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Quantum Gravity and the Distance-Velocity Profile of a Black Hole

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ABSTRACT

Newton's law of universal gravitation implies that the circular velocity of a low-mass body at a distance *r* from the center is given by the relation $v^2=GM/r$, while the velocity v remains unbounded. According to Einstein's theory of relativity, the relative velocity of motion is limited by a maximum speed, which is the speed of light. Furthermore, a massive object cannot reach the speed of light. In General Relativity, the circular velocity from a gravitational center is also determined by the same expression used to describe the gravitational field according to Newton.

By using this fundamental relationship in combination with the relativistic limit of maximum velocity to derive the magnitude of binding energy—and consequently the binding mass of a bound state of two massive objects—we obtain two equations. According to the first, the value of the gravitational coupling parameter *K* can be determined as a function of the distance from the center; according to the second, the velocity of circular motion at that distance can be established.

The derived equation for the coupling parameter, which we shall call the "vacuum equation," can be expressed as the sum of three equations. The first is the fermion equation, the second is the boson equation, and the third is the gravitation equation. Solving these equations individually, as well as their partial sums for the parameter *K*=2, leads to determining the volume ratios of selected regular polytopes in four-dimensional space (with properties described by L. Schläfli, which are in agreement with the mass ratios of selected elementary particles. The concept of regular polyhedra in three-dimensional space in connection with planetary orbits was previously used by J. Kepler.

For the parameter $K\in(0,1)$, the solution of the gravitation equation leads to determining the mass of the resulting so-called black hole after the collapse of two massive objects. The first set of solutions to the equation for the gravitational coupling parameter determines the velocity profile based on circular velocity at a distance from the gravitational center both outside and inside the black hole. This gravitational parameter is established by solving the gravitation equation, which depends on a parameter that distinguishes massive objects based on their composition—whether they are fermionic, bosonic, or a mixture of these basic components. The Planck black hole, which lies on the boundary between the masses of elementary particles and massive objects such as black holes, can be considered the limiting threshold.

The second set of solutions for the parameter $K > 2$ determines the mass of a particle equivalent to the anti-gravitational effect expected in dark energy. Simultaneously, this group of solutions allows for the determination of conditions for the quantum mechanical evaporation of black holes based on particle-antiparticle creation with rest mass. This tri-particle creation involves a dark energy particle, a dark matter particle, and a baryonic particle, whose masses are in the expected ratio corresponding to their observed distribution in the universe.

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Introduction

Derivation of the Velocity Equation Determining Velocity as a Function of the Coupling Parameter

An analogy with the equation for electromagnetic interaction is used for the derivation, specifically for the bound state of an electron to a proton in a hydrogen atom [1,2]. In this case, the resulting mass of the hydrogen atom is given by the sum of the rest masses of the participating particles, reduced by the mass that represents the binding intensity. In the case of this electromagnetic bond, it is represented by the parameter I_e, which depends on quantum numbers and electric charge as determined by the solution to the Dirac equation [4],[5]. Furthermore, this parameter is multiplied by the reduced mass of the participating particles.

$$m_H = m_p + m_e - I_e \frac{m_p m_e}{m_p+m_e}$$

Analogously Determined Relationship for the Coupling Parameter of Gravitational Interaction

$$m_g = m_1 + m_2 - I_g \frac{m_1 m_2}{m_1+m_2} = m_1 + m_2 - m_{gv} \quad (1)$$

Equation (1) determines the resulting gravitational mass m_g, defined by the sum of the rest masses m_1 and m_2 reduced by the mass deficit given by the expression:

$$m_{gv} = I_G \frac{m_1 m_2}{m_1 + m_2} \qquad (2)$$

Equivalence of the Mass Deficit

In a reference coordinate system where the origin lies at the center of a massive object m_1, an object m_2 moves in an ideal circular orbit with velocity v at a distance r from the center, where $m_1 \geq m_2$. To determine the equivalence of the mass deficit, a second idealized velocity is important: the parabolic (escape) velocity. The parabolic velocity for a given distance r determines the separation of object m1 from object m_2. The difference between the kinetic energy determined by the parabolic velocity and the kinetic energy determined by the circular velocity of object m_2 (or the equivalent mass of this energy) is equal to the mass deficit m_{gv}.

The equivalent mass m_p for the kinetic energy at the parabolic velocity v_p will be:

$$m_p = \frac{m_2}{\sqrt{1 - \frac{v_p^2}{c^2}}} - m_2,$$

similarly, the equivalent mass mk for the kinetic energy at the circular velocity v_k will be:

$$m_k = \frac{m_2}{\sqrt{1 - \frac{v_k^2}{c^2}}} - m_2$$

according to the model [6].

Then: $m_p - m_k = m_{gv} = I_G \frac{m_1 m_2}{m_1 + m_2}$

$I_G \frac{m_1 m_2}{m_1 + m_2} = m_2 \left(\frac{1}{\sqrt{1 - \frac{v_p^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v_k^2}{c^2}}} \right)$, therefore $I_G \frac{m_1}{m_1 + m_2} = \frac{1}{\sqrt{1 - \frac{v_p^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v_k^2}{c^2}}}$

Derivation of the Vacuum Equation, its Additive Components, and their Solutions

The quantity $K = I_G \frac{m_1}{m_1 + m_2}$ is called the coupling parameter, and for $v_p = \sqrt{2}\, v_k$ we then obtain the relationship

$K = \frac{1}{\sqrt{1 - \frac{2v_k^2}{c^2}}} - \frac{1}{\sqrt{1 - \frac{v_k^2}{c^2}}}$. Using the substitution $q = \frac{v_k^2}{c^2}$, we receive

$K = \frac{1}{\sqrt{1-2q}} - \frac{1}{\sqrt{1-q}}$.

After removing fractions and square roots, we obtain the implicit form of the equation:

$$f(K, q) = K^4(4q^4 - 12q^3 + 13q^2 - 6q + 1) + 2K^2(6q^3 - 13q^2 + 9q - 2) + q^2 = 0 \quad (3)$$

which we shall call the velocity equation.

Another expression for K is obtained by using the substitutions $v_k^2 = \frac{Gm_1}{r}$, $r_g = \frac{Gm_1}{c^2}$, $R = \frac{r}{r_g}$.

Then $K = \frac{1}{\sqrt{1 - \frac{2}{R}}} - \frac{1}{\sqrt{1 - \frac{1}{R}}}$ and implicitly

$$K^2(K^2 - 4)R^4 + 6K^2(3 - K^2)R^3 + (13K^4 - 26K^2 + 1)R^2 + 12K^2(1 - K^2)R + 4K^4 = 0 \quad (4)$$

or briefly $f_V(R, K) = 0$. We shall name this equation the vacuum equation.

The vacuum equation (4) determines the value of the coefficient K as a function of the homogeneous distance R (or the homogeneous coordinate relative to the introduced substitution).

To utilize the derived equation (4) and considering the hierarchy of the four interactions and their corresponding elementary particles, the solution assumes that equation (4) is a sum of partial functional dependencies on the parameters R, K, and L. We then define the equation of sums as $f_V(R, K) = f_F(R, K) + f_B(R, K, L) + f_G(R, K, L)$.

Furthermore, the partial sums of the functions also hold significance:

$f_{FB}(R, K, L) = f_F(R, K) + f_B(R, K, L)$, $f_{FG}(R, K, L) = f_F(R, K) + f_G(R, K, L)$, $f_{BG}(R, K, L) = f_B(R, K, L) + f_G(R, K, L)$.

The introduced parameter L determines the properties of the individual functional dependencies and their sums. A special value of the coupling parameter, K=2, which determines a resulting zero mass for the binding of two particles of equal mass, is designated as the annihilation parameter. For this parameter, the solutions of the individual functional dependencies determine numerical values that are considered to be the sides of regular four-dimensional polytopes. The ratios of the volumes of these selected polytopes then equal the mass ratios of elementary particles with high precision.

Solution of the Vacuum Equation

The vacuum equation (4) as a function:

$f_V(R, K) = K^2(K^2 - 4)R^4 + 6K^2(3 - K^2)R^3 + (13K^4 - 26K^2 + 1)$. $R^2 + +12K^2(1 - K^2)R + 4K^4 = 0$

For K=2, it satisfies a single value R_V=2.19496887746. In four-dimensional space, a 16-cell (orthoplex) has a volume of $V^{(16)} = \frac{1}{6}a^4$. The dual of this 16-cell is the 8-cell (tesseract) with a volume of $V^{(8)} = a^4$.

Then, the volume of the orthoplex determined by the value R_V is $V_{Rv}^{(16)} = \frac{1}{6}R_V^4$=3.868674726 and the volume of the tesseract is

$V_{Rv}^{(8)} = R_V^4$=23.21204836.

Solution of the Fermion Equation

The fermion equation is defined as:

$$f_F(R, K) = (13K^4 - 26K^2 + 1)R^2 - 12K^2(K^2 - 1)R + K^4 = 0 \qquad (5)$$

For K=2 it satisfies R_{F1}=1.249472327 and R_{F2}=0.1219562443.

The volume of the orthoplex determined by the number R_{F1} is denoted as:

$V_{F1}^{(16)} = \frac{R_{F1}^4}{6} = 0.406214402$.

The volume of the tesseract determined by the number R_{F2} is denoted as:

$V_{F2}^{(8)} = R_{F2}^4 = 0.000221215812$.

Three Solutions of the Boson Equation
The boson equation is defined as:

$$f_B(R,K,L) = K^2(18-L)R^3 - (13K^4 - 26K^2 + 1)R^2 - 12K^2(K^2-1)R - K^4 = 0 \quad (6)$$

The first solution for K=2 satisfies the values R_{B1}=-1.249450927771, R_{B2}=-0.121956287072672 and R_{B3}=94423.575, when L=L_p=17.999721993355355.

The volume of the orthoplex determined by the number R_{B1} is denoted as

$$V_{pB1}^{(16)} = \frac{R_{pB1}^4}{6} = 0.406186573.$$

The volume of the tesseract determined by the number R_{B2} is denoted as

$$V_{pB2}^{(8)} = R_{pB2}^4 = 0.000221216121.$$

The volume of the orthoplex determined by the number R_{B3} is denoted as

$$V_{pB3} = \frac{R_{pB3}^4}{6} = 1{,}324861586 \cdot 10^{19}.$$

The second solution for K=2 satisfies the values R_{B1}=-1.2498812721, R_{B2}=-0.121956287072672, and R_{B3}=-3738.29278447371 , when L=L_n =18.007019345924666.

The volume of the orthoplex determined by the number R_{B1} is denoted as:

$$V_{nB1}^{(16)} = \frac{R_{nB1}^4}{6} = 0.406746469.$$

The volume of the tesseract determined by the number R_{B2} is denoted as:

$$V_{nB2}^{(8)} = R_{nB2}^4 = 0.000221216121.$$

The volume of the orthoplex determined by the number R_{B3} is denoted as:

$$V_{nB3}^{(16)} = \frac{R_{nB3}^4}{6} = 3.254932595 \cdot 10^{13}.$$

Note: The difference $L_n - L_p$ = 0.007297352569310078 is equal to the fine-structure constant of electromagnetic interaction α = 0.0072973525693 = 1/137.035999084.

The third solution for K=2 satisfies the values R_{HB1}=-1.224673408801; R_{HB2}=-0.934505939909; R_{HB3}=4.245808351, when L_H=9.76815375457.

The volume of the orthoplex determined by the number R_{HB3} is denoted as:

$$V_{HB3}^{(16)} = \frac{R_{HB3}^4}{6} = 54{,}16145182.$$

Sum of the Fermion and Boson Equations

$$f_{FB}(R,K,L) = f_F(R,K) + f_B(R,K,L) = K^2(18-L)R^3 - 12K^2(K^2-1)R = 0 \quad (7)$$

For $K=2$, R_{FB1}=0 and $R_{FB2} = \sqrt{\frac{72}{18-L}} = 2{,}888300267,$ when L=L_{FB} = 9.369264334, then:

The volume of the tesseract determined by the number R_{FB1} is $V_{FB1}^{(8)} = 0.$

The volume of the tesseract determined by the number R_{FB2} is $V_{FB2}^{(8)} = 69{,}59360944.$

If L= L_H=9,76815375457, then $R_{FB3} = \sqrt{\frac{72}{18-L}} = 2{,}957451445.$

Sum of the Fermion and Gravitation Equations

$$f_{FG}(R,K,L) = f_F(R,K) + f_G(R,K,L)$$
$$K^2(K^2-4)R^4 + +K^2(L-6K^2)R^3 + 2(13K^4 - 26K^2 + 1)R^2 + 5K^4 = 0 \quad (8)$$

For K=2 and L=10.5625 the result is R_{FG1}=4. Then, the volume of the orthoplex is: $V_{FG1}^{(16)} = \frac{R_{FG1}^4}{6}$=42.66666667

Sum of the Boson and Gravitation Equations

$$f_{BG}(R,K) = f_F(R,K,L) + f_G(R,K,L) = \mathrm{K}^2(\mathrm{K}^2-4)\mathrm{R}^4 + \mathrm{K}^2(18-6\mathrm{K}^2)\mathrm{R}^3 + 3\mathrm{K}^4 = 0 \quad (9)$$

For K=2 the result will be $24R^3-48=0$ and therefore $R_{BG1} = \sqrt[3]{2}$. Then, the volume of the tesseract is: $V_{BG1}^{(8)} = R_{BG1}^4 = 2.5198421.$

Solution of the Gravitation Equation

$$f_G(R,K,L) = K^2(K^2-4)R^4 + K^2(L-6K^2)R^3 + (13K^4 - 26K^2 + 1)R^2 + 12K^2(K^2-1)R + 4K^4 = 0 \quad (10)$$

Solutions of the gravitation equation (10) for K=2 and the values of L mentioned in the previous sections 2.3 to 2.5:

For L=0 R_{G1}=2.0067660007923693

Then the volume of the tesseract is $V_{G1}^{(8)} = R_{G1}^4 = 16.21741944$ and the volume of the orthoplex is $V_{G1}^{(16)} = \frac{R_{HB3}^4}{6} = 2.70290324.$

For L=L_{FB}=9,369264334 is L_{G2}=2.808804821952889

Then the volume of the tesseract is $V_{G2}^{(8)} = R_{G2}^4 = 62.24238814$ and the volume of the orthoplex is $V_{G2}^{(16)} = \frac{R_{G2}^4}{6} = 10.37373136.$

For L= L_H=9,76815375457 is R_{G3}=2.86452300307827445.

Then the volume of the tesseract is $V_{G3}^{(8)} = R_{G3}^4 = 67.33009953$

and the volume of the orthoplex is $V_{G3}^{(16)} = \frac{R_{G3}^4}{6} = 11.22168326.$

For L= L_p =17,999721993355355 is R_{G4}=5.54380005535420219

Then the volume of the tesseract is $V_{G4}^{(8)} = R_{G4}^4 = 944.5614866$

and the volume of the orthoplex is $V_{G4}^{(16)} = \frac{R_{G4}^4}{6} = 157.4269144.$

For L= L_p =18,007019345924666 is R_{G5}=5.5493036157352756

Then the volume of the tesseract is $V_{G5}^{(8)} = R_{G5}^4 = 948.317897$ and the volume of the orthoplex is $V_{G5}^{(16)} = \frac{R_{G5}^4}{6} = 158.0529828.$

For L= L_p+6= L_{Pl} =23.999721993355341 is R_{G6}=94423.557

Then the volume of the tesseract is $V_{G6}^{(8)} = R_{G6}^4 = 7.949163437 \cdot 10^{19}$

and the volume of the orthoplex is $V_{G6}^{(16)} = \frac{R_{G6}^4}{6} = 1.324860573 \cdot 10^{19}.$

Rest Masses of Elementary Particles
Rest masses of elementary particles are considered independent parameters of the Standard Model, and their values are determined through measurement. Likewise, the mass ratios of these particles expand the number of free parameters of physical reality. To determine these ratios, it is best to use the mass of the electron as the fundamental parameter, as it has been established by

measurement with a precision of nine decimal places.

Rest Masses of Elementary Particles [9]

m_e=0.51099895000±0.00000000015 MeV … electron mass.
m_e=938.27208816±0.00000029 MeV … electron mass.
m_e=939.5654205±0.0000005 MeV … neutron mass.
m_W=80369.2±13.3 MeV … mass of the intermediate boson W±.
m_Z=97188.0±0,0020 MeV … mass of the intermediate boson Z0.
m_H=125200±110 MeV … Higgs boson.
m_f=0 … photon mass.

$M_P = \sqrt{\frac{hc}{G}}$=3.06031777 10^{22}MeV … Planck mass, unreduced.

Mass Ratios of Particles to the Electron Mass

$\frac{m_p}{m_e} = 1836.152673$; $\frac{m_n}{m_e}$ =1838.683662; $\frac{m_w}{m_e} = 157278.60$

$\frac{m_z}{m_e} = 190192.1716$; $\frac{m_H}{m_e} = 245010.2882$; $\frac{m_f}{m_e} = 0$; $\frac{M_P}{m_e} = 5.988892482 \cdot 10^{22}$

Correlation of Volume Ratios of Regular Four-Dimensional Polytopes with Particle Mass Ratios

The solutions to the equations provided above determine numerical values that can be considered the sides of regular polytopes in four-dimensional space. The volume ratios of these four-dimensional bodies coincide, with precision limited only by computational constraints, with selected particle mass ratios.

$\frac{V_{Rv}^{(16)}}{V_{F2}^{(8)}} = 17488.21337 = \frac{m_{de}}{m_e}$; $m_{de} = 8936.458672$ MeV, particle component of dark energy.

$V_{F1}^{(16)}=\frac{R_{F1}^4}{6} = 0.406214402$; $V_{F2}^{(8)} = R_{F2}^4$ =0.000221215812;

$\frac{V_{F1}^{(16)}}{V_{F2}^{(8)}} = 1836.281043$ that is 100.007% of $\frac{m_p}{m_e} = 1836.152673$

$V_{pB1}^{(16)}=\frac{R_{pB1}^4}{6} = 0.406186573$; $V_{pB2}^{(8)}=R_{pB2}^4$ =0.000221216121; $V_{pB3}^{(16)}$ =$\frac{R_{pB3}^4}{6}$ 1,324861586 $\cdot 10^{19}$

$\frac{V_{pB1}^{(16)}}{V_{pB2}^{(8)}} = 1836.152678$ that is 100.0000003% z $\frac{m_p}{m_e} = 1836.152673$

$\frac{V_{pB3}^{(16)}}{V_{pB2}^{(8)}} = 5.988992019 \cdot 10^{22}$ that is 100.002% of $\frac{M_P}{m_e} = 5.988892484 \cdot 10^{22}$; $M_P = \sqrt{\frac{hc}{G}}$

$V_{nB1}^{(16)}=\frac{R_{nB1}^4}{6} = 0.406746469$; $V_{nB2}^{(8)}=R_{nB2}^4$=0.000221216121; $V_{nB3}^{(16)}=\frac{R_{nB3}^4}{6} = 3.254932595 \cdot 10^{13}$

$\frac{V_{nB1}^{(16)}}{V_{nB2}^{(8)}} = 1838.683669$ that is 100.0000004% of $\frac{m_n}{m_e}$ =1838.683662

$\frac{V_{nB3}^{(16)}}{V_{nB2}^{(8)}} = 1.471381281 \cdot 10^{17}$; $\frac{m_M}{m_e} = 1.471381281 \cdot 10^{17}$

Hypothetical particle "magnetic monopole" $m_M = 7.518742896 \cdot 10^{16}$ MeV.

$V_{HB3}^{(16)}=\frac{R_{HB3}^4}{6} = 54,16145182$

$\frac{V_{HB3}^{(16)}}{V_{pB2}^{(8)}} = 244830.012$; that is 99.93% of $\frac{m_H}{m_e} = 245010.2882$

$\frac{V_{FB1}^{(8)}}{V_{pB2}^{(8)}} = 0$ that is in agreement with $\frac{m_f}{m_e} = 0$

$\frac{V_{FB2}^{(8)}}{V_{pB2}^{(8)}} = 314595.5599$ that is 100.012% of $\frac{2m_W}{m_e} = 314557.2021$

$\frac{V_{FG1}^{(5)}}{V_{pB2}^{(8)}} = 192873.2249$ that is 101.407% of $\frac{m_Z}{m_e}$=190192.1716

$\frac{V_{BG1}^{(8)}}{V_{pB2}^{(8)}}$=11390.86107=$\frac{m_{dm,}}{m_e}$, m_{dm}=5820.718045 MeV Hypothetical "dark matter" particle.

$\frac{V_{G1}^{(16)}}{V_{pB2}^{(8)}} = 12218.38276 = \frac{m_{x3}}{m_e}$, m_{x3}=6243.580761 MeV

$\frac{V_{G2}^{(16)}}{V_{pB2}^{(8)}} = 46894.10208 = \frac{m_{x5}}{m_e}$, m_{x5}=23962.83692 MeV

$\frac{V_{G3}^{(16)}}{V_{pB2}^{(8)}} = 50727.23999 = \frac{m_{x6}}{m_e}$, m_{x6}=25921.56637 MeV

$\frac{V_{G4}^{(16)}}{V_{pB2}^{(8)}} = 711643.047 = \frac{m_{x8}}{m_e}$, m_{x8}=363648.8498 MeV

$\frac{V_{G5}^{(16)}}{V_{pB2}^{(8)}} = 714473.1681 = \frac{m_{x10}}{m_e}$, m_{x10}=365095.0387 MeV

$\frac{V_{G6}^{(16)}}{V_{pB2}^{(8)}} = 5.988987453 \cdot 10^{22}$ that is 100.002% of $\frac{M_P}{m_e} = 5.988892484 \cdot 10^{22}$; $M_P = \sqrt{\frac{hc}{G}}$

Solution of the Gravitation Equation for 0≤K≤1

$$f_G(R,K,L) = K^2(K^2-4)R^4 + K^2(L-6K^2)R^3 + (13K^4 - 26K^2 + 1)R^2 + 12K^2(K^2-1)R + 4K^4 = 0$$

For L=L_H=9.76815375457 and R=2, when R=r/r_g, then r=2r_g.

For r determined by the Schwarzschild radius and the parameter L_H, the equation is satisfied by K_1 = 0.188861988988. This constant K_1 is also assigned the value R_2 = 0.0123639616308 by the equation, which determines K_2 = 0.032134464594 using the same equation.

The constants K_1 and K_2 determine the resulting mass of a black hole after the gravitational collapse of two such objects.

This involves a superposition of two possible states. Using the constants K_1 and K_2 and the mixing angle ω, the mixing parameter C_s of two possible gravitationally bound states of massive objects m_1 and m_2 can be defined.

$$C_s = 1 - \frac{K_1 + K_2}{2}\sqrt{1 + 2 sin\omega cos\omega}$$

Under the condition $m_1 \geq m_2$, we obtain for the final mass of the black hole M_{BH}= m_1+C_s m_2 and also for the total mass of the radiated gravitational waves M_{gw}= m_1+m_2- M_{BH}.

The values determined by this procedure for selected black hole objects are arranged in Tab.1.

Table 1

Object [8]	m1	m2	ω	Cs	MBH	Mgw	MBH[8]	Mgw[8]
GW150914	35,6	30,6	101,90	0,8987	63,0997	3,1003	63,1	3,1
GW151012	23,3	13,6	98,54	0,8897	34,4006	1,4994	35,7	1,5
GW151226	13,7	7,7	90,60	0,8702	20,4005	0,9995	20,5	1
GW170104	31	20,1	98,85	0,8906	48,9002	2,1998	49,1	2,2
GW170608	10,9	7,6	95,35	0,8816	17,6003	0,8997	17,8	0,9
GW170729	50,6	34,3	86,05	0,8601	80,101	4,799	80,3	4,8
GW170809	35,2	23,8	97,30	0,8865	56,2998	2,7002	56,4	2,7
GW170814	30,7	25,3	99,90	0,8933	53,301	2,699	53,4	2,7
GW170817	1,46	1,31	125,53	0,9695	2,73001	0,04	2,73	0,04
GW170818	35,5	26,8	102,1	0,8992	59,5993	2,7007	59,7	2,7
GW170823	39,6	29,4	97,75	0,8877	65,6985	3,3015	65,6	3,3

Velocity Profile of the Gravitational Field Determined by Equations (3) and (10)

By solving the gravitational equation (10), it is possible to determine the coefficient K which characterizes the intensity of the gravitational bond, or the mass deficit of the gravitationally bound state of two objects. This coefficient depends on the distance given by the homogeneous coordinate R=r/r_g.

For this purpose, a rewrite is necessary where the sought variable dependent on R is K. Due to the quadratic nature of the dependency, the equation will have two solutions. These two solutions serve to establish the circular velocities for R using the velocity equation (3) rewritten for the variable q dependent on the parameter K determined by equation (10).

Values of Parameters *K* and *q* as a function of *R* for various values of *L* shown Graphically
Equation (10) rewritten with respect to the dependence of *K* on *R* has the form

$$K^4(R^4\text{-}6R^3\text{+}13R^2\text{+}12R\text{-}4)\text{-}\ K^2(4R^4-LR^3+26R^2+12R)+R^2=0 \tag{11}$$

Similarly, equation (3) rewritten with respect to the dependence of *q* on *K* has the form

$$4K^4q^4+12K^2(1-K^2)q^3+(13K^4-26K^2+1)q^2+6K^2(3-K^2)q+K^2(K^2-4)=0 \tag{12}$$

For $R\in$<0,10>, with the introduced substitution $R=r/r_g$, there are two solutions to the quadratic equation (11).

The fourth-degree equation (12) determines, according to the previously introduced substitution $q=v^2/c^2$ the ratio of the circular velocity to the speed of light. The selected interval of *R* values covers the inner part of the black hole as well as the outer parts up to a distance of ten times the gravitational radius.

The following graphs display the values K_1 and K_2, for the two solutions of equation (11), along with their assigned ratio *v*/*c*. These values are either real numbers or the real component of a complex number is shown. If a value of *R* in the graph is not assigned a point on the curve, it indicates a purely imaginary value.

Equation (12) is a fourth-degree equation; therefore, four solutions exist, two of which are purely complex over the entire domain of *R*. For the graphs, solutions were selected that are real in at least a part of the domain *R*. The range of numerical values for *R* can be extended to a scale corresponding to the distances of planets from a central star, or stars from the center of a galaxy. In these cases, the ratio *v*/*c* is identical to the ratio determined according to Newton's law of universal gravitation.

The solution of the equations and the representation of the graphs are provided for eight selected special values of *L* within the range of 0 to 24.

The graph for *L*=0 corresponds to a hypothetical object that can be considered a false vacuum filled only with energy. The graph for *L*=17.999… and 18.007… respectively corresponds to a standard black hole composed of fermionic matter. The graph for *L*=23.999… then corresponds to a hypothetical object formed by the Planck mass. For other values of *L*,these are objects of a bosonic nature.

Standard Universe
Current ideas regarding the material or energetic components of the universe lean toward a percentage distribution of approximately 5% baryonic matter, 30% dark matter, and 65% dark energy. According to the discovered values of the volume ratios of selected regular polytopes, which have counterparts in the elementary particle ratios of the real world, it is possible to determine the particles whose masses represent these components of the universe.

Baryonic Matter
To determine the share of baryonic matter in the total density of matter and energy within the observed limits of the universe's expanse, the mass of the neutron m_n=939.5654205 MeV is sufficient.

Dark Matter
To determine the share of dark matter, we use the value mentioned above, m_{dm}=5820.718045 MeV.

Dark Energy
If we use the above value m_{de}=8936.458672 MeV to determine the share of dark energy, assuming the same numerical density for these three particles, we obtain a discrepancy between the percentage shares of the individual components. If 939.5654205+5820.718045+8936.458682=15696.74192 represents 100% of the mass (energy) content of the vacuum, then the individual components represent 5.97%, 37,08% and 56.93%. This apparent discrepancy with values established by observation can be resolved if we assume that dark energy particles are carriers of an anti-gravitational effect. The particles m_{de} acquire this property if a pair of particles is in a mutual gravitational bond according to equation (1), where I_g is determined from the established relationship $K=I_G\frac{m_1}{m_1+m_2}$.

In this case, for the value of *K* we use the determined maximum of 3.48801 according to the graph K_1 at the value *L*=0. Then for $m_1=m_2=m_{de}$=8936.458672, the result will be $m=m_1+m_2\text{-K}\,m_2$=-13297.5 MeV. The resulting negative mass represents an anti-gravitational [9] effect in relation to baryonic and dark matter.

If we consider the sum of the absolute values 939.5654205 + 5820.718045 + 13297.5 = 20057.783 to be 100%, then the individual components in this case represent 4.68%, 29.02%, and 66.29%, in agreement with the values determined from observational data.

Black Hole Evaporation

In the work of Anshul Saini and Dejan Stojkovic [10], *Radiation from a Collapsing Object is Manifestly Unitary*, it is admitted that during the "evaporation" of black holes, massive particles are emitted in addition to electromagnetic quanta. In connection with the solution presented here for the parameter L and the corresponding K_1 determined by equation (11), it is possible to determine the kinematics for given masses of particles in a particle-antiparticle pair. This includes their maximum velocity of motion during their spontaneous creation at a given distance $R=r/rg$, where K_1 reaches its maximum value. This therefore concerns the evaporation of particles with rest mass. The probability of this process is not discussed in this article; however, it is non-zero.

Electron-Positron Pair Creation

$m_g = m_1 + m_2 - I_g\frac{m_1m_2}{m_1+m_2}$, as stated above, for $m_1 >>> m_2$, in the case of a black hole versus an elementary particle will be

$$m_g = m_1 + m_2 - I_g\frac{m_1m_2}{m_1+m_2} = m_1 + m_2 - Km_2, \text{ because } K = I_g\frac{m_1}{m_1+m_2}, \text{ therefore } K = I_g \text{ because } \frac{m_1}{m_1+m_2} = 1..$$

During the spontaneous creation of a particle-antiparticle pair, the mass of the black hole will be reduced by the mass of the particle, including the mass equivalent to its kinetic energy. Under this assumption, the velocity of the emitted particle can be determined; if this velocity is higher than the escape velocity for the given distance, it becomes part of the cosmic radiation.

The mass balance is determined by the creation equation:

$$m_{BH2} = m_{BH1} + m_2 - K_1m_2 \qquad (13)$$

where m_{BH2} is the resulting mass of the secondary black hole after the act of creation, mBH1 is the primary mass of the black hole before creation, m_2 is the radiated particle, and K_1 is the gravitational coupling coefficient.

In the case of the creation of an electron (positron) at $K_1=2.1379$, then $m_{BH2} = m_{BH1} + m_e - 2m_e - 0.1379m_e = m_{BH1} - (1 + 0.1379)m_e$

and the mass deficit of the secondary black hole is $(1 + 0.1379)m_e = \frac{m_e}{\sqrt{1-\frac{v^2}{c^2}}}$. From this, $1.1379 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, then: $\frac{v}{c} = \sqrt{1-\frac{1}{1.1379^2}} = 0.477968$.

During the spontaneous creation of an electron (positron), its velocity relative to the speed of light will be 0.477968 at the expense of the black hole's mass loss.

Creation of Baryonic Matter, Dark Matter, and Dark Energy

In the context of the above description of the creation of massive particles, it is also possible to describe the simultaneous creation of a triplet of particles representing the three components of the universe. If we denote $m_1 = m_{DE}$, $m_2 = m_{DM}$ and $m_3 = m_{BM}$ then at the distance $R = r/r_g$, where the value of parameter K_1 reaches its maximum of 2.1379, the simultaneous creation of these three particles occurs.

In this process, the particle m_{DE} remains at the center of gravity of the departing particles m_{DM} and m_{BM}. Each of this pair of particles carries away one half of the momentum, which is the complement to the total mass loss of the secondary black hole. This process can be expressed by the equation:

$$\begin{aligned} m_{BH2} &= m_{BH1} - m_{DE} + m_{DM} + m_{BM} - 2m_{DM} - 0.1379m_{DM} - 2m_{BM} - 0.1379m_{BM} = \\ &m_{BH1} - m_{DE} - (1 + 0.1379)m_{DM} - (1 + 0.1379)m_{BM} \end{aligned} \qquad (14)$$

In order to preserve the total energy of the emitted particles during the process of spontaneous creation, it is desirable that:

$$m_{DE} - (1 + 0.1379)m_{DM} - (1 + 0.1379)m_{BM} = 0 \qquad (15)$$

Assuming that m_{DE} is located at the center of gravity of the three-particle decay, equation (15) will be satisfied if each of the particles m_{DM} and m_{BM} carries one half of the total momentum. By substituting the rest masses into equation (15), we obtain a difference of 7692.525571 MeV. Half of this value then represents the momentum of each of the particles m_{DM} and m_{BM}. Relativistic velocities, or the ratio of the particle velocity to the speed of light, correspond to these momenta. This ratio is obtained from the equation:

$\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} - m_0 = \frac{7692.525571}{2}$ when we sequentially substitute m_{DM} and m_{BM} for m_0. For m_{DM}, the ratio is v/c = 0.798403, and for m_{BM}, it is v/c = 0.980535. These values are related to the distance $4.9R$, where K_1 reaches its maximum. In such a region, at five times the gravitational radius of a galactic black hole, the creation of baryons occurs. Their interaction with the surrounding environment creates collimated jets oriented, for example, along the rotation axis of the black hole.

The examples of particle creation in the gravitational vicinity of a black hole provided above were selected for objects defined by the parameter $L = 18.00...$. Similarly, the v/c ratio can also be obtained for the other mentioned values of L.

It is possible to propose the hypothesis that a similar line of reasoning can lead to understanding the essence of the formation of the solar corona. This would involve using the mass of the Sun and determining the parameter K_1 for the distance $R=r_z/r_{g\odot}$ at the value $L = 18$, which corresponds to the fermionic nature of the object.

Velocity Profiles

The appendix contains a set of graphs that assign the parameters K_1 and K_2 and their corresponding velocity ratios v/c as a function of the distance from the gravitational center given by the ratio $R=r/r_g$. The set of graphs in the supplement is structured according to the parameter L which distinguishes the solutions of the equations based on the bosonic or fermionic nature of the particles.

It is assumed that the solution for the parameter K_1 determines the velocities relative to the non-rotating object, namely the velocity v_3/c of the falling observer and the velocity v_4/c of the distant observer. While the solution for the parameter K_2 determines the velocities relative to the rotating object, namely the velocity v_2/c of the falling observer and the velocity v_4/c of the distant observer. In this case, the merger of the objects is accompanied by the emission of gravitational waves. In the range of values of R from zero to ten times the gravitational radius, the mentioned velocities are in the graphs.

Summary

Based on the solution of the equations, a very precise correlation is found between the mass ratios of the elementary particles of the Standard Model and the volume ratios of selected regular four-dimensional polytopes, whose sides are expressed by the ratio r/rg. This correlation allows for the determination of further ratios related to entities possessing the properties of dark matter and dark energy, respectively.

In connection with the description of the gravitational field of macro-objects such as black holes using a velocity profile both outside and inside the black hole, agreement with Newtonian gravity is achieved at large distances. Simultaneously, at distances in the immediate vicinity and also inside black holes, it is found that velocities are lower than the speed of light, and related quantities do not reach infinite values, as inferred by the General Theory of Relativity in the so-called singularity.

The above is achieved based on a single solution with a characteristic quantity K_2 determining the magnitude of the gravitational coupling between the participating objects. An alternative solution with the quantity K_1 then determines the conditions for the mass loss of gravitational objects through spontaneous creation—not only of electromagnetic field quanta but also the creation of elementary particles with rest mass.

This process is also applicable to the simultaneous creation of a baryonic matter particle, a dark matter particle, and a dark energy particle. Furthermore, the dark energy particle possesses the property of anti-gravity, as it has the nature of negative mass. Assuming the same numerical density of these three types of particles, agreement is reached regarding the percentage representation of matter and energy in the universe. Spontaneous particle creation can be considered a manifestation of quantum gravity, as permitted by the solution described in work [10].

Through a special solution of the relevant equations concerning black holes, it is possible to determine the resulting mass of an object following the collapse of two bodies and to calculate the equivalent mass of the produced gravitational waves. The determining factor for establishing the final mass is the superposition of two states: one corresponds to the event horizon of the Schwarzschild radius, and the other to a radius smaller than the gravitational radius itself. The participation of two states in determining the resulting mass is equivalent to the superposition of states according to quantum mechanics.

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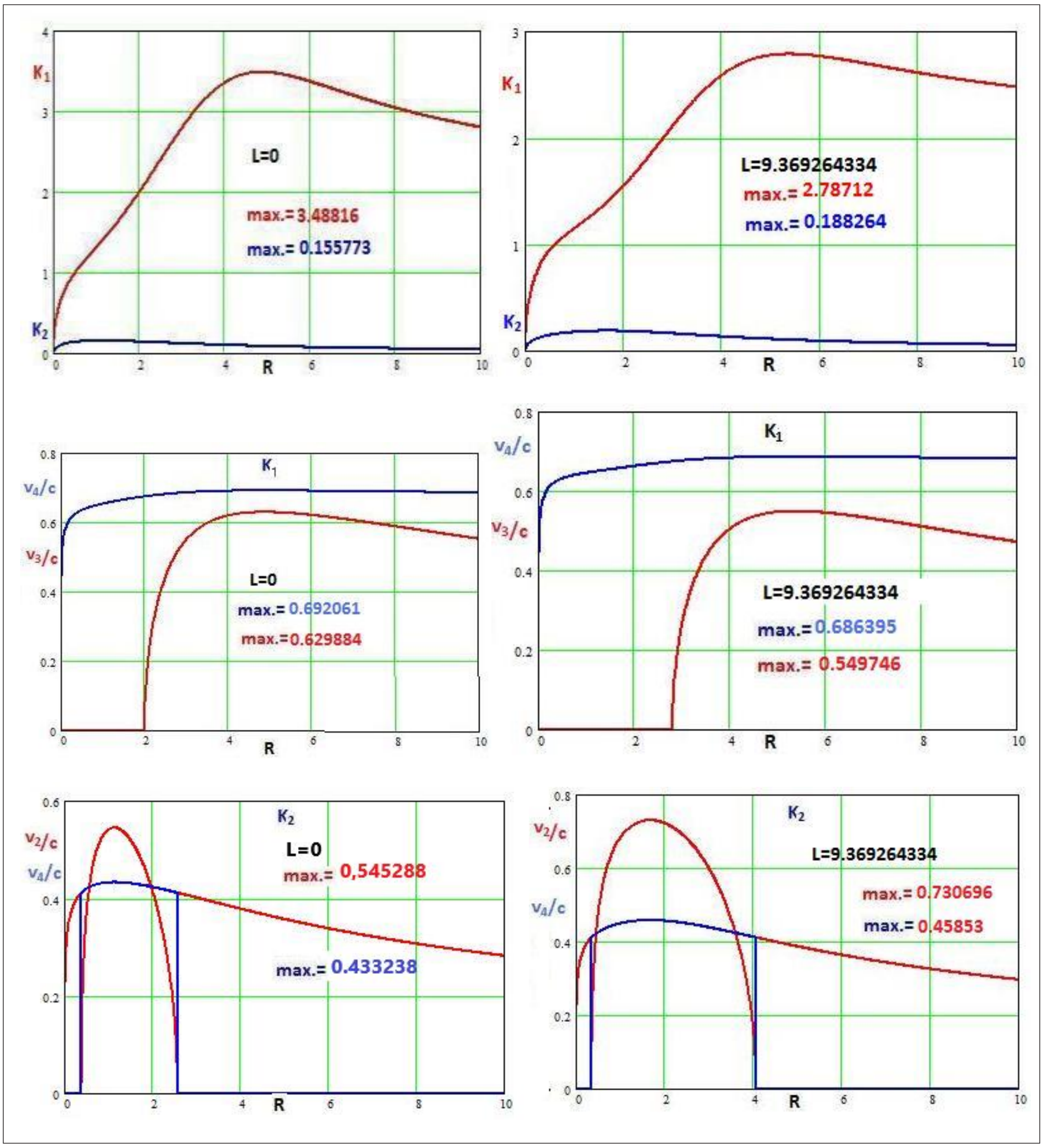
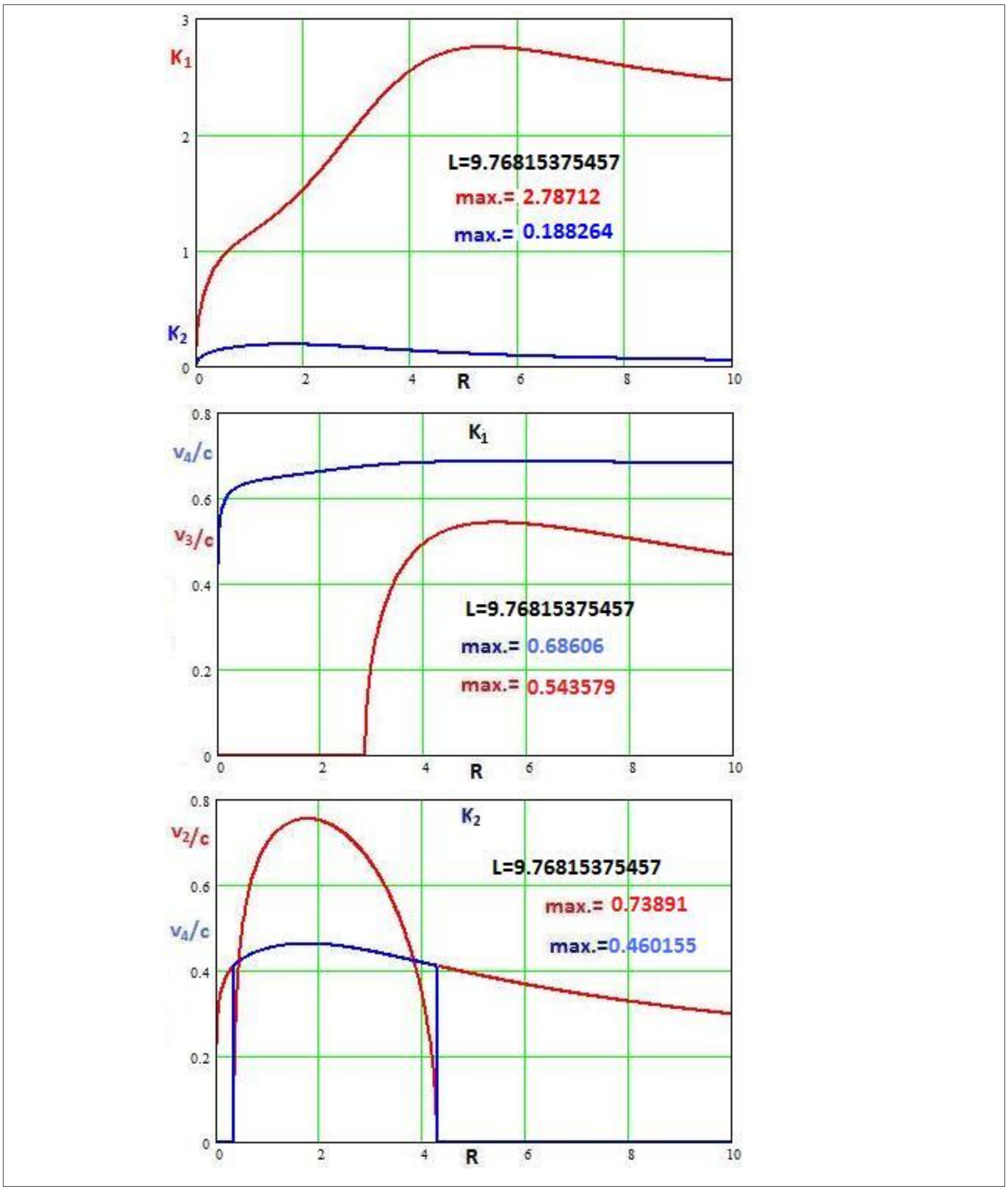
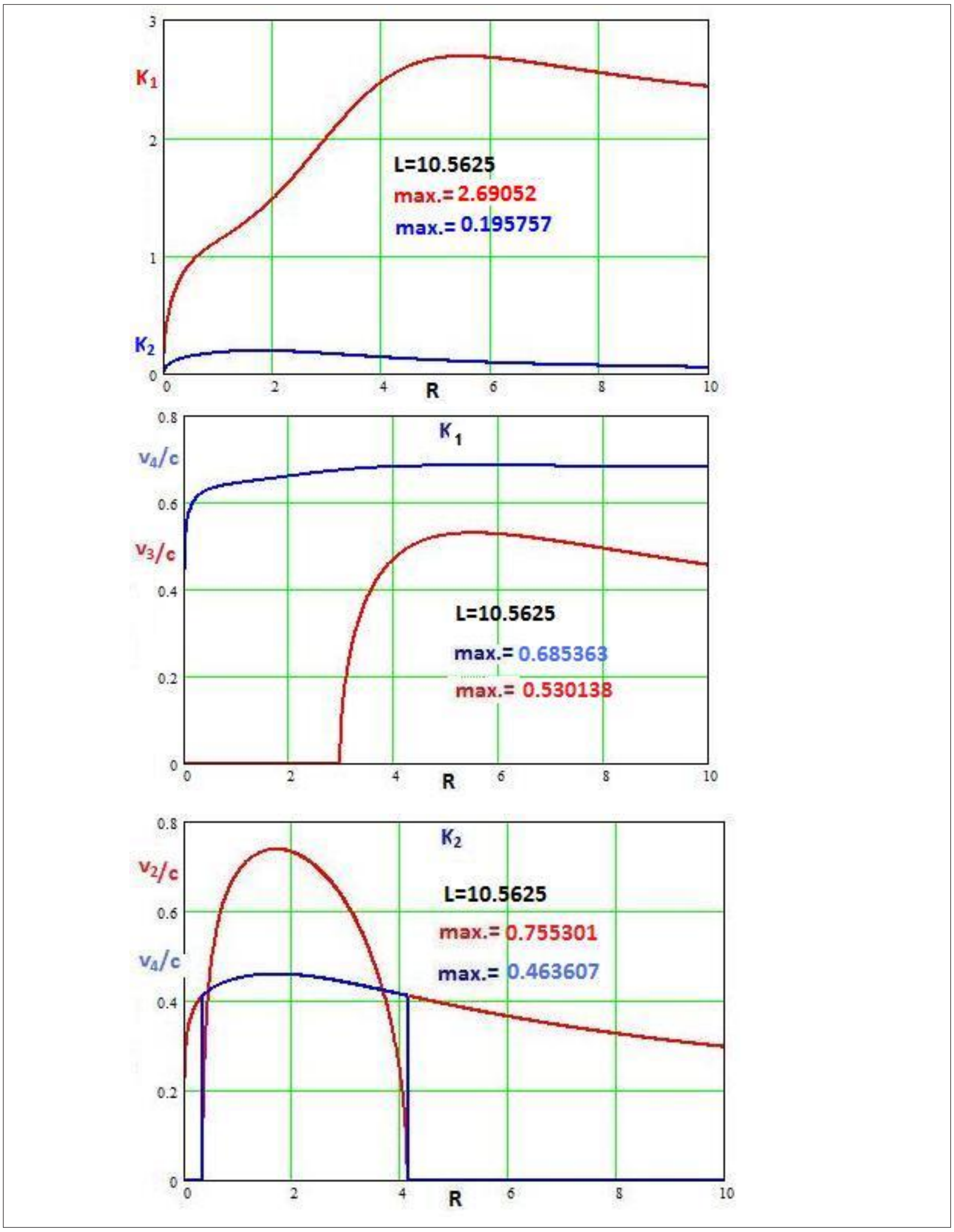
Appendix

A collection of Graphs of *K* Parameters and Velocity Ratios *v*/*c* for Selected *L* Values




K1
K2
L=9.76815375457
max.= 2.78712
max.= 0.188264
R
v4/c
v3/c
K1
L=9.76815375457
max.= 0.68606
max.= 0.543579
R
v2/c
v4/c
K2
L=9.76815375457
max.= 0.73891
max.=0.460155
R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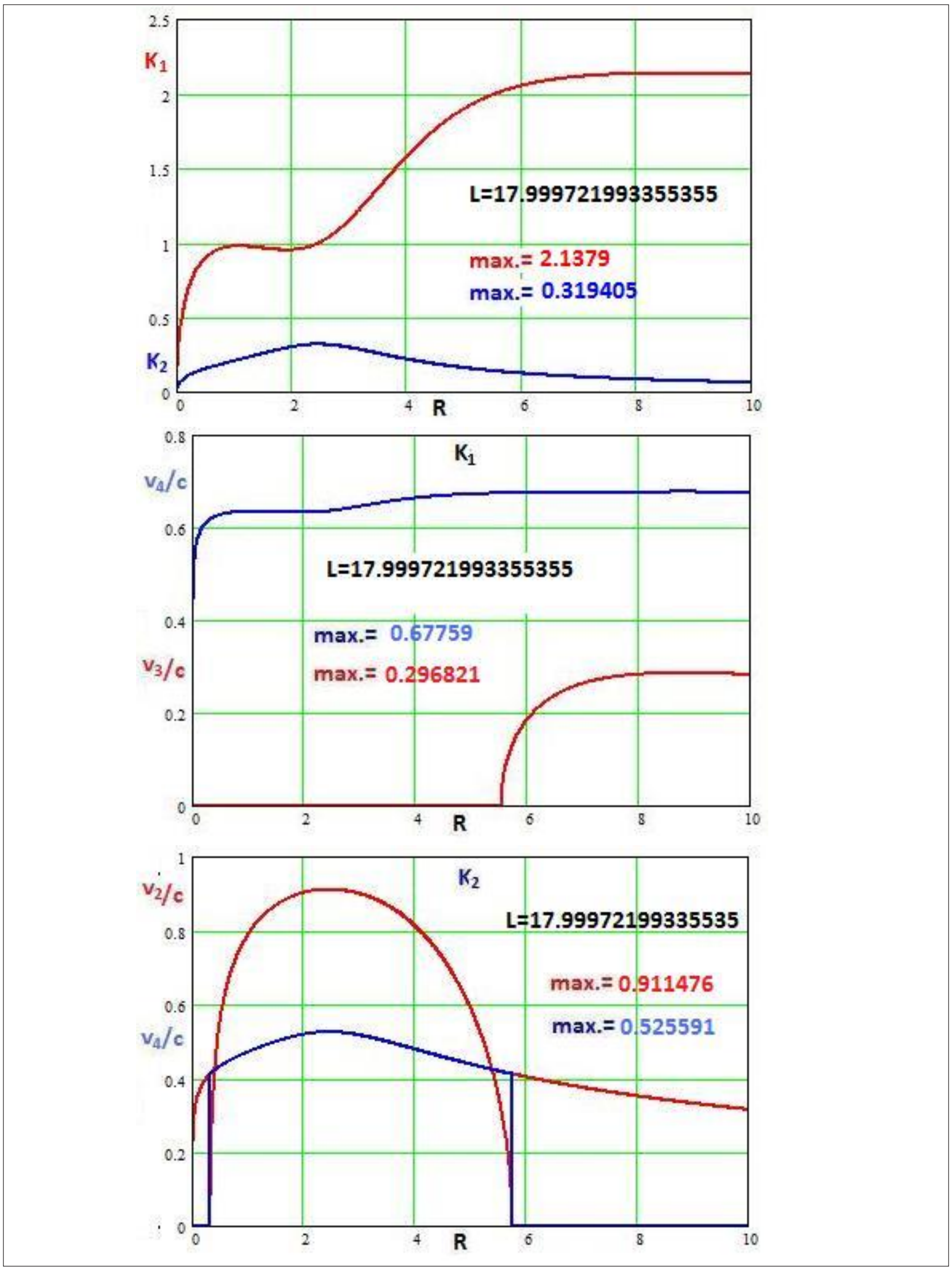
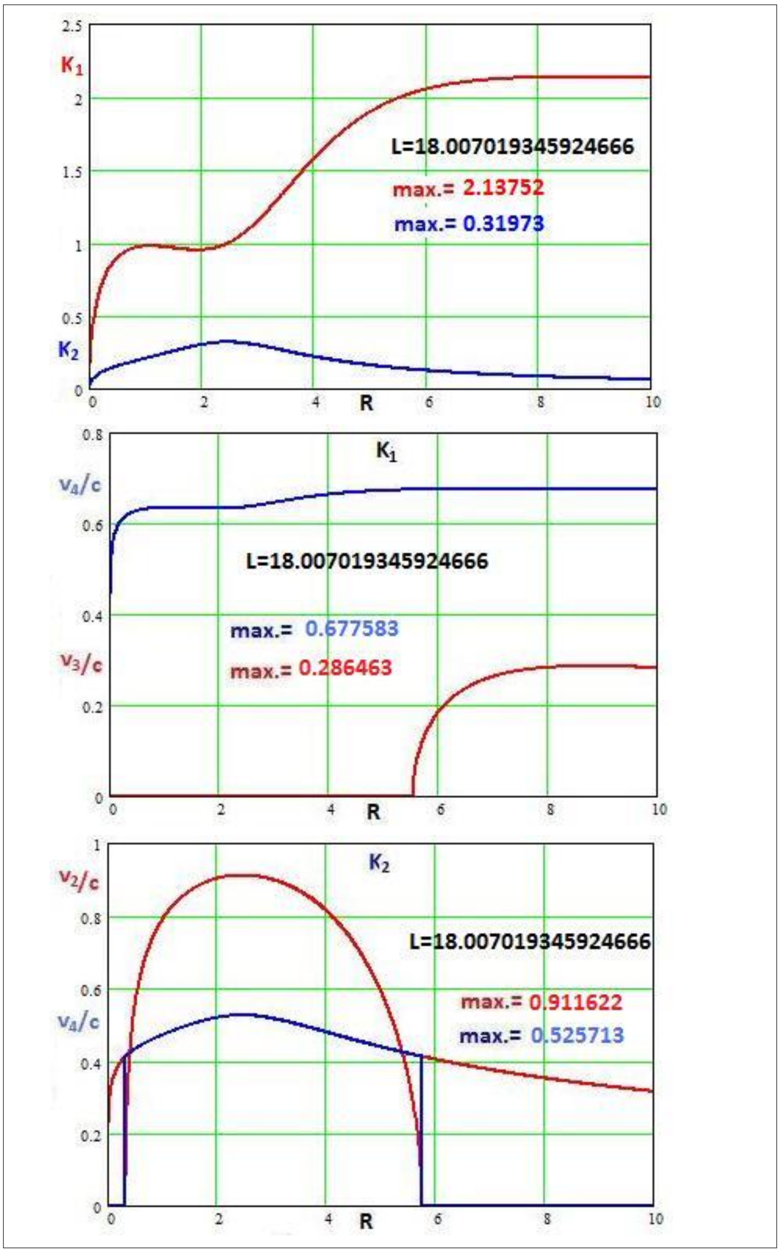
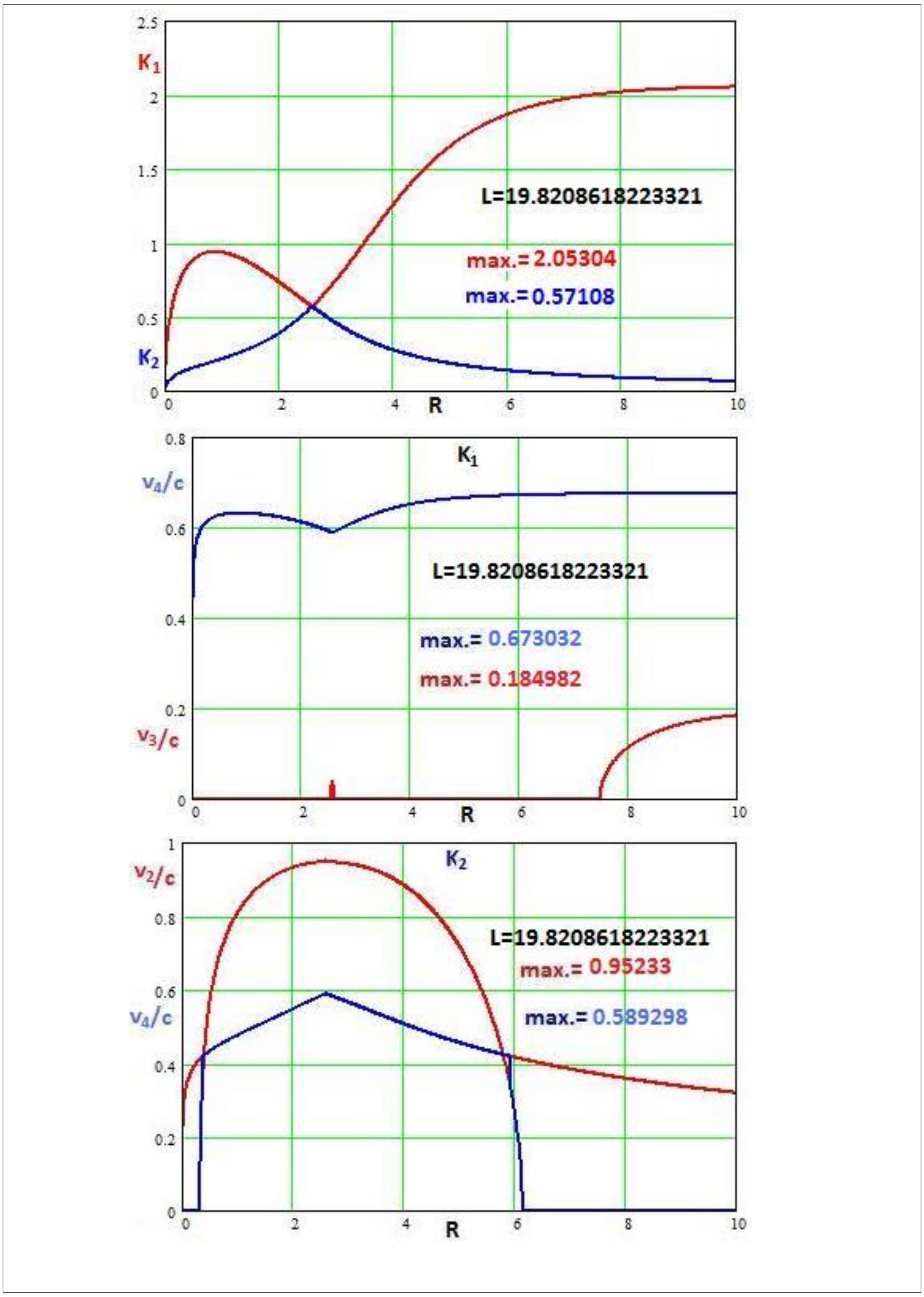
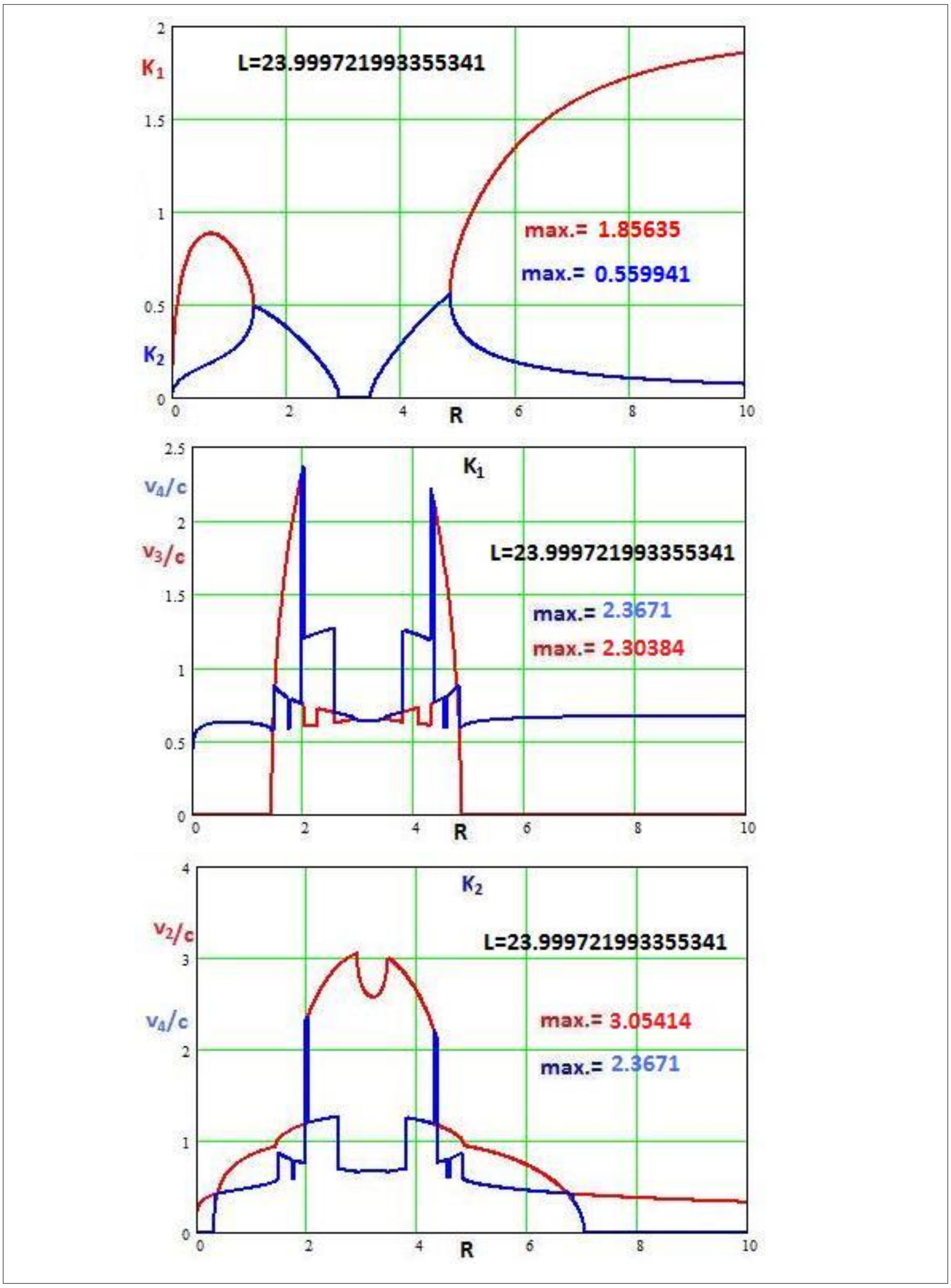
K2
L=10.5625
max.= 2.69052
max.= 0.195757
R
v4/c
v3/c
K1
L=10.5625
max.= 0.685363
max.= 0.530138
R
v2/c
v4/c
K2
L=10.5625
max.= 0.755301
max.= 0.463607
R


K1
K2
L=17.999721993355355
max.= 2.1379
max.= 0.319405
R
v4/c
v3/c
K1
L=17.999721993355355
max.= 0.67759
max.= 0.296821
R
v2/c
v4/c
K2
L=17.99972199335535
max.= 0.911476
max.= 0.525591
R


K1
K2
L=18.007019345924666
max.= 2.13752
max.= 0.31973
R
K1
v4/c
v3/c
L=18.007019345924666
max.= 0.677583
max.= 0.286463
R
K2
v2/c
v4/c
L=18.007019345924666
max.= 0.911622
max.= 0.525713
R



K_1
K_2
R
L=19.8208618223321
max.= 2.05304
max.= 0.57108
K_1
v_4/c
v_3/c
L=19.8208618223321
max.= 0.673032
max.= 0.184982
K_2
v_2/c
v_4/c
L=19.8208618223321
max.= 0.95233
max.= 0.589298



K1
K2
L=23.999721993355341
max.= 1.85635
max.= 0.559941
R
v4/c
v3/c
K1
L=23.999721993355341
max.= 2.3671
max.= 2.30384
R
K2
v2/c
v4/c
L=23.999721993355341
max.= 3.05414
max.= 2.3671
R

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