

Mathematical Models for Epidemiology: Prevention and Control of Infectious Diseases

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Introduction

Infectious diseases remain a major global health challenge, especially in developing countries with limited healthcare resources. Malaria, transmitted by mosquitoes, continues to cause significant harm in tropical and subtropical regions like Mozambique.

To design effective prevention and control strategies, it is essential to understand how diseases spread. Mathematical modeling provides a powerful approach to simulate disease dynamics and evaluate intervention outcomes.

This study develops and analyzes functional differential equation (FDE) models for malaria transmission. These models incorporate key epidemiological parameters and aim to support public health decision-making through simulations and parameter sensitivity analysis.

General Objective

To develop and analyze functional differential equation models to study the dynamics of malaria transmission.

Specific Objectives

- Identify and analyze the parameters that contribute to the transmission of malaria.
- Develop an FDE model to study the dynamics of malaria transmission.
- Adjust the model to reflect variations in parameters affecting malaria transmission.
- Use the models to evaluate strategies for the prevention and control of infectious diseases.

Methodology

This work was conducted through bibliographic research, which consists of a review of the literature related to the addressed topic. Articles, theses, dissertations, and other scientific publications relevant to the study were utilized.

Malaria

- Malaria is an infectious disease transmitted by mosquitoes to humans and other animals, caused by parasites (a type of microorganism) of the genus *Plasmodium*.
- The parasites ingested by the mosquito develop in its gut

for more than 10 days, until they mature and move to the mosquito's salivary glands.

Basic Concepts

- Incidence is the rate of new cases of a disease in a population during a specific period.
- Prevalence is the rate of the total number of recorded cases of a disease during a certain period.
- Epidemic curve is a graph that shows the number of disease cases over time.
- Epidemic refers to a sudden increase in the number of cases of a disease within a population.
- Endemic is a disease that is constantly present in a population or specific geographic area.
- Pandemic is an epidemic that spreads across multiple geographic regions.

SIR Model

$$\begin{cases} \frac{dS}{dt} = -\beta SI \\ \frac{dI}{dt} = \beta SI - \alpha I \\ \frac{dR}{dt} = \alpha I \end{cases}$$

$S(t)$: Number of susceptible individuals at time t

$I(t)$: Number of infected individuals at time t

$R(t)$: Number of recovered individuals at time t

β : Transmission rate (contact rate)

α : Recovery rate (inverse of average infection duration)

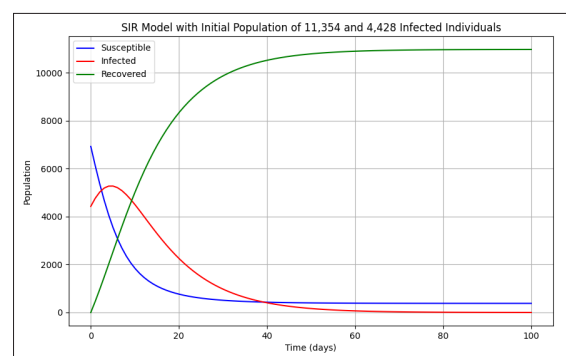


Figure 1: Dynamics of Malaria Transmission

SEIR Model

- The SEIR model extends the classical SIR framework by introducing an additional compartment for exposed individuals.
- Those who have been infected but are not yet capable of transmitting the disease.
- This model is particularly suitable for infectious diseases that feature a significant incubation period, during which individuals are infected but asymptomatic and non-contagious.

SEIR Model

$$\begin{cases} \frac{dS}{dt} = -\beta SI \\ \frac{dE}{dt} = \beta SI - \sigma E \\ \frac{dI}{dt} = \sigma E - \alpha I \\ \frac{dR}{dt} = \alpha I \end{cases}$$

$S(t)$: Susceptible individuals
 $E(t)$: Exposed (infected but not infectious)
 $I(t)$: Infectious individuals
 $R(t)$: Recovered individuals
 β : Transmission rate
 σ : Progression rate (1/incubation)
 α : Recovery rate (1/infectious period)

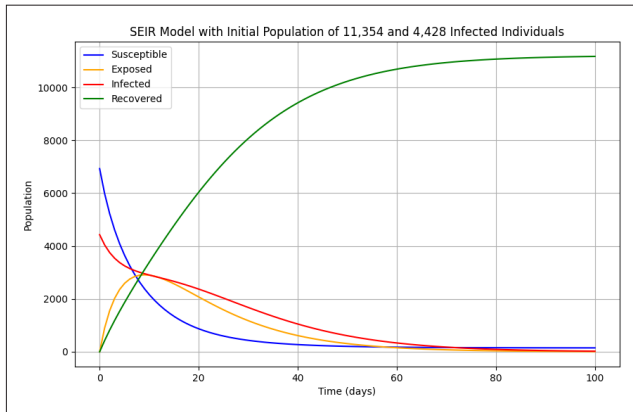


Figure 2: Dynamics of Malaria Transmission

Network-Based Propagation Model

- Describes the infection dynamics between humans and vectors (mosquitoes).
- Infection rates depend on the network of contacts linking individuals and vectors.

Key Concept: The structure of the contact network critically influences how malaria spreads within populations.

Variation Equations for Humans and Vectors Humans

$$\begin{cases} \frac{dS_h}{dt} = -\beta_h S_h \sum_j A_{hj} I_v \\ \frac{dE_h}{dt} = \beta_h S_h \sum_j A_{hj} I_v - \sigma_h E_h \\ \frac{dI_h}{dt} = \sigma_h E_h - \alpha_h I_h \\ \frac{dR_h}{dt} = \alpha_h I_h - \delta_h R_h \end{cases}$$

Vectors (Mosquitoes):

$$\begin{cases} \frac{dS_v}{dt} = \mu_v (N_v - S_v) - \beta_v S_v \sum_i A_{vi} I_h \\ \frac{dI_v}{dt} = \beta_v S_v \sum_i A_{vi} I_h - \mu_v I_v \end{cases}$$

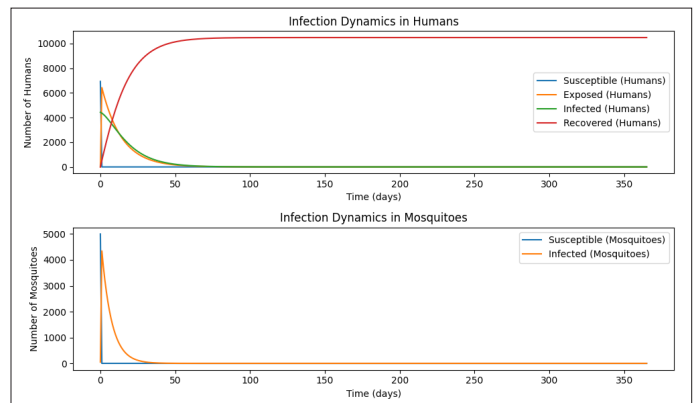


Figure 3: Dynamics of Malaria Transmission on Networks

Data Collection

Summary of Collected Data

Parameter	Value
Population	Children aged 0 to 5 years
Total population size	11,354 children
Infected individuals	4,428 children
Infection rate	0.39
Data source	2018 Malaria Indicator Survey (National Institute of Health)

Basic Reproduction Number Theorem (R_0)

Interpretation of R_0

The basic reproduction number, denoted by R_0 , represents the average number of secondary infections caused by a single infected individual in a completely susceptible population.

The theorem relating to R_0 establishes its dependence on the transmission rate β and the recovery rate α , as expressed by the following formula:

$$R_0 = \frac{\beta}{\alpha}$$

SIR Epidemic Theorem

✓ **If $R_0 < 1$:** The epidemic will die out. The disease-free equilibrium is **stable**.

✗ **If $R_0 > 1$:** The epidemic will spread. The disease-free equilibrium becomes **unstable**.

Population Immunization Theorem

Key Idea

Mass vaccination reduces the number of susceptible individuals, thereby decreasing the probability of an outbreak.

- $v \in [0, 1]$ denotes the proportion of the population that is successfully vaccinated.
- The effective contact rate is reduced by a factor of $(1 - v)$.

Modified SIR Equation with Vaccination

$$\frac{dS}{dt} = -\beta \frac{SI}{N} \cdot (1 - v)$$

Treatment Protective Effect Theorem

The introduction of treatment measures can influence disease spread and reduce the overall disease burden in the population.

$$\beta_{\text{eff}} = \beta \cdot (1 - p)$$

where $p \in [0, 1]$ represents the proportion of the population effectively treated.

SIR Model with Immunity Loss Rate

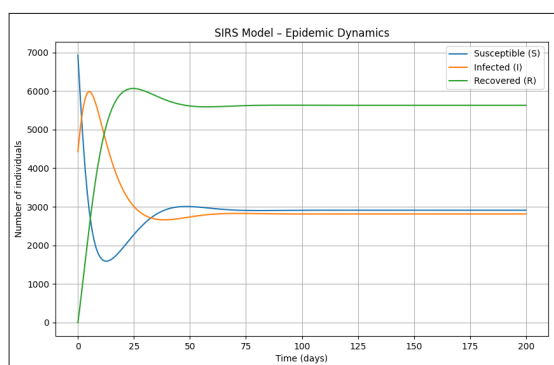


Figure 4: Simulation of Malaria Dynamics with the SIR Model

Stochastic Models

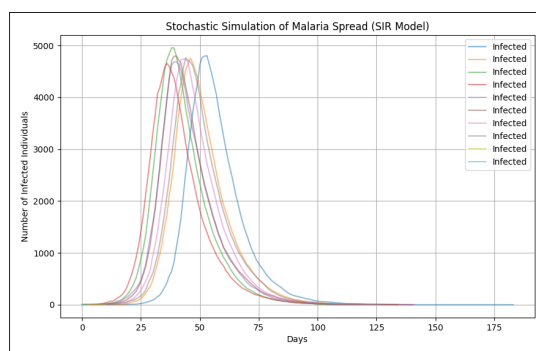


Figure 5: Simulation of a Stochastic Epidemic Model

Intervention and Vaccination Model

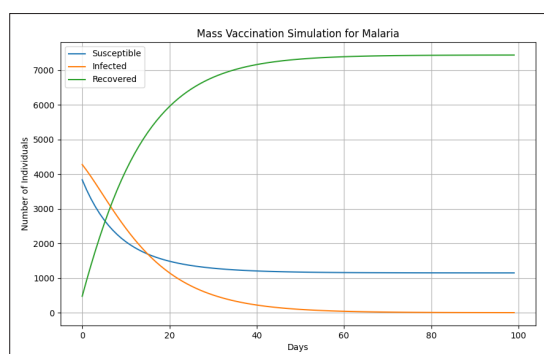


Figure 6: Simulation of a Population Vaccination Intervention

Herd Immunity Model

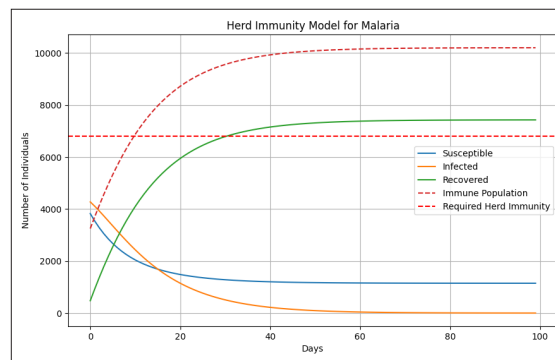


Figure 7: Simulation of the Herd Immunity Model

SIR Model with Vaccination for COVID-19

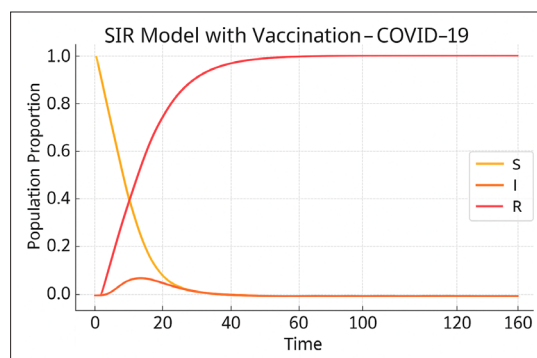


Figure 8: Dynamics of COVID-19 Spread

SI Model for HIV/AIDS

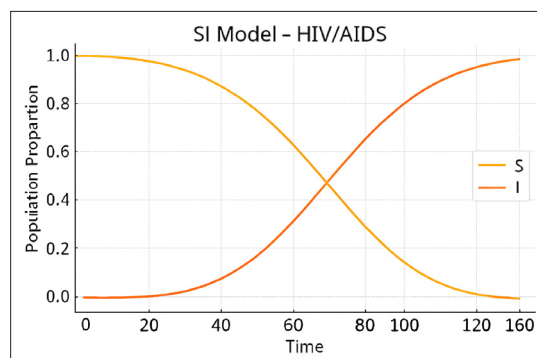


Figure 9: Dynamics of HIV/AIDS Spread

Contributions of this Work

- These epidemiological models help predict how diseases spread in populations, enabling more effective preventive interventions.
- They identify high-risk areas and groups, allowing for more precise allocation of prevention resources.
- Supports real-time outbreak monitoring and adaptation of preventive measures.
- Helps understand the necessary immunization levels to control diseases in a population.
- Assists efficient allocation of healthcare resources such as hospital beds and medical supplies.

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