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A Challenge to Solve Complicated Nonlinear Differential Equations in the Engineering Fields and Basic Sciences by New Approach AKLM

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ABSTRACT

In this manuscript, we claim that many complicated applied problems in nonlinear differential equations can be answered analytically.And as far as the researchers know, the application of analytical (not numerical) solution in the field of engineering and basic sciences is very important.These virgin methods can be successfully applied in various engineering fields such as mechanics (solid, fluid,heat and mass transfer), electronics, petroleum industry, designing chemical reactors, and also in applied sciences, economics and so on. It is worth noting that these methods are convergent at any form of differential equations, including any number of initial and boundary conditions. During the solution procedure, it is not required to convert or simplify the exponential, trigonometric and logarithmic terms, which enables the user to obtain a highly precise solution This method can be very valuable in solving engineering and basic science problems, and even economics, as a result, we present this method in this article for academic researchers.

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Introduction

In the manuscripts, we investigate and solve the most complicated applied nonlinear differential equations in mathematics and so our aim introduce of accuracy, capabilities and power for solving complicated non-linear partial differential (Pde) in the engineering fields and basic sciences. These methods can be successfully applied in various engineering fields such as mechanics (solid and fluid), electronics, petroleum industry, industrial design, and also in basic sciences and applied sciences (physics and chemistry), economics and so on. It is worth noting that these methods is convergent at any form of differential equations, including any number of initial and boundary conditions. During the solution procedure, it is not required to convert or simplify the exponential, trigonometric and logarithmic terms, which enables the user to obtain a highly precise solution. Besides, the methodology behind these techniques are completely understandable, easy to use, and users with common knowledge of mathematics will be capable of solving the most complicated equations at low calculation cost. As all experts know most of engineering actual systems behavior in practical are nonlinear process and analytical scrutiny these nonlinear problems are difficult or sometimes impossible. Our purpose is to enhance the ability of solving the mentioned nonlinear differential equations at chemical engineering and similar issues with a simple and innovative approach which entitled “Akbari Kalantari Leila Method” or “AKLM”. He’s Amplitude Frequency Formulation method which was first presented by Ji-Huan He gives convergent successive approximations of the exact solution and Homotopy perturbation technique HPM [1-5].It is necessary to mention that the above methods do not have this ability to gain the solution of the presented problem in high precision and accuracy so nonlinear differential equations such as the presented problem in this case study should be solved by utilizing new approaches like AGM that created by Mohammadreza Akbari(in 2015) [6-13].In recent years, analytical methods in solving nonlinear differential equations have been presented and created by Mohammadreza Akbari, these methods are called and AKLM (Akbari Kalantari Leila Method)and ASM (Akbari Sara’s Method) and AYM (Akbari Yasna’s Method) and IAM[20] (Integral Akbari Method) [14-20]. These example somehow can be considered as complicated cases to deal with for all of the existed analytical methods especially in the design slides engineering, which means old methods cannot resolve them precisely or even solve them in a real domain.These examples somehow can be considered as complicated cases to deal with for all of the existed analytical methods especially in the engineering fields design and basic sciences ,which means old methods cannot resolve them precisely or even solve them in a real domain.

Example 1

We consider a nonlinear partial differential equation loas folws:

$$\frac{\partial u(x,t)}{\partial t} = \alpha \frac{\partial^2 u(x,t)}{\partial x^2} + \cos\left(\sqrt[q]{1 + \eta\, u(x,t)} + \mu \sqrt[p]{1 + \beta \frac{\partial u(x,t)}{\partial x}} \right)$$

The boundary and initial conditions are:

$$u(0,t) = u1,\ u(L,t) = u2,\ u(x,0) = uo$$

AKLM solution process (Akbari Kalantari Leila's Method)

At first, we write polynomial equation for the boundary conditions Eq.(2) as follows:

The following values have been used for the physical parameters of this problem as:

$$u1 := 10;\ u2 := 50;\ uo := 20;\ L := 5;\ \alpha := 0.3;\ \eta := 0.5$$
$$\varepsilon := 0.4;\ \beta := 0.2;\ p := 3;\ q := 5;\ \mu := -0.5$$

The output answer the Eq.(12) of Maple software with AKLM method is obtained as follows:

$$u(x,t) = 10 + \frac{5844719291\, x^5}{6000000000000} - \frac{525921149\, x^4}{90000000000} + \frac{13451513\, x^3}{187500000} - \frac{1226913777\, x^2}{1000000000} + \frac{71784892411\, x}{5760000000} + \frac{1}{\pi^5} \sum_{n=1}^{\infty} \left\{ \frac{1}{n^5} \sin\left(\frac{n \pi x}{5}\right) e^{-\frac{3 \pi^2 n^2 t}{250}} \left[(-1)^n \left(60 \pi^4 n^4 - 19.0322 \pi^2 n^2 + 555.283\right) + 20 \pi^4 n^4 - 29446 \pi^2 n^2 + 175.30705 \right] \right\}$$

We compare solution of *AKLM* method with the numerical method of the Maple software code Eqs.(14),at the different times as shown below:

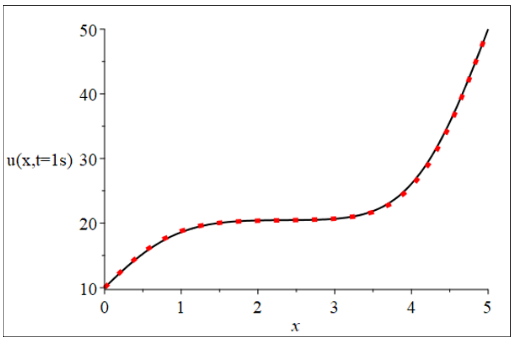

Figure 1(A): comparison between *AKLM* and Numerical solution

Graphs at different times are displayed as follows:

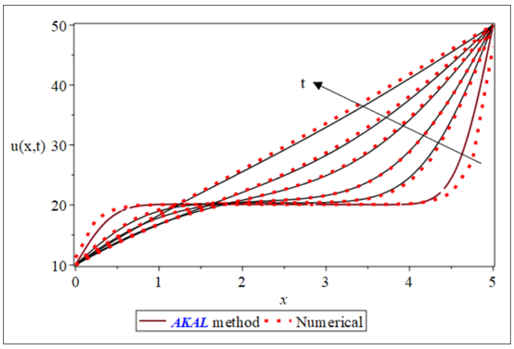

Figure 2: Graphs at different times for *t=0.2,0.8,1.5,5,10*,and high times

The time-space diagram is shown below in location x=3 as:

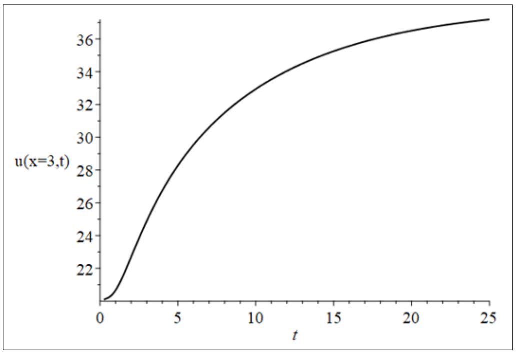

Figure 3: Time chart for location x=3.

Example 2

We consider a nonlinear partial differential equation as follows:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + \beta \cdot (u)^{\varepsilon u} \quad ; \ u = u(x,t)$$

The boundary and initial conditions are:

$$u(0,t) = u1,\ u(L,t) = u2,\ u(x,0) = uo$$

AKLM solution process (Akbari Kalantari Leila's Method)

By selecting the physical values at below:

$$u1 := 1;\ u2 := 2;\ uo := 1;\ L := 2;\ \alpha := 0.2;\ \beta := 0.2;\ \varepsilon := -0.5$$

The output answer the Eq.(12) of Maple software and the boundary and initial conditions Eqs.(13)with *AKLM* method is obtained as follows:

$$u(x,t) = 1 + \frac{x^4}{384} + \frac{x^3}{24} - \frac{x^2}{2} + \frac{21\, x}{16} + \frac{2}{\pi^5} \sum_{n=1}^{\infty} \left\{ \frac{1}{n^5} \sin\left(\frac{n \pi x}{2}\right) e^{-\frac{\pi^2 n^2 t}{20}} \left[(-1)^n \left(1 + \pi^4 n^4 + \frac{3}{2} \pi^2 n^2\right) - 4 \pi^2 n^2 - 1 \right] \right\}$$

We compare solution of *AKLM* method with the numerical method of the Maple software code Eqs.(15),at the different times as shown below:

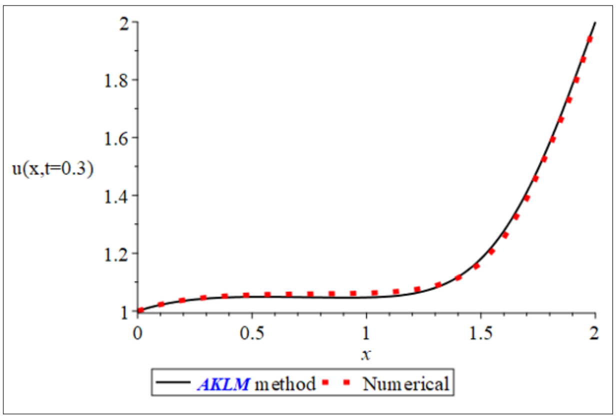

Figure 1(A): comparison between *AKLM* and Numerical solution

Graphs at different times are displayed as follows:

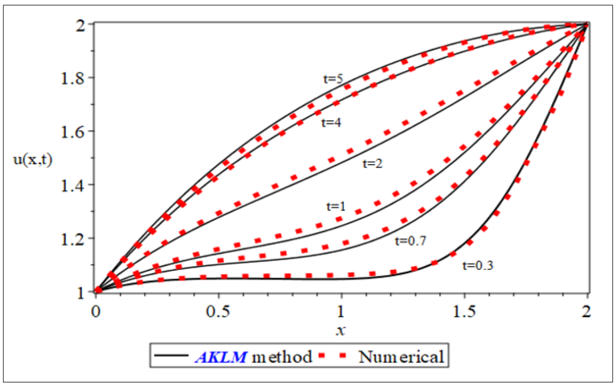

Figure 2: Graphs at different times for *t=0.3,0.7,1,2,4*,and high times

The time-space diagram is shown below in location x=0.8 as:

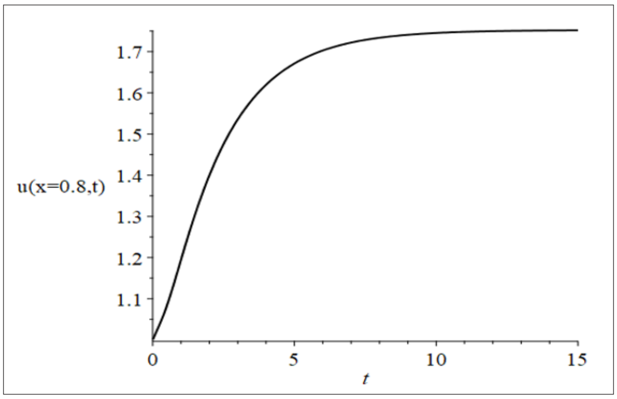

Figure 3: Time chart for location *x*=0.8

Example 3

We consider a nonlinear partial differential equation in the spherical coordinates as follows:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\alpha}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial u(x,t)}{\partial x}\right) + \sqrt[q]{1+\beta\left(\frac{\partial u(x,t)}{\partial x}\right)^2} + \mu\sqrt[p]{1+\eta\, u(x,t)^2}$$

The boundary and initial conditions are:

$$\frac{\partial u(0,t)}{\partial t} = u1\,,\ u(L,t) = u2\,,\ u(x,0) = uo$$

AKLM solution process (Akbari Kalantari Leila's Method)

The following values have been used for the physical parameters of this problem as:

$$u1 := 0;\ u2 := 1;\ uo := 1;\ \eta := 0.2;\ \alpha := 0.4$$
$$\beta := 0.2;\ \mu := 0.3;\ p := 3;\ q := 5;\ n := 1;\ L := 1$$

$$u(x) := -\frac{1521134691\,\lambda^2 x^4}{400000000000} - \frac{691424859\,\lambda x^3}{80000000000} - \frac{1037137289\,x^2}{8000000000} - \frac{25\,\lambda^2 x^2}{3\,(100\,\lambda^2+125)^{2/3}} + \frac{657973627\,\lambda^2 x^4}{80000000\,(100\,\lambda^2+125)^{2/3}} - \frac{125\,x^2}{12\,(100\,\lambda^2+125)^{2/3}} + \frac{1}{1200000000000\,(4\,\lambda^2+5)}\big(18253616292\,\lambda^4 + 5215823800\,(100\,\lambda^2+125)^{1/3}\lambda^2 + 41485491540\,\lambda^3 + 5445099393765\,\lambda^2 + 500000000000\,(100\,\lambda^2+125)^{1/3} + 51856864425\,\lambda + 6777852966750\big)$$

Or according to the time function as:

$$\lambda = e^{\frac{-\alpha n^2 \pi^2}{L^2}t}$$

The answer of the complicated nonlinear partial differential Eq.(1), *u*(*x*,*t*) is obtained.

We compare solution of *AKLM* method with the numerical method of the Maple software code Eqs.(18),at the different times as shown below:

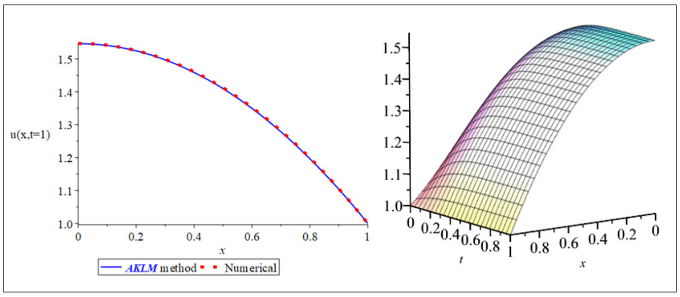

Figure 1(A): comparison between *AKLM* and Numerical solution

Graphs of contour for AKLM solution are displayed as follows:

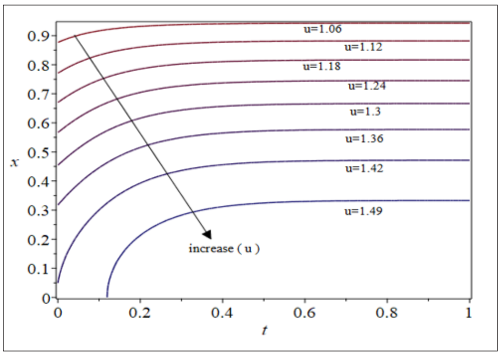

Figure 2: Charts of contour of *AKLM* solution for u(x,t)

The time-place diagram is shown below in location x=0.4 as:

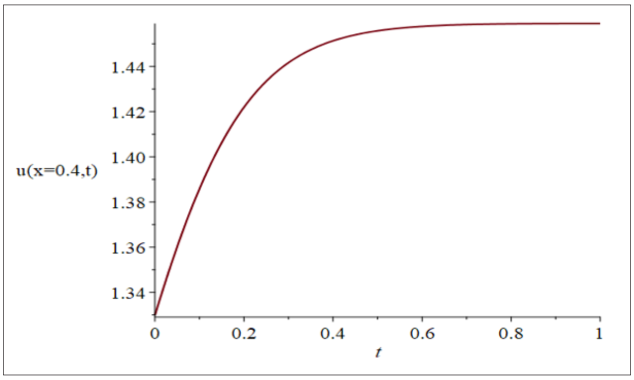

Figure 3: Time chart for location x=0.4

Conclusion

In this manuscript, we were able to prove that this method can easily solve the most complex mathematical differential equations, and also we proved conclusively whche new method (***AKLM***) can all kinds of complicated practical problems related to nonlinear partial and ordinary differential equations in the engineering field and basic sciences an they can be easily solved analytically. Obviously,most of the phenomena of physically are nonlinear, so it is quite difficult to study and analyze nonlinear mathematical equations in this area,also we wanted to demonstrate the strength, capability and flexibility of the new methods.This methods are newly created and they can have high power in analytical solution of all kinds of industrial and practical problems in engineering fields and basic sciences for complicated nonlinear differential equations.

Acknowledgment, History of AGM , ASM , AYM , AKLM , MR.AM and IAM, WoLF, a methods:

AGM (Akbari-Ganji Methods), ASM (Akbari-Sara’s Method), AYM (Akbari-Yasna’s Method) AKLM (Akbari Kalantari Leila Method), MR.AM (MohammadReza Akbari Method)and IAM (Integral Akbari Methods),WoLF,a method (Women Life Freedom,akbari), have been invented mainly by Mohammadreza Akbari (M.R.Akbari) in order to provide a good service for researchers who are a pioneer in the field of nonlinear differential equations.

*AGM method Akbari Ganji method has been invented mainly by Mohammadreza Akbari in 2014. Noting that Prof. Davood Domairy Ganji co-operated in this project.

*ASM method (Akbari Sara's Method) has been created by Mohammadreza Akbari on 22 of August, in 2019.

*AYM method (Akbari Yasna's Method)has been created by Mohammadreza Akbari on 12 of April, in 2020.

*AKLM method (Akbari Kalantari Leila Method)has been created by Mohammadreza Akbari on 22 of August, in 2020.

*MR.AM method (MohammadReza Akbari Method)has been created by Mohammadreza Akbari on 10 of November, in 2020.

*IAM method (Integral Akbari Method)has been created by Mohammadreza Akbari on 5 of February, in 2021.

*WoLF,a method (Women Life Freedom,akbari)has been created by Mohammadreza Akbari on 5 of February, in 2022.

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