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Otto H. Kegel's Beloved Contributions to Sylow Theory – In Memory of My Adored Masterly Teacher

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Abstract

Let p be a prime. A p -group is a group all of whose elements have order a power of p and a p' -group is a group all of whose elements have order prime to p , that is, not containing any element of order p . A p -group is finite if and only if its order is a power of p . Every group theorist knows what are well-known as the three Sylow theorems: any finite group G has

- 1) subgroups of order $|G|_p$, where $|G|_p$ is the highest power of p dividing the order $|G|$ of G , which are called Sylow p -subgroups and are not only maximal with respect to (w.r.t.) order but also w.r.t. inclusion, whence
- 2) every p -subgroup of G is contained in at least one Sylow p -subgroup, and
- 3) all the Sylow p -subgroups are conjugate, that is, if S_1 and S_2 are Sylow p -subgroups of G , then there exists an element x of G such that $x^{-1}S_1x = S_2$ ¹.

The set of all Sylow p -subgroups of a group G is denoted by $\text{Syl}_p(G)$. The theorems and the maximal p -subgroups are named after Ludvig Sylow, the great Norwegian mathematician who discovered them and published them (fortunately) in December 1872 (see [26.] and https://en.wikipedia.org/wiki/Peter_Ludvig_Sylow). A Sylow p -subgroup of an arbitrary group is a p -subgroup, which is maximal w.r.t. inclusion. Every p -subgroup of a group is contained in at least one Sylow p -subgroup. A group satisfies the Sylow Theorem for the Prime p or the Sylow p -Theorem, if all of its Sylow p -subgroups are conjugate, and it satisfies the Strong Sylow Theorem for the Prime p , if each of its subgroups satisfies the Sylow p -Theorem. A locally finite group is a group all of whose finitely generated subgroups are finite. Sylow Theory of Locally Finite Groups studies when they satisfy the Sylow p -Theorem and when even the Strong Sylow p -Theorem and determines the structure of those groups. A central concept to that end is the p -uniqueness subgroup of a locally finite group, which is a finite p -subgroup being contained in a unique Sylow p -subgroup.

¹When G is a finite group, P a p -subgroup of G and $S \in \text{Syl}_p(G)$, then the operation of P by conjugation on $C(G, S) := \{ S^x \mid x \in G \}$ has at least one fixed point, that is $(\exists x \in G) (P^x \subseteq S)$, and for $P \in \text{Syl}_p(G)$ exactly one, which means

4) $|\text{Syl}_p(G)| = |G : N_G(S)| = |C(G, S)| \equiv 1 \pmod{p}$; hence G satisfies the Strong Sylow Theorem for the Prime p , that means, every subgroup U of G conjugates transitively on $\text{Syl}_p(U)$, and thus we have the Frattini argument for G (and p), that is, if N is a normal subgroup of G and $P \in \text{Syl}_p(N)$, then $N_G(P)$ covers G/N , that is, $G = N \cdot N_G(P)$.

Otto H. Kegel has since the swinging sixties of last century again and again showed interest in Sylow Theory and very especially in how to extend it from finite groups to locally finite groups. He summarised findings up to 1973 by him and by others in the book [22.], which became a standard book on locally finite groups. When the book was in press, he developed the new paper [11.] on Sylow Theory of Locally Finite Groups and presented it in lectures during 11 December 1973. This paper has two open questions until today. 13½ years later he presented on 8 June 1987 in four lectures [12.] a summary of results up to 1987 about Sylow Theory of Locally Finite Groups and extended them beautifully from locally finite and p -soluble groups for $p \neq 2$, according to results by Brian Hartley and Andrew Rae, to locally finite groups in general for $p \geq 5$. This paper has ten open questions until today.

The paper at hand presents Otto H. Kegel's achievements of and merits for Sylow Theory of Locally Finite Groups and communicates that it was a "Herzensangelegenheit" (matter close to one's heart) for him. Otto H. Kegel passed away on his birthday 20 July 2025 at the age of 91. Being in deepest mourning, I miss him dreadfully and will always honour his memory.

The paper presents references to articles on Sylow Theory of Locally Finite Groups by Otto H. Kegel and by the author. First it presents and summarises the twelve beloved contributions by Otto H. Kegel. Secondly it presents and summarises the ten+four

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enthusiastic contributions by the author, which add thorough comments to Otto H. Kegel's admirable contributions regarding wide generalisations, solve two of his open issues and state twelve still open issues most of which, however, the author solved already in yet unpublished work. Thirdly it draws some conclusions from Kegel's contributions and the author's contributions thereby suggesting fascinating topics for future research based upon the twelve open issues of [11.] and [12.].

In the meantime the author published his solution of Open Issue 6: see 28. Flemisch, F.F.: Characterising locally finite groups satisfying the strong Sylow Theorem for the prime p – Manuscript on Sylow theory in locally finite groups – Mathematics edition – Second edition. Journal of Mathematical & Computer Applications (JMCA), Volume 5, Issue 2 (April 2026), 1-40. <https://srcpublishers.com/mathematical-computer-applicatio/issue/view/2041>.