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Integral Problem for System of Partial Differential Equations of Higher Order

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ABSTRACT
We propose method of solving problem with integral conditions for system of partial differential equation of higher order.

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Introduction
The problem with integral condition for system of partial differential equation and partial differential equation have been widly investigated and found a lot of applications in important practical problems. The integral conditions are used in particular, in models of heat conduction [1-6]. Evolution problems with nonlocal boundary conditions have received attention in several papers [4,7]. Mixed problems with nonlocal boundary conditions or with nonlocal initial conditions were studied in [8-11]. In the work the research was carried out nonloval-integral problem for differential equations of third order. Problems for parabolic equations were studied in the works and for hiperbolic equations were studied in the works [7, 8, 13, 14].

This paper, which continues our investigation in [15, 16] and for partial differential equations study of the unique solvability with a problem with integral condition for equations first order [17-21]. problem with integral conditions in Sobolev space were investigated in several papers [22-27].

Statememt of the Problem
Let $H(\mathbb{R}_+ \times \mathbb{R}^n)$ be a class of entire functions on $\mathbb{R}$, K_L is a class of quasipolynomials of the form

$$\varphi(x) = \sum_{j=1}^{m} Q_j(x) \exp[a_j x], \qquad (1)$$

where $a_j \in M \subseteq \mathbb{C}, \alpha_j \neq \alpha_k,$ for $j \neq k, x \in \mathbb{R}, Q_j(x),$ are polynomials with complex coefficients. Each quasipolynomial $\varphi(x)$ of the form (1) defines differential operation

$Q_j\left(\frac{d}{d\lambda}\right)$ of finite order on the class of entirie functions $\Phi(\lambda)$, namely,

$$\varphi\left(\frac{d}{d\lambda}\right)\Phi(\lambda) = \sum_{j=1}^{m} Q_j\left(\frac{d}{d\lambda}\right)\Phi(\lambda\Big|_{\alpha_j} = \sum_{j=1}^{m} Q_j\left(\frac{d}{d\lambda}\right)\Phi(\lambda + \alpha_j). \qquad (2)$$

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In the strip $\Omega = \{(t,x) \in \mathbb{R}^{n+1} : t \in \{(0,T), x \in \mathbb{R}^n\},$

we consider of the system of partial differential equations of higher order:

$$\frac{\partial^n U_i}{\partial t^n} + \sum_{j=1}^{n} a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j} U_i}{\partial t^{n-j}} = 0, \quad i = \{1,...,n\}, \qquad (3)$$

$$\int_0^T t^k U_i(t,x)dt = \varphi_{ik}(x), \quad k = \{1,...,n\}, \quad t \in (0,T), x \in \mathbb{R}^n. \qquad (4)$$

Where $a_{ij}\left(\frac{\partial}{\partial x}\right)$, are differential expression with entire symbols $a_{ij}(\lambda) \neq 0$. Along the lines of equation (3), replacing $\frac{\partial}{\partial x}$ by $\lambda \in \mathbb{C}$ we wirte down the ordinary differential equation

$$\frac{d^n T}{dt^n} + \sum_{j=1}^{n} a_{ij}(\lambda)\frac{d^{n-j} T}{dt^{n-j}} = 0, \qquad (5)$$

satrisfies integral conditions

$$\int_0^T t^{n-1} T(t,\lambda)dt = 1, \quad \int_0^T t^{n-2} T(t,\lambda)dt = 0, \quad ..., \int_0^T T(t,\lambda)dt = 0, \qquad (6)$$

The solution of equation (5) is given by the formula

$$T(t,\lambda) = \sum_{i=1}^{n} C_i(\lambda) T_i(t,\lambda), \qquad (7)$$

Taking into account conditions (6) in equality (7) we obtain an algebraic system of equations, to designate a function $C_1(\lambda), C_2(\lambda), ..., C_n(\lambda),$

$$\begin{cases} \int_0^T t^{n-1} \sum_{i=1}^n C_i(\lambda) T_i(t,\lambda)dt = 1 \\ \int_0^T t^{n-2} \sum_{i=1}^n C_i(\lambda) T_i(t,\lambda)dt = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \int_0^T \sum_{i=1}^n C_i(\lambda) T_i(t,\lambda)dt = 0 \end{cases}. \qquad (8)$$

The main determinant of the system (8)

$$\Delta(\lambda)=\begin{vmatrix} \int\limits_0^1 t^{n-1}T_1(t,\lambda)dt & \dots & \int\limits_0^1 t^{n-1}T_n(t,\lambda)dt \\ \dots & \dots & \dots \\ \int\limits_0^1 T_1(t,\lambda)dt & \dots & \int\limits_0^1 T_n(t,\lambda)dt \end{vmatrix}.$$

To determine $C_1(t,\lambda), C_2(t,\lambda), ..., C_n(t,\lambda)$ we consider determinants

$$\Delta_{11}(\lambda)=\begin{vmatrix} 1 & \int\limits_0^1 t^{n-1}T_2(t,\lambda)dt & \dots & \int\limits_0^1 t^{n-1}T_n(t,\lambda)dt \\ \dots & & \dots & \dots \\ 0 & \int\limits_0^1 t^{n-1}T_1(t,\lambda)dt & ... & \int\limits_0^1 T_n(t,\lambda)dt \end{vmatrix}.$$

$$\Delta_{12}(\lambda)=\begin{vmatrix} \int\limits_0^1 t^{n-1}T_1(t,\lambda)dt & 1 & \int\limits_0^1 t^{n-1}T_n(t,\lambda)dt \\ ... & ... & ... \\ \int\limits_0^1 T_1(t,\lambda)dt & 0 & \int\limits_0^1 T_n(t,\lambda)dt \end{vmatrix}.$$

....

$$\Delta_{1n}(\lambda)=\begin{vmatrix} \int\limits_0^1 t^{n-1}T_1(t,\lambda)dt & \int\limits_0^1 t^{n-1}T_2(t,\lambda)dt... & 1 \\ ... & ... & ... \\ \int\limits_0^1 T_1(t,\lambda)dt & \int\limits_0^1 T_2(t,\lambda)dt... & 0 \end{vmatrix}.$$

$$C_1(t,\lambda)=\frac{\Delta_{11}(\lambda)}{\Delta(\lambda)}, \quad C_2(t,\lambda)=\frac{\Delta_{12}(\lambda)}{\Delta(\lambda)}, \quad ..., \quad C_n(t,\lambda)=\frac{\Delta_{1n}(\lambda)}{\Delta(\lambda)}.$$

Solution of the system (8) we have:

$$T(t,\lambda)=\sum_{i=1}^{n}\frac{\Delta_{in}(\lambda)}{\Delta(\lambda)}T_i(t,\lambda).$$

Denot be $P=\{\lambda\in\mathbb{C}:\Delta(\lambda)=0\}$ set of zeros of a function $\eta(\lambda)=\frac{\Delta_{in}(\lambda)}{\Delta(\lambda)}$, denote be $M=\{\lambda\in\mathbb{C}:\Delta(\lambda)\neq 0\}$ for which there is a solution to the problem (3)-(4).

Lemma 1. Let $T(t,\lambda)$ solution of the equation (5), for $\lambda\in M\subset C$ satisfies conditions:

$$\int\limits_0^T t^k T(t,x)dt=0, \qquad (9)$$

Proof. If $T(t,\lambda)$ is a solution to equation (6), then it can be expressed in the form (7). $C_i(\lambda)$ any functions of the parameter $\lambda\in M\subset C$ satisfying the conditions (9), we obtain a system of equations

$$\begin{cases} \int\limits_0^T t^{n-1}\sum_{i=1}^n C_i(\lambda)T_i(t,\lambda)dt=0 \\ \int\limits_0^T t^{n-2}\sum_{i=1}^n C_i(\lambda)T_i(t,\lambda)dt=0 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \int\limits_0^T \sum_{i=1}^n C_i(\lambda)T_i(t,\lambda)dt=0. \end{cases} \qquad (10)$$

Of course $\lambda\in M$, we get the condition $\Delta(\lambda)\neq 0$. It follows from this that the system (10) has a trivial solution $C_1(\lambda)=C_2(\lambda)=\cdots=C_n(\lambda)=0$, then $T(t,\lambda)=0$ na $(0, T)$ x M. Which concludes the proof.

Lemma 2. For any $t\in(0,T)$ and $\lambda\in\mathbb{C}\setminus P$, the following equality holds:

$$\left[\frac{d^n}{dt^n}+\sum_{j=1}^n a_{ij}(\lambda)\frac{d^{n-j}}{dt^{n-j}}\right]\left\{T_{kjp}(t,\lambda)\right\}=0.$$

Proof. The following equality we have:

$$\left[\frac{d^n}{dt^n}+\sum_{j=1}^n a_{ij}(\lambda)\frac{d^{n-j}}{dt^{n-j}}\right]\left\{T_{kjp}(t,\lambda)\right\}=$$

$$=\left[\frac{d^n}{dt^n}+\sum_{j=1}^n a_{ij}(\lambda)\frac{d^{n-j}}{dt^{n-j}}\right]\left\{\sum_{i=1}^n\frac{\Delta_{in}(\lambda)}{\Delta(\lambda)}T_i(t,\lambda))\right\}=$$

$$=\frac{d^n}{dt^n}\left\{\sum_{i=1}^n\frac{\Delta_{in}(\lambda)}{\Delta(\lambda)}T_i(t,\lambda)\right\}+\frac{d^{n-j}}{dt^{n-j}}\left\{\sum_{i=1}^n\frac{\Delta_{in}(\lambda)}{\Delta(\lambda)}T_i(t,\lambda)\right\}=$$

$$\left[\frac{d^n}{dt^n}+\sum_{j=1}^n a_{ij}(\lambda)\frac{d^{n-j}}{dt^{n-j}}\right]\left\{\sum_{i=1}^n\frac{\Delta_{in}(\lambda)}{\Delta(\lambda)}T_i(t,\lambda)\right\}=0.$$

Which concludes the proof.

Theorem. Let $\varphi_{ik}(x)\in K_L$, $i=\{1,...,n\}$, $j=\{1,...,n\}$ then the class $K_{L\setminus P}$ exist and unique solution of the problem (1)-(2), can be represented

$$U_i(t,x)=\sum_{k=0}^{n-1}\sum_{p=1}^{n}\varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)}T_{kjp}(t,\lambda\exp[\lambda x]\right\}\Bigg|_{\lambda=0}, \qquad (11)$$

Proof. If $\varphi_i(x)\in K_M$ for $i=\{1,...,n\}$, a $T(t,\lambda)$ solution of the equation (5) according with formul (2) function $\varphi_i\left(\frac{\partial}{\partial\lambda}\right)\{T(t,\lambda)\exp[\lambda x]\Big|_{\lambda=0}$ belongs to the class $K_{C\setminus P}$.

Then for some $\varphi_i(x)\in K_M$ for $i=\{1,...,n\}$ according to the formula (11), we have $U_i(t,x)$ dla $i=\{1,...,n\}$ is a quasipolynomial in the class $K_{M\setminus P}$ .Using the equationi $a_i\left(\frac{\partial}{\partial x}\right)=a_i(\lambda)$, for $i=\{1,...,n\}$, we will show that the function (11) satisfies the system of equations (3).

$$\frac{\partial^n U_i}{\partial t^n}+\sum_{j=1}^n a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j}U_i}{\partial t^{n-j}}U(t,x)=$$

$$=\frac{\partial^n}{\partial t^n}\left\{\sum_{k=0}^{n-2}\sum_{p=1}^{n}\varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)}T_{kjp}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\right\}+$$

$$+\sum_{j=1}^n a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j}}{\partial t^{n-j}}\left\{\sum_{k=0}^{n-2}\sum_{p=1}^{n}\varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)}T_{kjp}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\right\}=$$

$$=\sum_{k=0}^{n-2}\sum_{p=1}^{n}\varphi_{kp}\left(\frac{\partial}{\partial x}\right)\frac{\partial^n}{\partial t^n}\left\{\frac{1}{\eta(\lambda)}T_{kjp}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}+$$

$$+\sum_{k=0}^{n-2}\sum_{p=1}^{n}\varphi_{kp}\left(\frac{\partial}{\partial x}\right)\sum_{j=1}^n a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j}}{\partial t^{n-j}}\left\{\frac{1}{\eta(\lambda)}T_{kjp}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}=0.$$

We shall prove the realization conditions (4) we have:

$$\int_0^T t^k U_{ik}(t,x)dt =$$

$$= \int_0^T t^k \Big\{ \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kjp}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\Big\} dt =$$

$$= \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\int_0^T t^k \Big\{\frac{1}{\eta(\lambda)} T_{kjp}(t,\lambda)\exp[\lambda x]\Big\}\Bigg|_{\lambda=0} dt = \varphi_{ik}(x).$$

We proved the existence and unique solutions (11) of the problem (3)-(4). Let be $\varphi_k \in K_M$ for $k \in \{0,1,...,n-1\}$ exists two solutions U_1^1, U_2^2 of the problem (3)-(4) in the form:

$$U_i^1(t,x) = \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{ki1p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}$$

$$U_i^2(t,x) = \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{ki2p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}$$

dla $i = \{1,...,n\}$. Then the difference of these functions satisfies the equation $V_i^1 = U_i^1 - U_i^2 \in \mathbb{C}\setminus P$.

$$\frac{\partial^n V_i}{\partial t^n} + \sum_{j=1}^{n} a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j}V_i}{\partial t^{n-j}} = 0.$$

By calculating we have:

$$\frac{\partial^n}{\partial t^n}\left(U_i^1 - U_i^2\right) + \sum_{j=1}^{n} a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j}}{\partial t^3}\left(U_i^1 - U_i^2\right) =$$

$$= \frac{\partial^n}{\partial t^n}\Big\{\sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj1p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0} -$$

$$- \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj2p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\Big\} +$$

$$+ \sum_{j=1}^{n} a_{ij}\left(\frac{\partial}{\partial x}\right)\frac{\partial^{n-j}}{\partial t^{n-j}}\Big\{\sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj1p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0} -$$

$$+ \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj2p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\Big\},$$

$$\sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj1p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0} -$$

$$- \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj2p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0} = 0.$$

Satisfying the integral conditions:

$$\int_{T_1}^{T_2} t^k V_i(t,x)dt = 0.$$

By calculating we have:

$$\int_0^T t^k V_i(t,x)dt = \int_0^T t^k (U_i^1(t,x) - U_i^2(t,x))dt =$$

$$= \int_0^T t^k \left(\sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj1p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\right) dt -$$

$$- \int_0^T t^k \left(\sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\left\{\frac{1}{\eta(\lambda)} T_{kj2p}(t,\lambda)\exp[\lambda x]\right\}\Bigg|_{\lambda=0}\right) dt =$$

$$= \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\int_0^T t^k \Big\{\frac{1}{\eta(\lambda)} T_{kj1p}(t,\lambda)\exp[\lambda x]\Big\}\Bigg|_{\lambda=0} dt -$$

$$= \sum_{k=0}^{n-2}\sum_{p=1}^{n} \varphi_{kp}\left(\frac{\partial}{\partial x}\right)\int_0^T t^k \Big\{\frac{1}{\eta(\lambda)} T_{kj2p}(t,\lambda)\exp[\lambda x]\Big\}\Bigg|_{\lambda=0} dt = 0.$$

Of course $\varphi_{kp}(x) \in K_M$, $k \in \{0,...,n-2\}$, it according to the forms (11) quasipolynomial $V(t, x)$ belongs to class $K_{\mathbb{C},Z}$.
The quasipolynomial he has a property $T_{ki1p}(t,\lambda) - T_{ki2p}(t,\lambda)$, for $p \in \{1,...,n\}$,$T_{ki2p}(t,\lambda) - T_{ki1p}(t,\lambda)$, for $p \in \{1,...,n\}$, and their derivatives with respect to λ in point (t, λ), where $\lambda \in M$, $t \in (0,T)$. With regard to the condition (7) we receive, that difference function $T_{ki1p}(t,\lambda) - T_{ki2p}(t,\lambda)$, $T_{ki2p}(t,\lambda) - T_{ki1p}(t,\lambda)$ satysfies condition Lemma 1. It according to it Lemma 2 we receive the equality of the ones function $T_{ki1p}(t,\lambda) - T_{ki2p}(t,\lambda)$ $T_{ki2p}(t,\lambda) - T_{ki1p}(t,\lambda)$ and their derivatives with respect to λ in points (t, λ). So $V(t, x) = 0$ in strip Ω, then $U_i^1(t,x) = U_i^2(t,x)$.

Exempl and Applied of the Method

Exempl 1. We consider problem with integral conditions for syustem of equations fourth order. In the strip $\Omega = \{(t,x) \in \mathbb{R}^2 : t \in (0,1), x \in \mathbb{R}\}$ we consider equations

$$\frac{\partial^4 U_1(t,x)}{\partial t^4} - \frac{\partial^4 U_2(t,x)}{\partial x^4} = 0,$$

$$\frac{\partial^4 U_2(t,x)}{\partial t^4} - \frac{\partial^4 U_1(t,x)}{\partial x^4} = 0,$$

satysfies integreal conditions:

$$\int_0^1 U_1(t,x)dt = xe^x, \quad \int_0^1 tU_1(t,x)dt = x^2e^x \quad \int_0^1 t^2U_1(t,x)dt = xe^x + x^4e^{2x},$$

$$\int_0^1 t^3U_1(t,x)dt = xe^{2x}$$

$$\int_0^1 t^2U_3(t,x)dt = (1-x^2)e^x, \quad \int_0^1 t^2U_3(t,x)dt = (1+x)e^{2x}, \quad \int_0^1 t^2U_3(t,x)dt = e^{3x},$$

$$\int_0^1 t^2U_3(t,x)dt = (1+x+x^2+x^3)e^x,$$

Solution of the problem, according to formua (11)

$$U_i(t,x) = \sum_{k=0}^{1}\sum_{p=1}^{4} \varphi_{kp}\left(\frac{\partial}{\partial x}\right) T_{kjp}(t,\lambda\exp[\lambda x]\Big\}\Bigg|_{\lambda=0}.$$

Exempl 2. We consider problem with integral conditions for system of equations thirt order. In the strip $\Omega = \{(t,x,y) \in \mathbb{R}^3 : t \in (0,1), x \in \mathbb{R}^2\}$ we consider system of equations

$$\frac{\partial^3 U_1(t,x,y)}{\partial t^3} - \frac{\partial^3 U_2(t,x,y)}{\partial x^3} + \frac{\partial}{\partial t}\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y}\right)U_3(t,x,y) = 0,$$

$$\frac{\partial^3 U_2(t,x,y)}{\partial t^3} - \left(\frac{\partial^3 U_1(t,x,y)}{\partial x^3} + \frac{\partial^3 U_3(t,x,y)}{\partial y^3}\right) = 0,$$

$$\frac{\partial^3 U_2(t,x)}{\partial t^3} + \left(\frac{\partial^2 U_1(t,x,y)}{\partial x^2} + \frac{\partial^2 U_3(t,x,y)}{\partial y^2}\right) = 0,$$

satysfies integreal conditions:

$$\int_0^1 U_1(t,x,y)dt = (x^2+y^2+x-y)e^{x+y}, \quad \int_0^1 tU_1(t,x,y)dt = (x-y-1)e^{2x},$$

$$\int_0^1 t^3U_1(t,x)dt = (x+y)e^{2x-y},$$

$$\int_0^1 tU_2(t,x,y)dt = e^x, \quad \int_0^1 tU_2(t,x,y)dt = e^y, \quad \int_0^1 tU_2(t,x,y)dt = e^{x+y},$$

$$\int_0^1 t^2U_3(t,x)dt = (1+x+y)e^{x+y}, \quad \int_0^1 t^2U_3(t,x,y)dt = (x+y^2-y^2)e^x,$$

$$\int_0^1 t^2U_3(t,x)dt = (1+x)e^{3x+2y}.$$

Solution of the problem, according to formua (11)

$$U_i(t,x) = \sum_{k=0}^{2}\sum_{p=1}^{3}\varphi_{kp}\left(\frac{\partial}{\partial x}\right)T_{kjp}(t,\lambda\exp[\lambda x]\Big\}\Bigg|_{\lambda=0}.$$

Conclusion. In the present paper, we propose a method of solving problem with integral conditions for system of partial differential equations of higher order with respect to time. We prove the existence and uniqueness of a solution of the problem in a class of quasipolynomials of the specjal form. Solution problem of this problem with the use of the differential-symbol method [20, 21, 27-30].

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