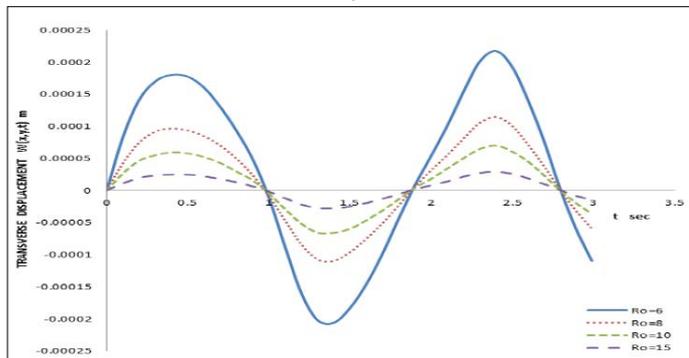


Figure 10: Displacement Profile of Cantilever Orthotropic Rectangular Plate with Varying R_o and Traversed by Moving Mass

Figure 11 displays the comparison between moving force and moving mass for fixed values of R_o , G_o , K_o , D_x and D_y

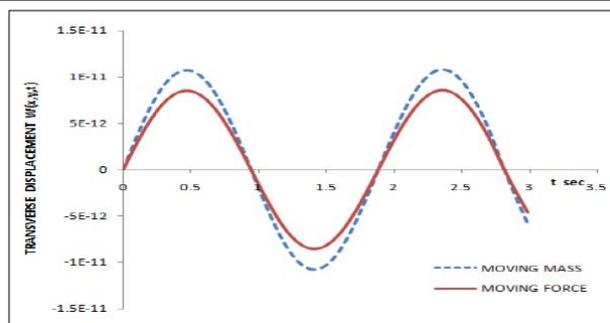


Figure 11: Displacement Profile of Comparison between Moving Force and Moving Mass

Conclusion

In this research, the dynamic characteristics of orthotropic rectangular plates under the influence of moving distributed masses and resting on a variable elastic Pasternak foundation has been studied. The closed form solutions of the fourth order partial differential equations with variable and singular coefficients governing the orthotropic rectangular plates is gotten for both cases of moving force and moving mass problems using a solution technique that is based on the separation of variables which was used to remove the singularity in the governing fourth order partial differential equation and thereby reducing it to a sequence of coupled second order differential equations. The modified asymptotic technique of Struble and Laplace transformation technique are then used to obtain the analytical solution to the two-dimensional dynamical problem.

The solutions are then analyzed. The analyses show that, for the same natural frequency, the critical speed for the moving mass problem is smaller than that of the moving force problem. This implies that resonance is attained earlier in the moving mass problem than in the moving force problem. That is to say the moving force solution is not an upper bound for the accurate solution of the moving mass problem.

The results represented in plotted curves show that as foundation modulus K_o and the shear modulus G_o increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems. As the rotatory inertia correction factor R_o increases, the amplitudes of plates decrease for both cases of moving force and moving mass problems. As the flexural rigidities along both the x-axis D_x and y-axis D_y increase, the amplitudes of plates decrease for both cases of moving force and moving mass problems.

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