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Professor Otto H. Kegel out of Great Gratitude for Opening a New and Grandiose Dimension of Mathematics to Me

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ABSTRACT

This paper presents my true Happy Birthday Greetings (in German) which I sent per express delivery on July 17, 2025 to Prof. Kegel together with a Happy Birthday Gift Box with tasty titbits to congratulate him on his 91st birthday wishing "À vôtre santé et bon appétit!". But Prof. Kegel could not receive let alone relish them ... ☹. I also added (in English) two slides of my beautiful talk at IGT 2024 and my rather thorough comments with quite a number of improvements on his beautiful and groundbreaking paper "Four lectures on Sylow theory in locally finite groups".

Mathematics Subject Classification (2020): 20D20, 20F50, 20D15

Keywords: Happy Birthday Greetings, Happy Birthday Gift Box, Schmankerl, Amalgams of p-groups, JMCA-paper (The Strong Sylow Theorem for the Prime p in Simple Locally Finite Groups), (thorough comments on) Four lectures on Sylow theory in locally finite groups, Chain conditions and Sylow's theorem in locally finite groups, List of Open Issues, Locally Finite Classical Groups, classical Hall-Higman theory, rectangle/tableau, AGTA-Paper (Characterising Locally Finite Groups Satisfying the Strong Sylow Theorem for the Prime p)

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Gauting, 20. Juli 2025

Geburtstagsgrüße und Nachrichten

Lieber Professor ***Otto H. Kegel****,*

ich wünsche Ihnen alles Gute und allerbeste Gesundheit zu Ihrem 91. Geburtstag. Nach dem Trubel um Ihren 90. Geburtstag letztes Jahr sollten Sie nun dieses Jahr Ihren Geburtstag ganz ruhig und vergnügt im Kreise Ihrer tollen Familie feiern können. Anbei eine ***Happy Birthday Gift Box*** *mit kleinen Schmankerln. Ich wünsche Ihnen und Ihrer Familie "À vôtre santé et bon appétit!".*

*Ich habe freudevoll im Juli 1974 im Seminar von **Martin Barner** († 31. Juli 2020) und **Friedrich Flohr** († 1. Oktober 2010) meinen herrlich inspirierenden Mathematiklehrer **Dr. Helmut Bergold** vom Gymnasium wiedergetroffen, der sich an meine damaligen Beiträge noch sehr gut erinnern konnte, u.a. einen Preis bei der Mathematik-Olympiade ☺. Leider ist der schöne Kontakt während meines M.Sc.-Abenteuers in London (September 1974 bis August 1975 mit Supervisor **Paul M. Cohn** [† 20. April 2006] – Ich bedauere sehr, dass ich Ihre Vorlesungen nicht besucht habe.) nicht aufrechterhalten worden, ebenso wie der Kontakt mit **Herbert Götz**, dem ich doch den Bacc.Math. verdanke, nach meiner Rückkehr von London nach Freiburg. Ich habe dann im Wintersemester 1975 Ihre so inspirierende Gruppentheorie-Vorlesung besucht und im Sommersemester 1976 in Ihrem Seminar den leider ziemlich schlecht angekommenen (weil einschläfernden …) Vortrag zu **Graham Higman**'s "Amalgams of p-groups" gehalten.*

*Nach Jahren der Enttäuschung, guter Arbeit als Mathematiklehrer und einem Nebenfach Physik mit "sehr gut" (!) habe ich schließlich den Kontakt mit Ihnen zum Schreiben einer Diplomarbeit aufnehmen können und wurde sehr herzlich willkommen geheißen. Ich erinnere heute noch ganz gerne unsere morgendlichen "Kaffeeklatsche" bei denen Sie sich so große Mühe gaben, mir Ihre Ideen zu erklären und mir dadurch eine völlig neue und grandiose Dimension der Mathematik eröffnet haben, die ich inzwischen, in Kürze, meinem damaligen Mathematiklehrer vermitteln konnte, denn ich habe seine aktuelle Adresse ausfindig machen (!) und ihn, leider nur allzu kurz, vor einigen Monaten besuchen können. Ich habe mich dann jedoch selbstverschuldet in der Klassifikation der endlichen einfachen Gruppen verloren (siehe https://en.wikipedia.org/wiki/Classification_of_finite_simple_groups) und dort leider kaum Unterstützung erhalten und mußte schließlich am 8. Oktober 1984, ganz genau zehn Jahre vor dem tragischen Tod von **Brian Hartley** …, meine ach so arg geliebte Diplomarbeit überstürzt und ziemlich unvollständig einreichen ☹.*

Mein JMCA-paper "The Strong Sylow Theorem for the Prime p in Simple Locally Finite Groups" löst die damals schier unlösbar erschienenen Probleme. Bis heute offen geblieben ist jedoch meine

"Überarbeitung" Ihrer ganz wundervollen "Four lectures on Sylow theory in locally finite groups" von 1987 (s.u.), deren Entstehung ich ja hautnah mitverfolgen durfte, und ein Vergleich mit Ihren ganz phantastischen "Chain conditions and Sylow's theorem in locally finite groups" von 1973, zwei große Paper, die allerhand noch offene Probleme enthalten (!). ***Über die tollen Ideen dieser beiden Paper würde ich mich nach wie vor sehr gerne mit Ihnen unterhalten.*** *Sie sorgen somit heute noch für faszinierende Forschungsprojekte! Das heißt, Sie sind mit 91 Jahren wirklich* ***junggeblieben.*** *So wie ich, der ich in diesem Monat vor genau 50 Jahren, im Juli 1975, in London meine Prüfungen für den Master of Science abgelegt habe.*

*Ich habe in meinem jüngsten (und wahrscheinlich bis auf weiteres letzten …) Paper eine "***List of Open Issues***" veröffentlicht, von denen ich die meisten (OI 1, OI 2, OI 3, OI 4, OI 5, OI 8, OI 9) in unveröffentlichten (draft and partly German) Arbeiten gelöst habe:*

- **OI 1** Extend Theorems 2, 3 and 4 of the JMCA-paper to the remaining Locally Finite Classical Groups (i.e., the Symplectic Groups, the Unitary Groups, the Orthogonal Groups in char ≠ 2 and the Orthogonal Groups in char 2).
- **OI 2** Summarise the work by **B. Hartley** and **A. Rae** regarding λ_p and p^{a_p} and the foregoing work on the classical **Hall-Higman theory** regarding λ_p and p^{b_p}, c_p, d_p, p^{e_p} and r_p by **P. Hall**, **G. Higman**, **A.H.M. Hoare**, **T.R. Berger**, **F. Gross**, **E.G. Bryukhanova** and **A. Turell**.
- **OI 3** Let p be a prime. Let G be a p-soluble finite group, $\lambda_p(G)$ be its p-length, and $a_p(G)$ be its p-uniqueness. Then (best possible) $\lambda_p(G) \le a_p(G) + 1$.
- **OI 4** Deduce the three rectangles/tableaux shown below and use them to prove **Lagrange**'s theorem and **Cauchy**'s concealed second and third group theorems:

complete right transversal for G in H	the **first row** consists of ***all*** elements z_k of G ($1 \le k \le M$) acting on H in the **following rows** via multiplication from the left by their inverses				correspondence	set_H **Orbi(G)** := $G \setminus H$ of ***all*** orbits of H under G acting by left translation
$t_1 := 1 =: z_1$	z_2	z_3	…	z_M	↔	$G = {}_1\text{Orb}(G)$
t_2	$z_2\, t_2$	$z_3\, t_2$	…	$z_M\, t_2$	↔	$Gt_2 = {}_{t_2}\text{Orb}(G)$
t_3	$z_2\, t_3$	$z_3\, t_3$	…	$z_M\, t_3$	↔	$Gt_3 = {}_{t_3}\text{Orb}(G)$
…	…	…	…	…	…	…
t_R	$z_2\, t_R$	$z_3\, t_R$	…	$z_M\, t_R$	↔	$Gt_R = {}_{t_R}\text{Orb}(G)$

rectangle $|G| \times [H{:}G]$ of elements

set of ***certain*** orbits of H under G acting by left translation	the **first row** consists of ***all*** right cosets Gx_1^k of G in H $(0 \le k \le p-1)$ with the powers of some p-blank x_1 of G in H; the **following rows** consist of right cosets of G in H with the powers of left conjugates of x_1				correspondence	$X := < x_1 >$; set of ***all*** orbits of H under G ↺ ↻ X, the **simultaneous** actions of G by left translation and of X by right translation
$Gx_1^0\, t_1 = G$	Gx_1	Gx_1^2	…	$Gx_1^{p\text{-}1}$	↔	cosets $G < x_1 > = GX$ = double coset $G\,1\,X$
$Gx_2^0\, t_2 = Gt_2$	$Gx_2\, t_2$	$Gx_2^2\, t_2$	…	$Gx_2^{p\text{-}1}\, t_2$	↔	cosets $G < x_2 > t_2$ = double coset $G\,t_2\,X$
$Gx_3^0\, t_3 = Gt_3$	$Gx_3\, t_3$	$Gx_3^2\, t_3$	…	$Gx_3^{p\text{-}1}\, t_3$	↔	cosets $G < x_3 > t_3$ = double coset $G\,t_3\,X$
…	…	…	…	…	…	…
$Gx_S^0\, t_S = Gt_S$	$Gx_S\, t_S$	$Gx_S^2\, t_S$	…	$Gx_S^{p\text{-}1}\, t_S$	↔	cosets $G < x_S > t_S$ = double coset $G\,t_S\,X$

tableau $\boldsymbol{p}$ x $[\boldsymbol{H}:\boldsymbol{G}]\,/\,\boldsymbol{p}$ of cosets

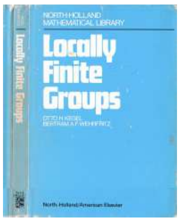
set of ***certain*** orbits of H under G acting by left translation	the **first row** consists of ***all*** right cosets $G\,x_{1c}$ of G in H $(0 \le c \le \lvert H\rvert_p\text{-}1)$ with the elements of some Sylow p-subgroup X of H, all of whose **elements of order** p are p-blanks of G in H; the **following rows** consist of right cosets of G in H with the elements of left conjugates of X				correspondence	$\lvert X\rvert = \lvert H\rvert_p = p^b$; set of ***all*** orbits of H under G ↺ ↻ X, the **simultaneous** actions of G by left translation and of X by right translation
$G\,x_{10}\,t_1 = G$	$G\,x_{11}$	$G\,x_{12}$	…	$G\,x_{1\,p^b\text{-}1}$	↔	cosets $G\,\{x_{1c} \mid 0 \le c \le p^b\text{-}1\}$ = $G\,X$ = double coset $G\,1\,X$
$G\,x_{20}\,t_2 = G\,t_2$	$G\,x_{21}\,t_2$	$G\,x_{22}\,t_2$	…	$G\,x_{2\,p^b\text{-}1}\,t_2$	↔	cosets $G\,\{x_{2c} \mid 0 \le c \le p^b\text{-}1\}\,t_2$ = double coset $G\,t_2\,X$
$G\,x_{30}\,t_3 = G\,t_3$	$G\,x_{31}\,t_3$	$G\,x_{32}\,t_3$	…	$G\,x_{3\,p^b\text{-}1}\,t_3$	↔	cosets $G\,\{x_{3c} \mid 0 \le c \le p^b\text{-}1\}\,t_3$ = double coset $G\,t_3\,X$
…	…	…	…	…	…	…
$G\,x_{T0}\,t_T = G\,t_T$	$G\,x_{T1}\,t_T$	$G\,x_{T2}\,t_T$	…	$G\,x_{T\,p^b\text{-}1}\,t_T$	↔	cosets $G\,\{x_{Tc} \mid 0 \le c \le p^b\text{-}1\}\,t_T$ = double coset $G\,t_T\,X$

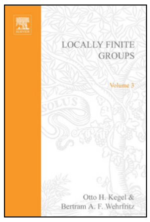
rectangle $\lvert\boldsymbol{H}\rvert_{\boldsymbol{p}}$ x $[\boldsymbol{H}:\boldsymbol{G}]\,/\,\lvert\boldsymbol{H}\rvert_{\boldsymbol{p}}$ of cosets

- **OI 5** Deduce **Cauchy**'s fundamental theorem of 1812/1815 from $[H{:}< x >] \ge \lvert G\rvert$, if x is a p-blank of G in H, that is, an element of H of order p with $x \notin G$.
- **OI 6** Determine all the (minimal) p-uniqueness subgroups for the known finite simple groups and their natural overgroups, the symmetric and the linear groups, and for the (locally) p-soluble groups, distinguishing $p \ge 5$, $p = 3$ and $p = 2$.
- **OI 7** Specify for a finite group the relationships of **the properties** of its p-subgroups which are **minimal** (w.r.t. order or w.r.t. inclusion) w.r.t. being contained in a unique **Sylow** p-subgroup to **the properties** (e.g. conjugacy) of its p-subgroups which are **maximal** w.r.t. being contained in a (unique) **Sylow** p-subgroup (i.e., its **Sylow** p-subgroups).
- **OI 8** Introduce for every prime p a generalised p-length for locally finite groups which is finite if and only if they satisfy the strong **Sylow** Theorem for p.
- **OI 9** Making a revision of **Kegel**'s (3.5) Theorem [44] thereby relating it to rarely known articles and extending it to $p \ge 3$, extend **Kegel**'s (4.4) Theorem [44] to the case $p = 3$.
- **OI 10** Summarise **Kegel**'s wonderful **Sylow** paper [44] thereby integrating **first** the AGTA-paper (see https://www.advgrouptheory.com/journal/Volumes/13/Flemisch.pdf) and **secondly** the JMCA-paper (see https://www.onlinescientificresearch.com/articles/the-strong-sylow-theorem-for-the-prime-p-in-simple-locally-finite-groups--de-luxe-edition.pdf) and extending **thirdly** his main Theorem (4.4) to $p = 3$ and $p = 2$ by using the Open Issues OI 9 and OI 3.

The reference [44] refers to the JMCA-paper as does reference [43]:

[43] O.H. KEGEL – B.A.F. WEHRFRITZ: “Locally Finite Groups”, North-Holland Mathematical Library, Volume **3**, North-Holland Publishing Company [, Ltd.], Amsterdam & London (1973) (see **MR0470081** [MR 57 #9848] and **Zbl 0259.20001** [by **Brian Hartley**]). ISBN 0-7204-2454-2. American Elsevier Publishing Company, Inc. (1973). ISBN 0-444-10406-2. ISBN 0-08-095413-8 (eBook, 2000).





[44] O.H. KEGEL: “Four lectures on Sylow theory in locally finite groups”, in: Group Theory, edited by K.N. CHENG and Y.K. LEONG, Walter de Gruyter, Berlin & New York (January 1989, reprinted November 2016), 3-27 (see **MR0981832** [MR 90c:20037 by **Brian Hartley** (March 1990)] and **Zbl 0659.20024** [by tough **Bernhard Amberg**]). ISBN 978-3-11-011366-2. ISBN 978-0-89925-406-7.
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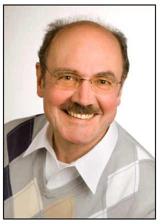
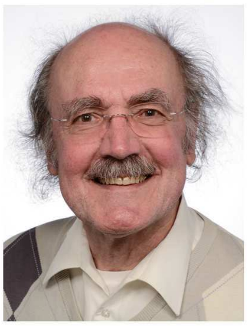
Herzliche Grüße an Sie Junggebliebenen, auch an Ihre liebe Frau Waltraut Kegel und von unserem Junggebliebenen-Stammtisch im Caritas-Marienstift Gauting Junggebliebenen-Stammtisch*, von Gauting/München nach Merzhausen/Freiburg,*



Ihr Bewunderer Felix F. Flemisch →



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Long live Group Theory!

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Felix F. Flemisch received his first degree Bacc.Math. in 1974 from the Albert-Ludwigs-Universität at Freiburg im Breisgau, his postgraduate degree M.Sc. in 1975 from the University of London, UK, and finally his degree Dipl.-Math. at Freiburg i.Br. in 1985. Since May 1985 he was working for the telecom industry. On April 11, 1992, he married beloved **Helga** in Florence in Tuscany in Italy. Since October 2016 he is retired and is still resp. is again loving to work on mathematics, in particular on the very beautiful Group Theory ☺.

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The Mathematical Institute in Freiburg im Breisgau

This is the **Mathematical Institute** at Albert-Ludwigs-University in **Freiburg im Breisgau** in Germany where from 1975 until 1999 **Prof. Kegel** occupied his chair, gave **beautiful** lectures and seminars, invited researchers over researchers, and hosted students in the morning **offering a cup of coffee** (or two) thereby doing careful supervision work and suggesting fascinating research topics.

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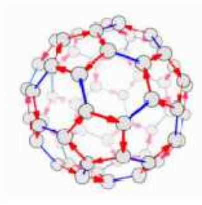
Citation: Felix F. Flemisch (2025) The Strong Sylow Theorem for the Prime *p* in Simple Locally Finite Groups. Journal of Mathematical & Computer Applications. SRC/JMCA-229. DOI: doi.org/10.47363/JMCA/2025(4)198

Prof. Kegel,
wie er leibt und
glücklich lebt,
auf einem Spaziergang in
Freiburg im Breisgau

Prof. Kegel,
that's him all over,
his happy spitting image,
on a cosy stroll in
Freiburg im Breisgau



Long live Group Theory and in particular Sylow Theory of Locally Finite Groups!

Citation: Felix F. Flemisch (2025) Professor Otto H. Kegel out of Great Gratitude for Opening a New and Grandiose Dimension of Mathematics to Me. Journal of Mathematical & Computer Applications. SRC/JMCA-257. DOI: doi.org/10.47363/JMCA/2025(4)216

Four lectures on Sylow theory in locally finite groups

O.H. Kegel

The AGTA **Research Paper** [15] "Characterising Locally Finite Groups Satisfying the Strong **Sylow** Theorem for the Prime p" (see https://www.advgrouptheory.com/journal/Volumes/13/Flemisch.pdf) and the JMCA **Research Article** "The Strong **Sylow** Theorem for the Prime p in Simple Locally Finite Groups" (see doi.org/10.47363/JMCA/2025(4)198) are both based on this **beautiful** paper [44] by Prof. **Otto H. Kegel** each one proving a conjecture of it: the AGTA-paper answers the question on Page 10 and the JMCA-paper finds a proof for the "inspection" of (2.4) Theorem on Page 13 both being centred around the gay concept of a ***p*-uniqueness subgroup** which is a finite p-subgroup being contained in a unique **Sylow** p-subgroup. We present the main result of **Kegel**'s paper and comment thoroughly his final considerations regarding p-length and $p = 3$ and $p = 2$.

Introduction

In their wording some theorems about finite groups also make sense for infinite groups; however, since there are counter examples, most of them cannot be proved. Narrowing down the class of groups considered more theorems of finite origin will become provable. In general, assuming the validity of some theorem about finite groups for some infinite group is a strong finiteness condition, and one might wonder what other finiteness conditions may be deduced from it. A good class in which to study such phenomena is the class of locally finite groups, i.e. the class of those groups in which every finite set of elements generates a finite subgroup.

The basic result on finite groups is the *Sylow Theorem* stating that for a fixed prime p the maximal p-subgroups of a finite group G are conjugate in G. This statement makes sense in arbitrary groups, but it is false in general, as we shall see even in locally finite groups. We shall refer to the discussion of the validity of the Sylow Theorem for the prime p or some variation or generalisation of it in some class of infinite groups as *Sylow Theory*. More than anyone else, Brian Hartley has contributed to Sylow Theory in locally finite groups. If not only in the locally finite group G itself, but also in every subgroup of G, the maximal p-subgroups are conjugate we shall say that G satisfies the *strong Sylow Theorem* for the prime p. For locally finite, locally p-soluble groups satisfying the strong Sylow Theorem for the prime p Hartley exhibited in [10] a very strong finiteness property, if $p \neq 2$. An extension of a weak form of Hartley's finiteness result is

If for the prime $p \geq 5$ the locally finite group G satisfies the strong Sylow Theorem then there is a finite series of normal subgroups N_i of G with

$$G = N_0 \supseteq \ldots \supseteq N_i \supseteq N_{i+1} \supseteq \ldots \supseteq N_k = \langle 1 \rangle$$

such that the factors N_i/N_{i+1} of this series are either a direct product of finitely many linear simple groups or locally p-soluble.

Hartley's result allows one to refine the series so that the locally p-soluble factors N_i/N_{i+1} are either p-groups or p'-groups.

It is this result that I want to explain in these lectures. I hope that on the way the audience will get a few glimpses of the landscape of locally finite groups in general and of the importance that definite results in the theory of finite groups may have for the theory of locally finite groups. Occasionally, questions on finite groups have been motivated by possible applications to locally finite groups.

Prof. **Kegel** then presents his four lectures on **Sylow** theory in locally finite groups

Lecture I : Variations on Sylow's Theorem

Lecture II : Singular p-subgroups and simple locally finite groups

Lecture III : The study of crucial configurations

Lecture IV : The strong finiteness results

which culminate in a detailed refinement of the result from Page 8:

(4.4) Theorem. *If the locally finite group G satisfies the strong Sylow Theorem for the prime $p \geq 5$, then there are characteristic subgroups*

$$\langle 1 \rangle \subseteq \mathbf{O}_p(G) \subseteq \mathbf{O}_{p,p'}(G) \subseteq \mathbf{O}_{p,p',p}(G) \subseteq \mathbf{S}_p(G) \subseteq S \subseteq A \subseteq P \subseteq G$$

such that $S/\mathbf{S}_p(G) = \mathrm{soc}(G/\mathbf{S}_p(G)$ is a direct product of finitely many locally finite simple linear groups, A/S is an abelian group of rank bounded by the number of simple direct factors of $S/\mathbf{S}_p(G)$, the factor group P/A is a finite soluble group of order bounded by the number and a function of the types of the simple direct factors of $S/\mathbf{S}_p(G)$, and the factor group G/P permutes these direct factors faithfully. If none of the characteristics of the underlying locally finite fields of the infinite simple direct factors of $S/\mathbf{S}_p(G)$ is p, then the factor group $G/\mathbf{O}_p(G)$ satisfies the minimum condition for p-subgroups. In any case, the factor group $G/\mathbf{O}_{p,p',p}(G)$ is countable.

Here $\mathbf{S}_p(G)$ denotes the largest normal locally p-soluble subgroup of G.

Making a suitable definition for the p-length of the group $S/\mathbf{S}_p(G)$ – possibly simply the number of simple direct p-perfect factors – one gets that with this extended notion of p-length the locally finite group G satisfying the strong Sylow Theorem for the prime $p \geq 5$ will have finite p-length.

The author has defined in unpublished work for every prime p such a generalised p-length for locally finite groups which is finite if and only if they satisfy the strong **Sylow** Theorem for p.

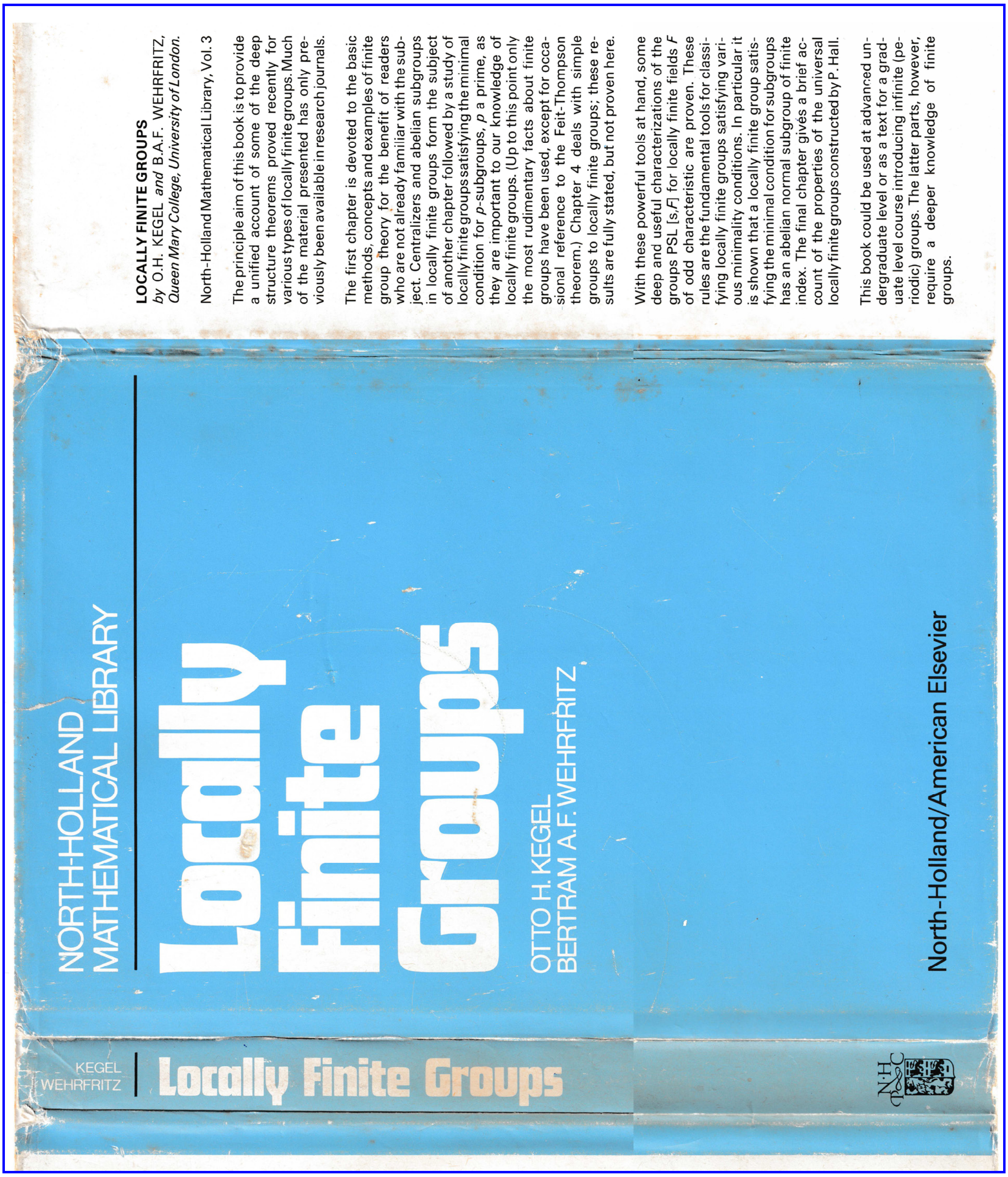
It seems desirable to remove the restrictions on the prime p in the above result. This would mean finding an argument in the finite soluble case to prove a qualitative result like (2.1) and finding a rather different argument to prove (3.5) which might use different properties of finite simple groups.

The qualitative result like (2.1) is **Open Issue 3**. Making a revision of **Kegel**'s (3.5) Theorem thereby relating it to rarely known articles and extending it to $p \geq 3$, the author was able to extend **Kegel**'s (4.4) Theorem to the case $p = 3$ without using special properties of finite simple groups. Note also that our results for simple groups are valid for all primes p.

Acknowledgements. **For the presentation of these ideas I am indebted to the efforts of my former student Felix Flemisch. – B. Hartley pointed out an embarrassing error in Lecture III.**

The **Bibliography** comprises 16 pages with 323 titles (!) from I. N. ABRAMOVSKIĬ to H. ZASSENHAUS.

The author learned **Locally Finite Group Theory** in the 1970ies through the **beautiful** book [43] (see below) and in the 1980ies through personal education over years by adored Professor **Otto H. Kegel** in the course of developing this **beautiful** paper [44].



Professor Otto. H. Kegel giving one of his such fascinating lectures on Group Theory with an outlook to challenging and beautiful research topics thereby offering his careful supervision ☺

This JMCA-paper has been published under https://www.onlinescientificresearch.com/journals/jmca/archives/2025/4/4 with its PDF at https://www.onlinescientificresearch.com/articles/professor-otto-h-kegel-out-of-great-gratitude-for-opening-a-new-and-grandiose-dimension-of-mathematics-to-me.pdf and with its short Abstract at https://www.onlinescientificresearch.com/journals/jmca/abstract/professor-otto-h-kegel-out-of-great-gratitude-for-opening-a-new-and-grandiose-dimension-of-mathematics-to-me-6787.html .

The Albert-Ludwigs-University in Freiburg i.Br. published an obituary under https://bztrauer.de/traueranzeige/92113/prof-otto-kegel:

Die Albert-Ludwigs-Universität Freiburg und ihre Fakultät für Mathematik und Physik trauern um

Prof. Dr. rer. nat.
Otto H. Kegel

* 20. Juli 1934 † 20. Juli 2025

langjähriger und ordentlicher Professor für Algebra an der Fakultät für Mathematik und Physik.

Wir verlieren mit ihm einen engagierten Hochschullehrer, einen Wissenschaftler von großem Ansehen und einen sehr geschätzten Kollegen.

Unser Mitgefühl gilt seinen Angehörigen. Wir werden ihm ein ehrendes Andenken bewahren.

Prof. Dr. Kerstin Krieglstein, Rektorin

Prof. Dr. Michael Růžička, Dekan

universität freiburg

We know (see Page 7) that Prof. Otto H. Kegel occupied his chair at the Mathematical Institute from 1975 to 1999, gave beautiful lectures and seminars and invited researchers over researchers.

Prof. Otto H. Kegel's magnificent family published an obituary as well under https://bztrauer.de/traueranzeige/92049/otto-h-kegel:

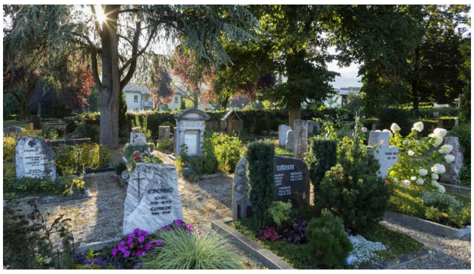
Otto H. Kegel
* 20. 7. 1934 † 20. 7. 2025

Wir nehmen Abschied

Waltraut Kegel
Hannah Kegel und Gela Samsonidse
Niko Samsonidse
Elene Samsonidse

Die Trauerfeier mit anschließender Beisetzung findet
am Dienstag, den 29. Juli 2025 um 14.00 Uhr
auf dem Friedhof in Merzhausen statt.

Coming from London, UK, in 1975 he found a beautiful domicile in Merzhausen, an endearing straight southern suburb of Freiburg i.Br., and he found now his final resting place on Merzhausen's cemetery



(see https://www.merzhausen.de/leben-wohnen/friedhof). The burial was very recently, on July 29, 2025, whence the grave is in a provisional state: no marble or granite tombstone yet with the major dates of his eventful academic and private life.

<table>
<tr><td>Von guten Mächten wunderbar geborgen,
erwarten wir getrost, was kommen mag.
Gott ist mit uns am Abend und am Morgen
und ganz gewiß an jedem neuen Tag.</td><td>By loving forces wonderfully sheltered,
we are awaiting fearlessly what comes.
God is with us at dusk and in the morning
and most assuredly on ev'ry day.</td></tr>
<tr><td>http://www.berlinertourguide.com/dietrich-bonhoeffer-von-guten-maechten-translation-by-loving-forces.htm;
https://de.wikipedia.org/wiki/Dietrich_Bonhoeffer</td><td>http://www.berlinertourguide.com/dietrich-bonhoeffer-von-guten-maechten-translation-by-loving-forces.htm#english;
https://en.wikipedia.org/wiki/Dietrich_Bonhoeffer</td></tr>
<tr><td colspan="2">Dietrich Bonhoeffer, evangelischer Theologe, Mitglied des anti-nazistischen Widerstands. Geschrieben kurz vor dem Jahreswechsel 1944/45 an seine Mutter. Dietrich Bonhoeffer wurde am 9. April 1945 von den Nazis ermordet. https://de.wikipedia.org/wiki/Von_guten_M%C3%A4chten_treu_und_still_umgeben; https://en.wikipedia.org/wiki/Von_guten_M%C3%A4chten; http://ingeb.org/spiritua/vonguten.html</td></tr>
</table>

This document "*Professor Otto H. Kegel out of Great Gratitude for Opening a New and Grandiose Dimension of Mathematics to Me*" has also been published under the name "*Felix_F._Flemisch_–_Happy_Birthday_and_Gratitude_Greetings_to_Professor_Otto_H._Kegel_on_July_17,_2025*" as a Historical at the Web site of the great Advances in Group Theory and Applications (AGTA) journal (see under https://www.advgrouptheory.com/GTArchivum/historicals.html) but without the scientific, i.e., the group-theoretic, part (List of Open Issues and thorough comments on "*Four lectures on Sylow theory in locally finite groups*"). However, it includes the English translation of the German text. We allow us to copy it:

FELIX F. FLEMISCH – HAPPY BIRTHDAY AND GRATITUDE GREETINGS TO PROFESSOR OTTO H. KEGEL ON JULY 17, 2025

HAPPY BIRTHDAY TO YOU

Let the coming year of age bring many wonderful adventures, successes and amazing memories.

Gauting, July 20, 2025

Happy Birthday Greetings and News

Dear Professor **Otto H. Kegel,**

I wish you all the very best and, above all, good health on the occasion of your 91st birthday. After all the excitement surrounding your 90th birthday last year, I hope you can now enjoy a peaceful and joyful celebration this year in the company of your wonderful family. Enclosed is a little **Happy Birthday gift box** with some tasty treats. I wish you and your family "*À vôtre santé et bon appétit!*".

In July 1974, I was delighted to meet again my wonderfully inspiring mathematics teacher **Dr. Helmut Bergold**, from high school, at the seminar of **Martin Barner** († July 31, 2020) and **Friedrich Flohr** († October 1, 2010). He still remembered my contributions well – including a prize in the Mathematics Olympiad. Unfortunately, I lost contact with him during my M.Sc. adventure in London (September 1974 to August 1975), where I was supervised by **Paul M. Cohn** († April 20, 2006) – I regret very much that I did not attend your lecture courses –, just as I did with **Herbert Götz**, to whom I owe the Bacc.Math., and still after my return from London to Freiburg. In the winter semester of 1975, I attended your truly inspiring lecture course on Group Theory and, in the summer semester of 1976, I gave a talk in your seminar (unfortunately not very well received – too sleep-inducing …) on **Graham Higman**'s really fantastic "*Amalgams of p-groups*".

After years of disappointment, good work as a mathematics teacher, and a minor in physics completed with "*very good*" (!), I was finally able to get back in touch with you to write my Diplom thesis – and was warmly welcomed. I still fondly remember our morning coffee chats, where you made such a great effort to explain your ideas to me. Through them, **you opened up an entirely new and grandiose dimension of mathematics**, which I was later able to convey – briefly but with joy – to my former mathematics teacher. I had tracked down his current address and, a few months ago, managed to visit him, if only all too briefly.

However, I then got really hopelessly lost – entirely through my own fault – in the classification of finite simple groups (see https://en.wikipedia.org/wiki/Classification_of_finite_simple_groups), where I received very little support. In the end, I had to submit my ever so dearly loved Diplom thesis on October 8, 1984 – hastily and quite incomplete – exactly ten years before the tragic death of **Brian Hartley** … ☹.

My JMCA-paper "*The Strong Sylow Theorem for the Prime p in Simple Locally Finite Groups*" solves the problems that seemed unsolvable at the time. Still pending, however, is my "revision" of your wonderful "*Four lectures on Sylow theory in locally finite groups*" from 1987, whose creation I was privileged to witness up very close, and a comparison with your fantastic "*Chain conditions and Sylow's theorem in locally finite groups*" from 1973. These are two great papers that still contain a wealth of open problems! **I would still love to discuss the brilliant ideas in both of them with you.** They continue to inspire fascinating research projects, which means that, at 91, you have truly remained **forever-young**. Just like me because this very month marks exactly 50 years since I took my M.Sc. exams in London, in July 1975.

Warmest greetings to you, dear **forever-young** Professor, also to your beloved wife **Waltraut Kegel**, and from our **Forever-Young**-Stammtisch at the Caritas-Marienstift Gauting, from Gauting/Munich to Merzhausen/Freiburg,

Your eternal admirer, **Felix F. Flemisch**

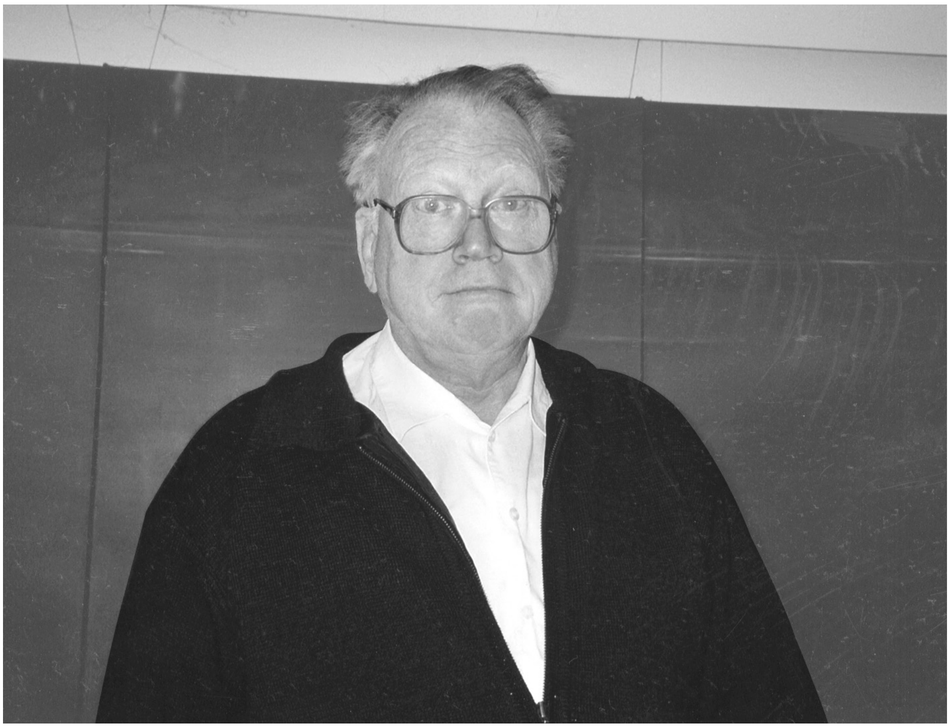
Archiv der Mathematik

Volume 85, Issue 1

July 2005

https://link.springer.com/journal/13/volumes-and-issues/85-1

The present issue is dedicated to Professor Dr. Otto H. Kegel,
our friend, colleague, and former co-editor.
Dedicated to Prof. Dr. Otto H . Kegel on the occasion of his seventieth birthday on July 20, 2004
Herrn Professor Dr. Otto H. Kegel zum 70. Geburtstag am 20. Juli 2004 gewidmet



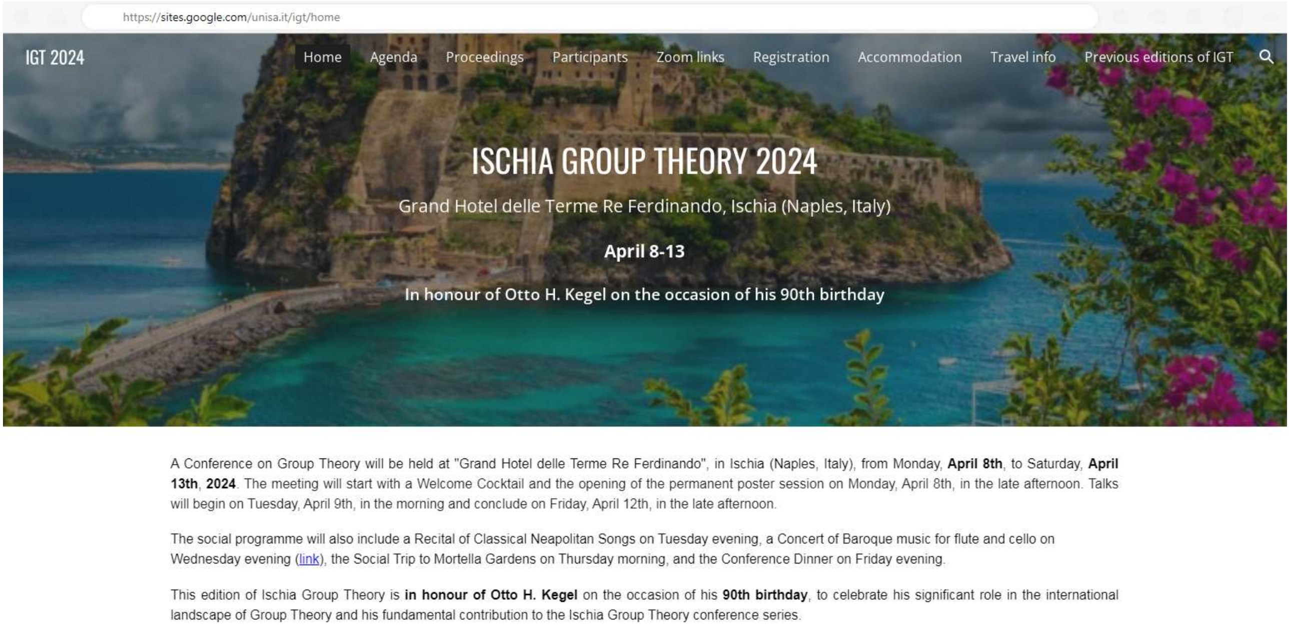
Dem Archiv der Mathematik ist **Prof. Otto H. Kegel** besonders verbunden. Zunächst als Autor, dann als ein langjähriger Herausgeber, dessen tatkräftiger Unterstützung sich die Redaktion stets sicher sein konnte, und anschließend als Beiratsmitglied bis 2002. Bereits mit seiner ersten im Archiv publizierten Arbeit "*Produkte nilpotenter Gruppen*" hat **Prof. Kegel** entscheidend dazu beigetragen, dass der Fluss der Arbeiten über faktorisierbare Gruppen bis heute noch nicht versiegt ist. Die Anregungsstärke ist übrigens eines seiner Markenzeichen. Ein anderes Charakteristikum ist seine sprudelnde Phantasie, die es ihm erlaubt, zur Lösung algebraischer und geometrischer Probleme Konstruktionen aus Theorien einzusetzen, die eher dem Modelltheoretiker offen stehen. Seine Arbeiten über lokal endliche und lineare Gruppen zeigen aber, dass er trotzdem die Bodenhaftung nie verloren hat. Als Mensch besitzt **Prof. Kegel** eine Eigenschaft, die nicht allzu viele Erfolgspersönlichkeiten auszeichnet: Es ist die Konstanz seines liebenswerten, freundlichen Charakters.

The collaboration of **Prof. Kegel** with Archiv der Mathematik dates back to his student days with Reinhold Baer at Frankfurt am Main. He served as a referee and author, then (1975–1990) as a co-editor, and finally (till 2002) as a member of the advisory board. Already his first paper published in Archiv, "*Produkte nilpotenter Gruppen*", was a significant contribution to the investigation of factorised groups, stimulating a stream of research that still continues with wide ramifications. Stimulating curiosity is one of his traits. Another one is his creative imagination, enabling him to use ideas and methods from other mathematical areas, e.g., model theory, in the treatment of algebraic or geometric problems. **Prof. Kegel**'s work on linear and locally finite groups proves that he keeps in touch with basic problems. An important property of **Prof. Kegel** as a colleague and teacher is his unassuming and friendly personality.

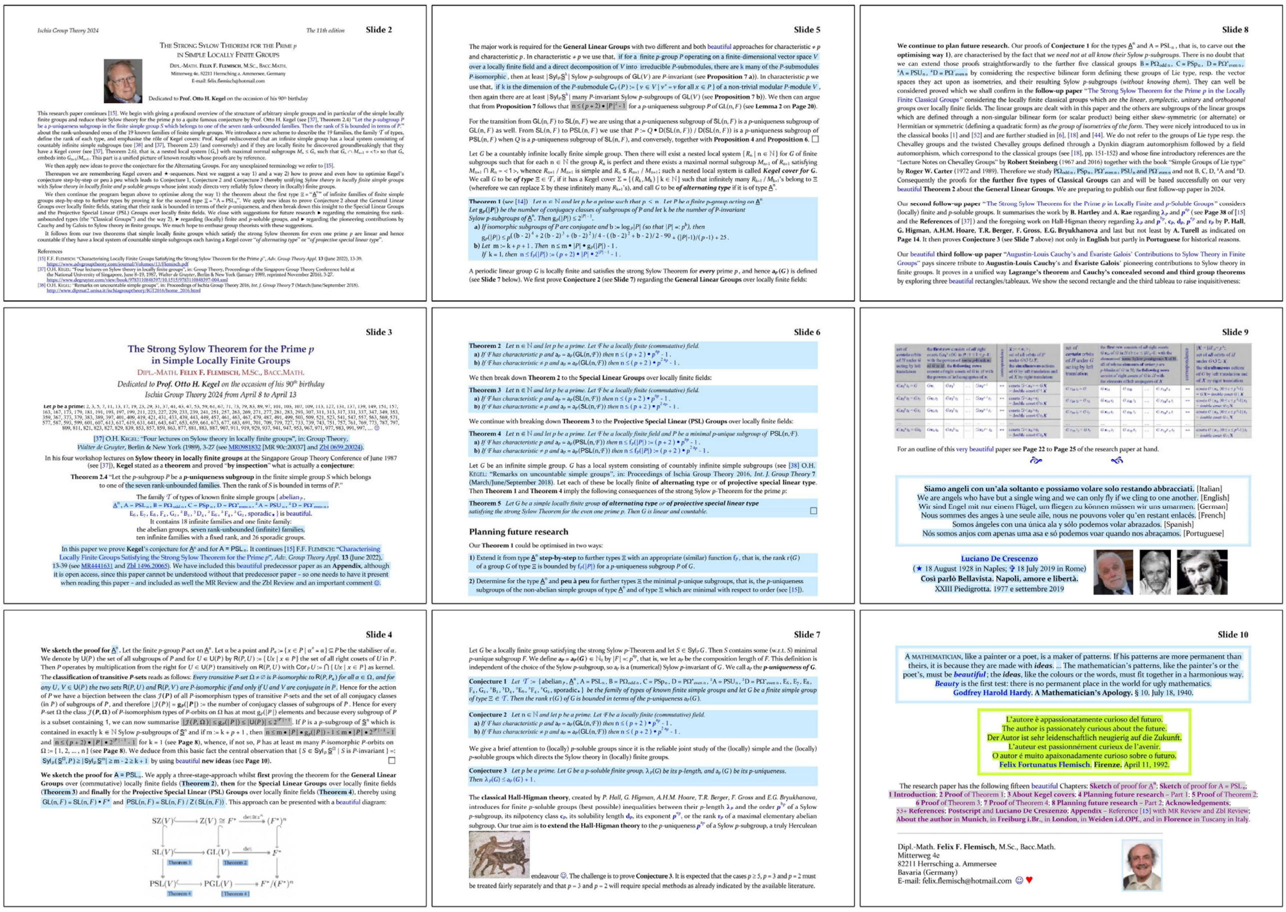
Ten and almost a half years later **Prof. Kegel** visited the Ischia Group Theory 2016 (IGT 2016) conference which took place from March 29, 2016 to April 2, 2016 in Italy at the beautiful Isola d'Ischia (see https://en.wikipedia.org/wiki/Ischia) directly opposite of Napoli (see https://www.dipmat2.unisa.it/ischiagrouptheory/IGT2016/home_2016.html). He there gave a talk entitled "REMARKS ON UNCOUNTABLE SIMPLE GROUPS", a topic which he discovered together with **Prof. Philip Hall**, and which is presented in detail in the JMCA-paper "*The Strong Sylow Theorem for the Prime p in Simple Locally Finite Groups*" (see https://www.onlinescientificresearch.com/articles/the-strong-sylow-theorem-for-the-prime-*p*-in-simple-locally-finite-groups.pdf and https://www.onlinescientificresearch.com/journals/jmca/abstract/the-strong-sylow-theorem-for-the-prime-p-in-simple-locally-finite-groups-6110.html) with a number of references to the "known" finite simple groups according to the Classification of Finite Simple Groups (see https://en.wikipedia.org/wiki/Classification_of_finite_simple_groups).

Otto H. Kegel (Ischia, 2016) ● courtesy of **Nikolay Aleksandrovich Vavilov**
(see https://www.advgrouptheory.com/GTArchivum/Pictures/gtphotos/OttoKegel.jpg)

Another eight years later the Ischia Group Theory 2024 (IGT 2024) conference took place, again at the nice Isola d'Ischia, from April 8, 2024 to April 13, 2024 with April 11, 2024 being the 120th birthday of **Prof. Hall**, but **Prof. Kegel** could not participate (see https//sites.google.com/unisa.it/igt/home and https://sites.google.com/unisa.it/igt/agenda):



However, **Prof. Kegel** was very friendly honoured on the occasion of his 90th birthday on July 20, 2024. At the conference **Dipl.-Math. Felix F. Flemisch** gave a talk having 18 slides entitled "*Talk by Felix F. Flemisch at Ischia Group Theory 2024*", being a part of his JMCA-paper "*The Strong Sylow Theorem for the Prime p in Simple Locally Finite Groups*", which included a presentation of **Prof. Kegel**'s contributions to **Sylow theory in locally finite groups** (see https://www.onlinescientificresearch.com/journals/jmca/archives/2025/4/1) ☺. Nine slides were presented as a poster at the Permanent Poster Session of the conference:



Otto H. Kegel & **Andrea Caranti** (Ischia, 2016) • courtesy of **Francesco de Giovanni**
(see https://www.advgrouptheory.com/GTArchivum/Pictures/gtphotos/KegelCaranti.jpg)

Mahmut Kuzucuoğlu & **Otto H. Kegel** (Ischia, 2016) • courtesy of **Nikolay Aleksandrovich Vavilov**
(see https://www.advgrouptheory.com/GTArchivum/Pictures/gtphotos/kuzuKegel.jpg)

Otto H. Kegel (Mathematisches Forschungsinstitut Oberwolfach (MFO), 2010) • courtesy of **Renate Schmid** (see https://opc.mfo.de/detail?photo_id=12422)