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Mathematical Modelling and Optimal Control Strategies in Drug Addiction Dynamics

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ABSTRACT

Drug abuse is a complex social and health problem that involves the individuals, their families, and societies at the international level. The tools that should be used in the understanding of its development should be based on the nonlinear and dynamic nature of addictive behavior. This is analyzed in terms of mathematical modeling. These models explain the changes in state, like susceptible, addicted, rehabilitated, and relapsed, depending on changes in the rate of contact, the efficacy of treatment, and the tendency to relapse. Through bifurcation behavior, stability analysis, and sensitivity studies, it is clear that a slight intervention or awareness change can cause dramatic changes in outcome. The basic reproduction number is another threshold measure that explains the continuation or the decline of addiction in a population. This review gives a summary of some of the key nonlinear modelling techniques, their assumptions, the way of analyzing them, and their use. It also indicates limitations, including a lack of real-time data and appropriate focus on psychological and social aspects. It is important to emphasize that synthesis presented the significance of additional data and more realistic models that can increase drug abuse prevention and rehabilitation and policy interventions.

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Introduction

Drug abuse is a major global health and social issue. Reports indicate that more than 280 million people use drugs each year, and millions develop chronic substance-use disorders. These impacts extended beyond physical and mental health to include education, employment, family stability, and community safety. The burden is further intensified by rising addiction rates among adolescents, women, and the vulnerable socio-economic groups. These trends create an urgent need to understand how addiction spreads and persists across populations.

Mathematical models give a systematic approach to the study of drug abuse dynamics. These models are used to turn behavioural and social processes into systems of differential equations. They define the changes between the most important states, which are non-user, user, addicted individual, rehabilitated person, and relapse. The evolution of addiction is dependent on parameters like peer pressure, the likelihood of relapse, treatment effectiveness, and awareness. Key indicators such as the basic reproduction number (R_0) help determine whether addiction will persist or decline. Analysis tools like stability analysis, bifurcation studies, and sensitivity analysis show how changes in intervention strategies influence addiction outcomes.

Drug abuse has been a rising issue of concern in the world, which has prompted researchers to come up with mathematical models that demonstrate how it is initiated, progresses, and is controlled. Simulated the effects of drug abuse in society, whereas focused on the type of treatment and the degree of risk [1,2]. Applied sensitivity analysis to high-low risk susceptible groups [3]. Social extensions include studies such as who linked crime and substance abuse, and who have modelled drug-crime relationships [4,5]. Included broader public-health factors, including such variables as unemployment, stress, and interactions of drug addictions [6]. According to, the idea of adolescent-centered modeling was introduced [7]. Reviewed the dual-drug dependence, and conducted a review of the SLARS stability [8,9]. Analyzed patterns of drug abuse, and modeled the substance abuse among women with control policies, which is a gender-specific intervention requirement [10,11]. Compared drug addiction to counseling and incarceration, where backward bifurcation was established [12]. Focused on awareness-based intervention using a nonlinear approach to the drug-addiction model [13]. Reviewed the recycling of recovered people into vulnerable groups. Examined drug abuse among children under optimal control [14,15]. Analyzed the broader opioid processes in the context of relapses and treatment saturation [16].

Although progress has been made, there are still key gaps. Most of the models rely on small data sets, and psychological and social variables are often simplified. There is limited incorporation of heterogeneity in age groups, genders, and social networks. Not many models involve dual-drug interactions, the relapse cycles over time, or aftercare. Responses to policy changes are also not studied in real time in terms of behavior. These drawbacks suggest that more data sources, better behavioral hypotheses, and more coherent socio-economic assumptions are required.

The reviewed article is a synthesis of mathematical and optimal-control models of drug addiction published between 2020 and 2025. It examines model equations, assumptions, and analysis. It also determines constraints, critical parameters, and policy-appropriate conclusions. This review aims to provide an in-depth explanation of how prevention, rehabilitation, and evidence-based policymaking can be used to deal with substance abuse through the use of mathematical models.

Material & Methods

A systematic search was performed to identify the publications in the field of mathematical modeling of drug abuse. The search was performed in four electronic databases, including Google Scholar, Scopus, ScienceDirect, and Web of Science, with combinations of keywords such as "mathematical modelling," "drug abuse," "optimal control," "dynamical systems," "relapse dynamics," and "substance-use modelling." The search was limited to peer-reviewed articles published between 2019 and 2025. Studies were included if they presented a mathematical model of drug initiation, addiction, relapse, treatment, or behavioral transitions; used nonlinear ODEs, optimal control theory, or stochastic modelling; and included at least two relevant keywords in the title or abstract. Articles without model equations, review articles, conference papers, and articles that are not related to the dynamics of addiction were eliminated. Titles and abstracts were filtered to eliminate duplicates and irrelevant work, and a full-text assessment was done to verify the model formulation and the method of analysis. After applying all criteria, 30 studies were identified. These studies were then grouped into thematic categories, including treatment-based models, school and campus models, socio-economic co-dynamics, crime-linked models, unemployment and mental illness models, stress-related models, and multi-risk population models. This review extracted and synthesized key information from each article, including model structure, assumptions, equilibrium analysis, derivation of reproduction number, stability results, sensitivity analysis, and numerical simulations.

Mathematical Models

Focuses on addressing the growing issue of drug addiction in Bangladesh through a mathematical modelling approach [17]. The authors develop a dynamic model (1) that categorizes the population into five groups: those at-risk S(t), light drug users L(t), heavy drug users H(t), individuals in rehabilitation R(t), and those who have quit Q(t). They use nonlinear differential equations to examine how such groups transition to one another and how to minimize the level of addiction. To formulate effective intervention strategies, the researchers utilize optimal control theory and incorporate three control measures: awareness and educational initiatives, family support, and the effectiveness of rehabilitation. Using Pontryagin's Maximum Principle, they identify the optimal mix of these controls that would lead to a decrease in the number of addicts and mitigate the social and economic impacts of drug abuse. In addition, the research includes computer simulations and a sensitivity analysis; this demonstrates that reducing interactions between those who are vulnerable and those who are addicted will lead to a significant reduction in overall addiction rates. While all of the control measures were found to reduce addiction rates, it was determined that awareness and family involvement are more cost-effective methods than using only rehab centers for treatment. The findings further demonstrate that an additive approach to addressing drug addiction (education, community involvement, and medical treatment) is effective in addressing drug addiction and improving the health of communities by creating environments where people do not experience addiction.

$$\left.\begin{aligned}
\frac{dS}{dt} &= r-\alpha SH-\mu S, && S(0)>0\\
\frac{dL}{dt} &= \alpha SH-\beta L-\mu L-\delta L, && L(0)\geq 0\\
\frac{dH}{dt} &= \beta L-\mu H-\gamma H+pR, && H(0)\geq 0\\
\frac{dR}{dt} &= \gamma H-\mu R-\theta R-pR, && R(0)\geq 0\\
\frac{dQ}{dt} &= \theta R-\mu Q, && Q(0)\geq 0
\end{aligned}\right\}(1)$$

Population size N(t)=S(t)+L(t)+H(t)+R(t)+Q(t)

Present a mathematical model (2) to explain the spread and persistence of drug use and addiction among university students in Nigeria [18]. The researchers categorized the population into five compartments: susceptible S(t), exposed E(t), drug users D(t), addicts A(t), and recovered R(t). Using the next-generation matrix method, they derived the basic reproduction number (R0), which shows the potential for drug addiction. The analysis revealed that the drug-free equilibrium is stable when R0 is less than 1; drug use is likely to continue spreading. Factors such as increased student enrollment, relapse rates, and interactions between users and non-users significantly elevate R_0 and consequently the rates of drug addiction. The results indicate that social influences, poor academic performance, early exposure to drugs, and lack of parental supervision contribute to substance abuse among students. In order to diminish these risks, the study enlightens that preventive interventions like family-based intervention, early education, self-control training, and good institutional policies are crucial. In conclusion, the model shows that drug addiction among students can be effectively controlled if R_0 is reduced below one through policy enforcement, education, and rehabilitation programs. The authors recommend that both the government and tertiary institutions improve their efforts to limit drug availability and promote awareness to combat addiction among students.

The total population of the students to be taken is
N(t)=S(t)+E(t)+D(t)+A(t)+R(t)
The following system of equations is

$$\left.\begin{aligned}
\frac{dS}{dt} &= \Lambda-\beta S(t)D(t)(1+\alpha D)-(\mu+\omega)S(t),\\
\frac{dE}{dt} &= \omega S(t)-(\mu+\gamma)E(t),\\
\frac{dD}{dt} &= \gamma E(t)+\beta S(t)D(t)(1+\alpha D(t))-(\sigma+\delta+r+\mu)D(t)+\rho R(t),\\
\frac{dA}{dt} &= \sigma D(t)-(\theta+\tau+\mu)A(t),\\
\frac{dR}{dt} &= \tau A(t)+\delta D(t)-(\mu+\rho)R(t),
\end{aligned}\right\}(2)$$

Examine the propagation and regulation of drug addiction in Tanzania through a mathematical model (3) [19]. The authors categorize the population into six segments: individuals who are susceptible S(t), those who are exposed E(t), light drug users I1(t), heavy drug users I2(t), patients undergoing rehabilitation Q(t), and individuals who have recovered R(t). The model outlines how individuals transition between these categories based on social interactions, levels of addiction, treatment options, and rates of recovery. The mathematical model describes the underlying transitions with the help of the mathematical tool of differential equations and uses the next generation matrix method to compute the basic reproduction number (R_0), which is a significant parameter that defines whether the drug abuse will persist in a community or decline. When R_0 is below 1, drug abuse will eventually disappear; when R_0 is above 1, the abuse will persist and become endemic. In both analytic and numerical models, the study reveals that the increase in drug abuse in the society occurs hugely as a result of high interactions between the vulnerable people and the users as well as delayed treatment and low rates of recovery. The authors illustrate that implementing early interventions and rehabilitation can significantly curtail the emergence of new users and stabilize the situation. The model also looks at the local and international stability of the equilibrium points, meaning that without the control strategies, the light and heavy drug users will increase rapidly, whereas prevention and treatment strategies can reduce the problem. The outcomes highlight the importance of stringent laws, early detection, and social participation to prevent addiction. To conclude, the paper presents a scientific and systematic method of understanding the dynamics of drug abuse. It highlights the fact that with proper control measures and active involvement of the society, mathematical modeling as a resource in the field of public health policy and social planning can be significantly reduced.

$$\left.\begin{aligned}
\frac{dS}{dt} &= \lambda + \alpha_3 R - (\psi_1 I_1 + \psi_2 I_2)(1-\omega)S - \mu_1 S,\\
\frac{dE}{dt} &= (\psi_1 I_1 + \psi_2 I_2)(1-\omega)S - ((1-\rho)+\rho)\alpha_1 E - \mu_1 E,\\
\frac{dI_1}{dt} &= (1-\rho)\alpha_1 E - (\rho_1\sigma_1 + \rho_2\sigma_1 + \rho_3\sigma_1 + \mu_1)I_1,\\
\frac{dI_2}{dt} &= \rho\alpha_1 E + \rho_1\sigma_1 I_1 - (\sigma_2 + \mu_1 + \mu_2)I_2,\\
\frac{dQ}{dt} &= \rho_2\sigma_1 I_1 + \sigma_2 I_2 - (\alpha_2 + \mu_1)Q,\\
\frac{dR}{dt} &= \rho_3\sigma_1 I_1 + \alpha_2 Q - (\mu_1 + \alpha_3)R
\end{aligned}\right\}(3)$$

$$\rho_1 + \rho_2 + \rho_3 = 1$$

Where S, E, I_1, I_2, Q, and R are susceptible, exposed, light drug users, heavy drug users, patients undergoing rehabilitation, and recovered.

Introduce a mathematical model (4) aimed at understanding and mitigating drug abuse among secondary school students [20]. This research conceptualizes drug abuse similar to an infectious disease through the SEIGWR model, which comprises six categories: susceptible (S) students are vulnerable to drug use; exposed (E) are those influenced but not yet addicted; infected (I) are active users; guided and counselled (G) are students benefiting from counselling; rehabilitated (W) are individuals undergoing treatment; and recovered (R) are those who have successfully overcome addiction. The model allows the researcher to analyze how students are transitioning through the various phases of drug use over a period of time. Using mathematical models and assessing the equilibrium conditions of the model, the researchers were able to calculate the basic reproduction ratio (R_0), which is used to determine if drug abuse will increase or decrease. The findings suggest that when R0 exceeds 1, it escalates. Also, numerical simulations conducted by the researchers showed that frequent guidance and counselling sessions would result in lower levels of drug abuse and subsequently less need for rehabilitating students. Overall, the research demonstrates that guidance and counseling are effective strategies for reducing drug abuse among school-aged youth. Specifically, the earlier a guidance program is initiated, the more likely it is to be successful in controlling drug abuse. Therefore, the researchers conclude that continuous guidance programs, as well as awareness education programs and post-rehabilitation services for students experiencing substance abuse problems, are essential for maintaining a safe and drug-free environment within schools. Overall, the researchers conclude that the combination of mathematical modeling and educational intervention will provide a viable means of tracking and potentially preventing drug abuse trajectories among young students.

$$\left.\begin{aligned}
\frac{dS}{dt} &= (1-q)\gamma - (d + \beta_1 + \mu)S,\\
\frac{dE}{dt} &= q\gamma + \mu S - (\beta_2 + \delta)E,\\
\frac{dI}{dt} &= \beta_1 S + \beta_2 E - (\alpha + \omega)I,\\
\frac{dG}{dt} &= \alpha I - (m + \sigma)G,\\
\frac{dW}{dt} &= \sigma G - kW,\\
\frac{dR}{dt} &= kW + mG + \omega I + \delta E - dR
\end{aligned}\right\}(4)$$

N(t)= S(t)+E(t)+I(t)+G(t)+W(t)+R(t)

Present a mathematical model (5) that examines the interdependence between illicit drug use and terrorism in a population [21]. The author constructed a nonlinear system of ordinary differential equations to examine the proliferation and endurance of these two social issues. The model categorizes the population into seven distinct groups: susceptible individuals, drug users, potential terrorists, detainees, inmates, rehabilitated persons, and those who have quit. With the aid of this structure, the researcher defined the key parameters, such as the basic reproduction number (R0), which can be used to establish conditions under which drug use and terrorism can cease or continue. Results suggested that in the case where R_0 is lower than 1, there is a possibility of the population getting rid of drug use and terrorism; however, due to a phenomenon termed as a backward bifurcation, this might still persist even at a level below one unless some form of strict control measures are applied. The analysis of the model based on Lyapunov stability showed that in case R0 is greater than the number 1, both issues become globally stable and self-sustaining in society. A sensitivity analysis was able to point out the parameters that have the greatest effect on the spread, e.g., the rates of influence, rehabilitation, and relapse. Increasing rehabilitation work and reducing the relapse rates was also proven to be effective in reducing the dual threats. The analytical results were also investigated by the author through numerical simulation using MATLAB to confirm their findings. Finally, the paper brings into perspective the interdependence between drug abuse and terrorism and that the two problems should be dealt with concurrently using effective policy and rehabilitation programs. The model is an effective tool for the policymakers intending to develop policies that reduce the proliferation and social consequences of illicit drug use as well as terrorism.

$$\left.\begin{aligned}
&\frac{dS}{dt}=\Lambda-\frac{\beta(I(t)+\eta S_E(t))}{N(t)}S(t)+\sigma_6 Q(t)+\rho R(t)-\mu S(t), && S(0)=S_0\\
&\frac{dI}{dt}=\frac{\beta(I(t)+\eta S_E(t))}{N(t)}S(t)+\alpha R(t)-(\sigma_1+\tau_1+\gamma_1+\delta_1+\mu)I(t), && I(0)=I_0\\
&\frac{dS_E}{dt}=\gamma_1 I(t)+(\sigma_2+\tau_2+\gamma_2+\delta_2+\mu)S_E(t), && S_E(0)=S_{E_0}\\
&\frac{dD}{dt}=\sigma_1 I(t)+\sigma_2 S_E(t)-(\sigma_3+\tau_3+\gamma_3+\mu)D(t), && D(0)=D_0\\
&\frac{dC}{dt}=\gamma_2 S_E(t)+\gamma_3 D(t)-(\sigma_4+\tau_4+\delta_4+\mu)C(t), && C(0)=C_0\\
&\frac{dR}{dt}=\tau_1 I(t)+\tau_2 S_E(t)+\tau_3 D(t)+\tau_4 C(t)-(\sigma_5+\alpha+\rho+\mu)R(t), && R(0)=R_0\\
&\frac{dQ}{dt}=\sigma_3 D(t)+\sigma_4 C(t)+\sigma_5 R(t)-\sigma_6 Q(t)-\mu Q(t), && Q(0)=Q_0
\end{aligned}\right\}(5)$$

Where S(t), I(t), SE(t), D(t), C(t), R(t), and Q(t) are susceptible (someone who is in a correctional center due to the use of illicit drugs but moves with drug users), illicit drug users (someone who utilizes illicit drugs often or those that absolutely depend on the use of illicit drugs), suspected terrorists, individuals undergoing detection, prisoners (someone who is in a correctional center due to the use of illicit drugs and a suspected terrorist), the rehabilitated population, and quitters (individuals who quit taking illicit drugs and terrorism).

Aim to understand and predict the spread of drug addiction in the region using a mathematical model (6) [22]. The researchers used secondary data from the South Sulawesi National Narcotics Agency and its rehabilitation center. They developed a SEARC model that includes five groups: Susceptible- people prone to drug use, Exposed- those who try drugs, Addicted- individuals who become addicted, Rehabilitation- addicts undergoing treatment, and Clean- people who have successfully recovered. The model uses mathematical formulas to study the mobility between these levels and will assist in determining the balance points in which addiction can be maintained or reduced further. The analysis found a basic reproduction number (R_0) = 0.05, which indicates that drug addiction in South Sulawesi can be controlled and is not spreading uncontrollably. However, preventive actions are important. Simulation results show that over time, the number of people susceptible to drugs stabilizes, while those who are addicted or in rehabilitation decrease gradually and reach equilibrium. The study concludes that rehabilitation programs play a major role in reducing addiction rates, and enhancing these services can significantly lower drug abuse cases. In essence, the research highlights the power of mathematical modeling to study social health problems like drug addiction. Through data analysis and simulation, it provides useful information to the policy makers to strategize countermeasures to curb the drug abuse problem long term by providing effective rehabilitation and prevention strategies relevant to the South Sulawesi province.

$$\left.\begin{aligned}
&\frac{dS}{dt}=\mu N-\mu S-\beta SA,\\
&\frac{dE}{dt}=\beta AS-\alpha E-\mu E,\\
&\frac{dA}{dt}=\alpha E-\theta A-\mu A,\\
&\frac{dR}{dt}=\theta A-\sigma R-\mu R,\\
&\frac{dC}{dt}=\sigma R-\mu C
\end{aligned}\right\}(6)$$

Where N, S, E, A, R, and C are total population, susceptible, exposed, addicted, rehabilitation, and clean from drugs.

Discuss the interrelation between poverty and drug addiction in the society [23]. The study creates a mathematical model (7) that categorizes the population into four segments: affluent individuals, the poor, those with drug addictions, and individuals in recovery. The model analyzes the dynamics of movement between the two groups in relation to time using deterministic and stochastic differential equations. The deterministic approach gives emphasis on constant trends of population behavior, whereas the stochastic approach emphasizes random influences in society or the environment to produce more lifelike results. The authors distinguish three equilibrium conditions-drug-free, poverty-free, and endemic (both poverty and addiction coexist)-and determine their stability by the use of mathematical and numerical modeling. These are Python implementations that demonstrate that, given the right intervention and assistance, the system can stabilize to the point where poverty and addiction will reduce over time. Among important parameters studied are the recovery rate, the rate at which people are coming into poverty, and the effects of addiction to understand their effects on population dynamics. Stochastic analysis highlights the unpredictability of the social factors but also states that with the help of rehabilitation efforts, NGOs, and awareness campaigns, the level of addiction could be significantly reduced. The authors come to the conclusion that the problem of poverty and the problem of drug addiction are highly interconnected issues in society and require a combination of measures. Poverty often motivates people to turn to drugs and substances, thus making financial problems worse. Through mathematical modelling, the authors propose that effective strategies-such as counselling, job creation, and rehabilitation services-can mitigate the prevalence of addiction and assist communities in overcoming poverty. The paper emphasizes that by addressing these problems with both analytical modeling and social programs, it will be possible to have a more stable, successful society.

$$\left.\begin{aligned}
&\frac{dH}{dt}=\mu N-(\mu+\theta_1)H-\alpha_1 H\frac{D}{N},\\
&\frac{dP}{dt}=\theta_1 H-(\mu+\theta_2)P-\alpha_2 P\frac{D}{N}+I_2 R,\\
&\frac{dD}{dt}=\alpha_1 H\frac{D}{N}+\alpha_2 P\frac{D}{N}-(\gamma+\mu+\mu_1)D+I_1 R\frac{D}{N},\\
&\frac{dR}{dt}=\gamma D-I_1 R\frac{D}{N}-I_2 R-\mu R
\end{aligned}\right\}(7)$$

Where N, H, P, D, and R are the total population, high-class individuals, poor individuals, drug-addicted individuals, and recovered population.

Present a mathematical model (8) to investigate the spread of drug abuse among college students [24]. By establishing comparisons between substance abuse and the transmission of infectious diseases, the study utilizes susceptible students S(t)-who are not currently abusing drugs but are at high risk of doing so. Exposed students E(t)-This group includes individuals who have been introduced to drugs or drug-related behaviors but have not yet become regular users. Infected students I(t)-These students are actively abusing drugs and are the main source of transmission in the model. Rehabilitation P(t)-This compartment represents students who have entered a rehabilitation program to overcome addiction. Recovered students R(t)-These individuals have completed rehabilitation or naturally recovered from drug abuse. (SEIPR) model to analyze drug use dynamics on campus. The model divides students into five groups, including susceptible, exposed, infected, in rehabilitation, and recovered, and analyzes the changing state of the students between these categories depending on the factors of recruitment, transmission, recovery, and rehabilitation rates. The authors determine the basic reproduction number (R_0) to evaluate how fast drug abuse can develop. The results suggest that the drug-free equilibrium is stable at R_0 of less than 1, meaning that each abuser influences less than one new individual. On the other hand, the case of R0 being greater than 1 implies that drug abuse persists and becomes a spreader among the population. Simulations have shown that by improving the rehabilitation and recovery rates, it can be reduced significantly to reduce drug transmission. The paper also brings out the significance of the awareness programs, preventive education, and easily accessible rehabilitation programs in controlling drug abuse. The sensitivity analysis shows that the most important factors are the transmission and recruitment rates, and the rehabilitation and natural recovery rates help to control the spread. In conclusion, the paper asserts that while it is challenging to completely eliminate drug abuse on campus, a strategy combining rehabilitation programs, preventive education, and social support can significantly mitigate its effects and ensure long-term drug-free stability among students.

$$\left.\begin{aligned}
\frac{dS}{dt} &= \lambda - \alpha SE - \kappa SI - \eta S, && S(0)>0\\
\frac{dE}{dt} &= \alpha SE - \eta E - \beta EI, && E(0)\ge 0\\
\frac{dI}{dt} &= \beta EI + \kappa SI - I(\eta+\tau+\gamma), && I(0)\ge 0\\
\frac{dP}{dt} &= \tau I - P(\eta+\delta), && P(0)\ge 0\\
\frac{dR}{dt} &= \delta p + \gamma I - \eta R, && R(0)\ge 0
\end{aligned}\right\}(8)$$

Where S, E, I, P, and R are susceptible, exposed, infected, rehabilitation program, and recovered.

Offers a thorough mathematical model (9) for investigating the dynamics of substance abuse and its impact on society [25]. The study groups the people into five segments, including potential users, moderate users, heavy users, persons in rehabilitation, and those who do not use anymore. Using nonlinear differential equations, the model examines how individuals move between these categories depending on their various factors, such as contact rate, recovery rate, relapse rate, and recruitment rate. The study has laid stress on the basic reproduction number (R_0), equilibrium points, and stability of the system. The results show that when R_0 is less than 1, then eventually substance abuse will be eliminated; when R_0 is more than 1, then it will keep the population addicted. The sensitivity analysis illustrates recruitment (π) and contact rate (β) as the most important parameters that enhance R_0, whereas recovery and mortality rates enhance the reduction of R_0. This implies that any prevention strategy that strives to reduce exposure and contact among the prospective users is better than making rehabilitation efforts better. Numerical simulations also illustrate the dynamics of each population group, which can increase initially with heavy and moderate substance abuse, which can decrease in case of successful interventions. The research concludes that increasing recovery and rehabilitation rates can lead to a significant reduction in addiction levels. Overall, the study delivers a realistic and analytical approach to understanding the spread of substance abuse and crafting data-informed prevention and treatment methods.

$$\left.\begin{aligned}
\frac{dp}{dt} &= \pi + \sigma R - \beta PM(1+\alpha M) - \mu P,\\
\frac{dM}{dt} &= \beta PM(1+\alpha M) - (\phi+\psi+\mu)M,\\
\frac{dH}{dt} &= \phi M + \eta Q - (\tau+\delta_1+\mu)H,\\
\frac{dQ}{dt} &= \tau H - (\eta+\gamma+\delta_2+\mu)Q,\\
\frac{dR}{dt} &= \gamma Q + \psi M - (\sigma+\mu)R
\end{aligned}\right\}(9)$$

The total population size at time t is denoted by N(t) and can be obtained from the above model.
N(t)=P(t)+M(t)+H(t)+Q(t)+R(t)

Where P(t), M(t), H(t), Q(t), and R(t) are potential substance abuse, moderate substance abuse, heavy substance abuse, rehabilitation, and quitters.

Introduce the analysis of the spread of illegal drug dependency and its management in the community [26]. Drawing inspiration from the classic SIR epidemiological model, the authors categorize the population into four groups: non-susceptible (Pn), susceptible (Ps), addicted (Pa), and rehabilitated (Pr) individuals. The model (10) takes into account real-life elements such as peer pressure, public awareness initiatives, the probability of relapse, and the effectiveness of rehabilitation. The transitions between these groups are represented by a series of different nonlinear equations. The study introduces the fundamental reproduction number (R_0= σ1 /δ) to assess whether addiction will diminish or endure. When R_0 is smaller than 1, addiction will slowly diminish, whereas when it is bigger than 1, then it will become embedded. Through an analysis of the stability of equilibrium points, the paper outlines the criteria for both addiction-free and addiction-continuing states. The authors utilize Pontryagin's maximum principle to design an optimal control strategy for intervention strategies in search of an equilibrium between efficacy and cost. Numerical simulations have shown that high involvement of family, community, and government may considerably decrease the susceptibility and addiction rates. A high-intervention scenario depicts a rapid shift to a drug-free state, whereas low intervention or delayed interventions enable the addiction to stabilize or increase. Overall, it is important to note that the study reveals that addiction is just like a communicable disease. Timely, regular, and cost-effective interventions supported by education, awareness, and rehabilitation play a very important role in controlling and eventually eliminating drug misuse in the society.

$$\left.\begin{aligned}
\frac{dP_n}{dt} &= -\sigma_1(P_a+P_s)\frac{P_n}{P_t}+\delta P_s+fpP_r, && P_n(0)\ge 0\\
\frac{dP_s}{dt} &= \sigma_1(P_a+P_s)\frac{P_n}{P_t}-\sigma_2\frac{P_aP_s}{P_t}-\delta P_s, && P_s(0)\ge 0\\
\frac{dP_a}{dt} &= \sigma_2\frac{P_aP_s}{P_t}+(1-f)pP_r-\psi P_a, && P_a(0)\ge 0\\
\frac{dP_r}{dt} &= \psi P_a-pP_r, && P_r(0)\ge 0
\end{aligned}\right\}(10)$$

$$P_t = P_n + P_s + P_a + P_r$$

Where P_t, P_n, P_s, P_a, and P_r are the total population, neither current nor potential drug users, those most susceptible to illicit drug abuse, people with illicit drug addiction, and the recovered compartment.

Formulates a mathematical model (11) to understand and manage the spread of drug and substance abuse (DSA) [27]. Accepting the fact that addiction spreads similarly to an infectious disease, the study introduces a deterministic compartmental model dividing the population into six specific classes: susceptible individuals, exposed users, light drug abusers (LDAs), heavy drug abusers (HDAs), people undergoing rehabilitation, and recovered persons. This model integrates the impacts of the treatment interventions and law enforcement (policing) on the reduction of the addiction rates. By using the stability and bifurcation analysis, the researcher is able to determine the basic reproduction number (R_0), which is used to determine whether drug abuse is going to persist or be eradicated. With R_0 declining to 1, the state without DSA is stable, whilst with R_0 rising to above 1, the addiction becomes common. The study reveals that there is a forward (transcritical) bifurcation in the system, which implies that a change of critical parameters may alter the population between an endemic and drug-free state. The results of simulations performed with the help of Python indicate that law enforcement (κ) and the rate of addict identification and treatment (γ) can significantly reduce the prevalence of drugs. Sensitivity analysis shows that high initiation rates (β) and peer influence (ξ) promote addiction, and stronger policing and treatment reduce it. The results indicate that an elaborate approach incorporating intervention in time, education, rehabilitation, and effective enforcement of the law can drastically curtail drug abuse in Kenya and the global world.

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi+\tau R-\lambda S-\mu S, && S(0)=S_0\ge 0\\
\frac{dE}{dt} &= \lambda S-(\mu+\vartheta)E, && E(0)=E_0\ge 0\\
\frac{dL}{dt} &= (1-\eta)\vartheta E-(\mu+\zeta)L, && L(0)=L_0\ge 0\\
\frac{dH}{dt} &= \eta\vartheta E+\alpha_3\zeta L-(\mu+\gamma+\delta_1)H, && H(0)=H_0\ge 0\\
\frac{dQ}{dt} &= \alpha_2\zeta L+\gamma H-(\mu+\sigma+\delta_2)Q, && Q(0)=Q_0\ge 0\\
\frac{dR}{dt} &= \alpha_1\zeta L+\sigma Q-(\mu+\tau)R, && R(0)=R_0\ge 0
\end{aligned}\right\}(11)$$

Where S, E, L, H, Q, and R are the drug- and substance-abuse-susceptible population, individuals just initiated into drug abuse, light drug users, heavy drug users, drug users under treatment, and the fully recovered population.

Examines the possibility of mathematical models contributing to the management of drug addiction in the society [28]. By identifying the problem of addiction as a social problem that is influenced by the economical, psychological, and environmental factors, the paper integrates the epidemiological and mathematical approaches to identifying the effective intervention measures. The authors developed a compartmental model (12) by splitting the population into four groups-potential users, experimental users, addicts, and recovered individuals-and examined the flows between the four groups. This model is also known as the P-E-A-R (Potential-Experimental-Addicted-Recovered) model, and it is analogous to the disease transmission models such as SEIR, except that it focuses on dynamics of the addiction. Two main control variables are introduced: preventive measures (u_1), such as awareness initiatives and education, and punitive actions (u_2), which include legal enforcement and rehabilitation efforts. Using the maximum principle by Pontryagin and numerical analysis in MATLAB, it can be noted that a combination of prevention and punishment can significantly reduce rates of addiction and increase rates of recovery. The conclusion of the research is that early prevention, tough policy enforcement, and community-oriented rehabilitation are better used as a package and not individually. Conclusively, the paper affirms that the reduction of drug addiction can be achieved by the use of consistent and preferably balanced interventions that combine social, legal, and health-oriented interventions-the importance of mathematical modeling in shaping real-life policies on public health.

$$\left.\begin{aligned}
\frac{dP}{dt} &= \Lambda-\sigma PA-\mu P, && P\ge 0\\
\frac{dE}{dt} &= \sigma PA-\alpha E-\tau_D E-\mu E, && E\ge 0\\
\frac{dA}{dt} &= \alpha E-\tau_A A-\mu A, && A\ge 0\\
\frac{dR}{dt} &= \tau_D E+\tau_A A-\mu R, && R\ge 0
\end{aligned}\right\}(12)$$

Where P, E, A, and R are the potential, experimental, addicted, and recovered.

Introduce a mathematical model (13) aimed at exploring how drug addiction proliferates within a community [29]. The framework categorizes the population into five groups: susceptible individuals (those at risk), exposed individuals (those who are addicted), individuals who are quarantined or undergoing rehabilitation, and those who have recovered. It introduces a nonlinear incidence rate, which takes into consideration real-world factors, including prevention efforts and social influence, which reduce the spread with a rise in prevalence of addiction. One of the main points of the study is the basic reproduction number (R_0) that is used to determine diminishing or not diminishing drug addiction. If R_0 is below one, the community can potentially remain free of drugs; however, if it rises above one, addiction will keep spreading. The research also points out a forward bifurcation at R_0=1, suggesting that minor alterations in circumstances can lead to a transition from a stable drug-free environment to one experiencing extensive addiction. Through simulations, the researchers illustrate that enhancing the intervention factor (which signifies education, awareness campaigns, and rehabilitation programs) considerably decreases the number of addicted people. The model indicates that both prevention and rehabilitation efforts not only curb the increase of addiction but also improve recovery rates. Overall, the study shows that drug addiction is more like an infectious disease that is transmitted through social interactions but can be controlled through long-term public health interventions. The model gives important information to policymakers, insisting that education programs and rehabilitation activities are the key to the successful long-term management of the addiction and promotion of health in communities.

$$\left.\begin{aligned}
&\frac{dS}{dt}=\Lambda-\frac{\beta SA}{1+\alpha A}-\mu S, && S(t_0)=S_0\geq 0\\
&\frac{dE}{dt}=\frac{\beta SA}{1+\alpha A}-(\gamma+\mu)E, && \mathrm{E}(t_0)=E_0\geq 0\\
&\frac{dA}{dt}=\gamma E-(\mu+\theta_1+\pi)A, && A(t_0)=A_0\geq 0\\
&\frac{dQ}{dt}=\pi A-(\mu+\theta_2+\omega)Q, && Q(t_0)=Q_0\geq 0\\
&\frac{dR}{dt}=\omega Q-\mu R, && R(t_0)=R_0\geq 0
\end{aligned}\right\}(13)$$

Where S(t) represents individuals who are susceptible to addiction if they interact with addicted individuals, E(t) represents individuals exposed to drugs, A(t) represents individuals who are addicted to illegal drugs, Q(t) represents addicted individuals who are trying to quit, and R(t) represents individuals who have recovered from drug addiction.

Investigates the role of mathematical modeling in understanding and managing hard drug abuse among females aged 15-20 [30]. Acknowledging that addiction spreads through social dynamics and behavioral patterns, the authors created a compartmental model (14) categorizing the population into Susceptible, Exposed, Addicted, and Rehabilitated groups. By employing differential equations, they analyzed the influences of peer pressure, curiosity, and rehabilitation initiatives on the transitions among these categories. The model's basic reproduction number (R_0) functioned as a benchmark for forecasting whether drug addiction would continue or diminish. Findings indicated that when R0 exceeds one, addiction persists; when it falls below one, drug abuse gradually diminishes. Simulations demonstrated that addiction rises swiftly and then stabilizes unless preventive or rehabilitation rates considerably reduce the number of individuals with addiction. Specifically, prevention measures combined with effective rehabilitation initiatives were the most effective way of minimizing the frequency of addiction as well as the rate of relapse. The research brings out that drug abuse by young females is a social contagion with peer relationships and emotional vulnerability. Therefore, mathematical modelling offers a scientific foundational basis of intervention strategy evaluation and guidance to policy makers, educators, and medical practitioners. The authors recommend combined measures, which encompass learning, early prevention, community rehabilitation, and law enforcement to reduce drug initiation and accomplish recovery. This study highlights the possibility of mathematical instruments to transform social health problems into quantifiable and practical conclusions that can be applied to real-world policymaking.

$$\left.\begin{aligned}
&\frac{dS}{dt}=-\beta SA+\delta E,\\
&\frac{dE}{dt}=\beta SA-(\alpha+\delta)E,\\
&\frac{dA}{dt}=\alpha E+\rho R-\gamma A,\\
&\frac{dR}{dt}=\gamma A-\rho R
\end{aligned}\right\}(14)$$

Number of Where the interaction term βSA denotes the peer pressure effect.
N(t) = S(t) + E(t) + A(t) + R(t)

Where N(t), S(t), E(t), A(t), and R(t) are the total population, number of susceptible girls at time t, number of exposed girls at time t, number of addicted girls at time t, and number of rehabilitated girls at time t.

Discussion and Conclusion

The compartmental modeling frameworks can be seen to have a steady capacity for the behavioral and social processes that lead to drug abuse. In schools, campuses, and general communities, the models highlight one main finding: that peer pressure and socialization are the most influential drivers of drug use initiation and transmission. This finding in studies is interesting, as it supports the view of addiction as a social contagion, where the transmission of behavior is patterned in the same way as the transmission of infectious disease. Such similarity makes it possible to apply such epidemiological methods as the basic reproduction number (R_0), the analysis of stability, and the bifurcation theory to understand whether the use of drugs will increase or decrease.

In all of the reviewed models, R_0 is the key. At R_0 larger than one, addiction is self-sustaining, and it is hard to reduce in the long term. On pushing R_0 such that it becomes less than one, the addictions are reduced, and drug-free equilibrium could stabilize. Sensitivity analyses indicate that the highest impact on R0 is from such factors as peer contact, relapse probability, and treatment initiation rates. Changes in these parameters, even small in themselves, like those that are brought about by the impact of awareness programs, interventions in schools, or counselling, have large impacts on the risk of new-user initiation. Models like SEIGWR and SEARC structures focus on the fact that the exposure and addiction rates can be cut down greatly with the rise in guidance, rehabilitation opportunities, and education outreach.

The best outcomes in terms of control in all studies support the significance of prevention. It has been demonstrated in simulations that the initial, vigorous preventive measures are the most advantageous in the long-term perspective. Combination prevention and treatment models reveal that no rehabilitation will be sufficient to tame addiction unless preventive measures also lower the incidences of its initiation. The research, which takes into consideration relapse, is pointing to the necessity of continuing aftercare; without long-term support, individuals can get back to the addicted category even in the case when R0 is already less than one. This result highlights the relevance of long-term recovery mechanisms instead of short-term treatment periods.

Although they are strong, the existing models exhibit a number of methodological shortcomings. The majority of them are of the assumption of homogeneous mixing, i.e., that all people are mixed equally. This assumption does not show actual social networks, age-specific trends, and social-economic inequality. There are models that deal with poverty, unemployment, or even terrorism, but they are relatively few. The uncertainty of parameters is another issue, since trusted information on the initiation, relapse, and treatment effect is not always available. Not many studies calibrate parameters to actual regional data, making the theory less practicable. The concept of the repeated and extended nature of actual addiction cycles is also simplified in many models to one transition relapse.

Such constraints indicate a number of significant gaps in research. Network-based interaction, spatial heterogeneity, socio-economic layering, and gender-specific patterns should be involved in future working. Further modelling is also necessitated by realistic longitudinal behavior of relapses, multi-drug interactions, and mental-health connections. Predictive accuracy would be improved by stochastic radiation of improved parameter estimation by using machine learning, data assimilation, or stochastic techniques. The inclusion of real-world datasets in such frameworks will be extremely beneficial to improve their policy applicability.

Despite the limitations, the models reviewed present useful policy insights. They emphasize the fact that addiction is not only a medical problem but also a socially transmitted practice. The cycle can be broken at an early stage before it gets established by applying interventions aimed at adolescents, students, and young adults. The use of mathematical models is a cheap method of testing prevention and treatment measures prior to the intervention, and it assists the decision-maker in pinpointing areas of leverage to be effective in controlling. Most of the research points towards a single policy strategy, namely, social transmission by means of raising awareness and behavioral controls, rehabilitation services, and long-term follow-up must be the central activities in the reduction of addiction.

To sum up, the literature of analyzed studies indicates the high possibilities of compartmental and optimal-control models in reflecting the multifaceted processes of drug abuse. Although there are still difficulties in data, behavioral realism, and model structure, these methods continue to bring out the same message: prevention, available rehabilitation, and long-term support are the most correct ways of controlling and ultimately reducing drug addiction. Further development will demand models with more complex data and socio-economics and behavioral dynamics that are more realistic, making mathematical modeling a more effective instrument of health policy and societal health.

References

1. Lucas Matonya F, Kuznetsov D (2019) Mathematical Modelling on the Effects of Drug Abuse to the Societies. Annals of Pure and Applied Mathematics 19: 21-35.
2. Alfiniyah C, Puspitasari A, Fatmawati (2023) Mathematical Modelling of Drug Abuse Reduction Strategies taking into account the Treatment Type and Risks Level. Jambura Journal of Biomathematics 4: 23-30.
3. Yaseen L, Zanib SA, Abbas N, Shatanawi W (2025) Modeling the Spread and Control of Drug Addiction: A Mathematical and Sensitivity Analysis Approach. European Journal of Pure and Applied Mathematics 18: 6711.
4. Mamo DK, Kinyanjui MN, Teklu SW, Hailu GK (2024) Mathematical modeling and analysis of the co-dynamics of crime and drug abuse. Scientific Reports 14.
5. Nantomah KK, Seidu B, Bornaa CS (2022) A Mathematical Model of Drug-Crime Dynamics. In Asian J. Math. Appl https://scienceasia.asia/files/2022-14.pdf.
6. Musyoka AM, Ronoh M, Wanjau PM, Kitavi DM (2023) Mathematical Modeling of Drug Abuse, Unemployment and Mental Stress on Population Dynamics of Mental Illness. Communications in Mathematical Biology and Neuroscience 66.
7. Okoro SI, Fatokun JO, Abegye SY, Eyabi GT, Fenstermacher T (n.d.) (2023) Mathematical Model and Analysis of Drug Addiction among Adolescent's Populace in Nigeria. Mathematical Theory and Modeling 13: 2.
8. Colegate S, Liu C (2022) Mathematical Modeling of Drug Use: The Dynamics of Monosubstance Dependence for Two Addictive Drugs. arXiv http://arxiv.org/abs/2211.07921.
9. Zakiyyah A, Bahri S (2025) Stability Analysis of Drug Abuse Transmission Dynamics. Jurnal Matematika UNAND 14: 103-116.
10. Olanrewaju Ibrahim M, Olusoji A, Babatunde Akande K, Hafizat Ozichu M, Kabir Adewale G, et al. (2020). A Deterministic Mathematical Model in Understanding Drug-Abuse Related Crimes in Contemporary Society for Prevention. In Abacus Mathematics Science Series 47: 1.
11. Divya G, Athithan S, Jan R (2024) Modeling and stability analysis of substance abuse in women with control policies. Partial Differential Equations in Applied Mathematics 9.
12. Memarbashi R, Ebadi Z (2022) An epidemic model for drug addiction. Computational Methods for Differential Equations 10: 876-893.
13. Andrawus J (2024) Journal of Advanced Science & Optimization Research. www.ssaapublications.com
14. Solomon Isa R, Emmanuel S, Tanko Danat N, Sidi Abubakar S, Samuel Hwere T, et al. (2024) Mathematical Modeling of Illicit Drug Use Dynamics: Examining the Impact of Recycling Recovered Individuals into the Population. In Applied Mathematics and Computational Intelligence 13: 2.
15. Ifeoma E, And O, Stanley O (2025) Optimal Control Strategies for Reducing Drug Abuse in Children Using Differential Equations. Global scientific Journals 13: 9.
16. Cole S, Olive MF, Wirkus S (2024) The dynamics of heroin and illicit opioid use disorder, casual use, treatment, and recovery: A mathematical modeling analysis. Mathematical Biosciences and Engineering 21: 3165-3206.
17. Islam Md AI, Biswas Md HA (2020) Optimal Control Strategy Applied to Dynamic Model of Drug Abuse Incident for Reducing Its Adverse Effects. Medrxiv https://www.medrxiv.org/content/10.1101/2020.05.02.20088468v1.
18. Binuyo AO, Osuntokun OT (2021) Mathematical Modelling of the Addiction of Drug Substances among Students in Tertiary Institutions in Nigeria. Journal of Biomedical Research & Environmental Sciences 2: 851-856.
19. Matonya FL, Kuznetsov D (2021) Mathematical Modelling of Drug Abuse and it's Effect in the Society. Annals of Pure and Applied Mathematics 24: 159-176.
20. Muketo CK, Kimathi G, Wainaina M (2021) Mathematical Modeling of Drug Abuse as an Infectious Disease in Secondary Schools Incorporating Guidance and Counseling. International Journal of Mathematics Trends and Technology 67: 11-23.
21. Akanni JO, Adediipo DA, Kehinde OO, Ayanrinola OW, Adeyemo OA (2022) Mathematical Modelling of the Co-dynamics of Illicit Drug use and Terrorism. Information

Sciences Letters, 11: 559-572.
22. Magfirah DR, Sanusi W, Side S, Syam R, Pandjajangi AM (2023) SEARC Mathematical Modeling of Drug Addiction in South Sulawesi Province. E3S Web of Conferences 400.
23. Chinnadurai K, Athithan S (2023) Poverty and the Effects of Drug Addiction in a Deterministic and Stochastic Model. Mathematical Modelling of Engineering Problems 10: 352-359.
24. Gogovi GK, Qin L (2024). Analyzing Campus Drug Abuse Dynamics Through an Infectious Disease Framework. Journal of Applied Mathematics 1.
25. Dotia AK, Ibrahim MO, Gambo A, Musa AB, Lawani AO (2024) Mathematical Analysis of Substance Abuse and its Effect. International Journal of Development Mathematics 1: 179-190.
26. Alharbi AH, Alzahrani MSJ, Jday F, Alsehaimi A (2025). Mathematical analysis for a dynamic model of illicit drug consumption. AIMS Mathematics 10: 14784-14803.
27. Muli FM (2025) Mathematical Analysis of Drug and Substance Abuse Model with Treatment and Policing. Journal of Applied Mathematics 1.
28. Kamal M, Ahmed M, Sarkar MAH (2025) Optimal control strategies for mitigating drug addiction: A mathematical modeling approach. Results in Control and Optimization 19.
29. Gautama IPW, Utari IAPA, Widiastuti RS, Arimbawa KIP (2025) Modelling the Nonlinear Incidence Rate of Drug Addiction: A Mathematical Approach. International Journal of Mathematics and Computer Research 13.
30. Ifeoma OE, Omokoh ES (2025) Mathematical Modeling of the Effects of Hard Drug Abuse on Young Girls Aged 15-20 Years. Asian Journal of Pure and Applied Mathematics 7: 621-630.