ISSN: 3033-3652

Journal of Physical Mathematics & its Applications

SCIENTIFIC Research and Community

Mini Review

Open Access

Proof of the Conjecture about an Infinite Set of Twin Prime Numbers

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ABSTRACT

Some classical problems in number theory serve as a reliable platform for solving slightly more complex mathematical problems that have long been, not quite deservedly, elevated to the rank of open mathematical problems. As the author sees it, this is precisely the case with the well-known unsolved problem of twin prime numbers.

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Received: March 19, 2026; **Accepted:** March 24, 2026; **Published:** April 02, 2026

Keywords: Prime Numbers, Twin Prime Numbers, Euclid's Proof

Introduction

Twin prime numbers are known to be pairs of prime numbers that differ by 2. The first formulation of the conjecture about existence of an infinite set of such pairs of numbers belongs to the French mathematician Alphonse de Paulignac and dates back to 1846. Polignac later generalized this conjecture to cases where the difference between primes is equal to [1].

Almost two centuries after those early days, the problem remains among the unsolved problems of mathematics. It seems to the author that this period in years should not have been a three-digit number, at least not with respect to Polignac's generalized conjecture, but to the original conjecture about an infinity of twin prime numbers.

Euclid's Proof of the Infinity of Primes

As a reminder, we cite Euclid's well-known proof of an infinite number of primes [2]. It is assumed that the number of primes is finite, and a finite list of these numbers is given: $p_1, p_2, ..., p_k$. We form the following new number from these numbers

$$P = \prod_{i=1}^{k} p_i + 1 = p_1 \cdot p_2 \cdot ... \cdot p_k + 1, \qquad (1)$$

from which it clearly follows that P is either a prime number or factorizable beyond the given finite list of numbers. In both cases, the assumption of a finite set of prime numbers is refuted.

Proof of the Twin Prime Numbers Conjecture

We assume, as in the previous case, an existence of finite sets of twin prime numbers, namely: $p_1, p_2, ..., p_k$ and $q_1, q_2, ..., q_k$. By the definition of twin primes, these pairs are formulated as follows:

$$p_i = q_i + 2, \quad i = 1,..., k. \qquad (2)$$

Unlike Euclid's proof, we now form not a single number P, but two numbers: P and Q, combined into the system:

$$\begin{cases} P = \prod_{i=1}^{k} p_i + 1 = p_1 \cdot p_2 \cdot ... \cdot p_k + 1, \\ Q = \prod_{i=1}^{k} p_i - 1 = p_1 \cdot p_2 \cdot ... \cdot p_k - 1, \end{cases} \qquad (3)$$

from which it is clear that P - Q = 2.

Applying the same reasoning to the system of numerical expressions (3) as we did above for the ordinary set of prime numbers, we arrive at the same conclusion: the numbers P and Q are either prime (either both together or each separately) or they contain prime factors that extend beyond the given list of numbers (either both together or each separately). In both cases, the assumption of a finite set of twin prime numbers is refuted.

It is easy to see that the above proof is equally valid using the sequence $q_1, q_2,..., q_k$ instead of the list of numbers $p_1, p_2,..., p_k$; however, in this case, a different system of numerical expressions must be used, as follows:

$$\begin{cases} P = \prod_{i=1}^{k} q_i + 1 = q_1 \cdot q_2 \cdot ... \cdot q_k + 1, \\ Q = \prod_{i=1}^{k} q_i - 1 = q_1 \cdot q_2 \cdot ... \cdot q_k - 1. \end{cases} \qquad (4)$$

Consequently, the set of twin prime numbers is infinite.
What needed to be demonstrated.

Conclusion

In this note, a short proof of the conjecture about an infinite set of twin prime numbers is proposed, based on Euclid's classical proof about an infinite set of primes.

References

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