

Review Article

Open Access

Pulsed Weak-Value Tomography of Gauge-Invariant Electron Flow in an Aharonov--Bohm Interferometer: Protocol, Weak-Regime Certification, and Roadmap to Quantum Backflow

Ndenga Lumbu Barack

Independent Researcher, Kinshasa, Democratic Republic of the Congo

ABSTRACT

I develop a fully operational proposal to reconstruct (tomographically) a gauge-invariant electron-flow field in a coherent Aharonov--Bohm (AB) interferometer using pulsed weak measurements and postselected weak values. The protocol targets a local kinetic-momentum density operator, ensuring gauge consistency in the presence of AB phases. A time-gated (pulsed) von Neumann interaction couples this local flow observable to a spin pointer, enabling extraction of the real part of the postselected weak value from conditional pointer shifts. From these data I reconstruct a drain-conditioned local velocity field and define experimentally testable flow lines as its integral curves. A central technical point is that time gating avoids a known failure mode of static interaction regions, where momentum sensitivity may be reduced by time-of-flight cancellation. I provide an internal certification strategy for the weak regime (linear signal scaling in coupling strength and quadratic visibility loss), a falsifiability program based on AB flux periodicity and drain dependence, and a detailed "minimum viable experiment" (MVE) that does not require spatial scanning. Finally, I outline an experimental roadmap from calibrated AB-ring flow tomography to a decisive nonclassical target in electronic quantum optics: electron quantum backflow, i.e., negative local probability current despite preparation with support restricted to positive kinetic momenta.

***Corresponding author**

Ndenga Lumbu Barack, Independent Researcher, Kinshasa, Democratic Republic of the Congo.

Received: September 10, 2026; **Accepted:** September 18, 2026; **Published:** September 26, 2026

Keywords: Weak Measurement, Weak Values, Electron Interferometry, Aharonov--Bohm Effect, Gauge Invariance, Kinetic Momentum, Probability Current, Postselection, Pulsed (Time-Gated) Von Neumann Coupling, Mesoscopic Quantum Transport, Electronic Quantum Optics, Quantum Backflow

Introduction

Quantum interference is often interpreted as limiting the meaning of trajectories or local motion. Independently of interpretation, standard quantum mechanics provides a conserved probability current, which defines a flow field for the quantum probability density. The practical difficulty is that direct (strong) attempts to determine local momentum flow or which-path information typically destroy interference. Weak measurement addresses this by extracting limited information per trial while keeping the disturbance arbitrarily small in the weak-coupling limit [1-2].

Weak-value reconstructions of flow-like quantities have been demonstrated in optical platforms and connected to the probability current and Bohmian guidance structure [3,4]. Extending such reconstructions to electrons is attractive both for foundations and for quantum transport. However, electrons in AB geometries raise two additional non-negotiable requirements:

(i) the reconstructed quantity must be *gauge consistent* in the presence of electromagnetic potentials [5].

(ii) the measurement coupling must be *faithful* to local momentum flow. In particular, fixed-length static interaction regions can become effectively momentum-insensitive due to time-of-flight cancellation (Sec.~\ref{sec:pulsed}).

This paper provides an "ultra-complete" protocol-level proposal: I specify the gauge-invariant target observable, the postselected weak-value reconstruction, a pulsed spin-pointer meter model, weak-regime certification tests, a minimum viable experiment, and an extension strategy toward quantum backflow. This text consolidates my earlier Zenodo releases \cite{NdengaV1,NdengaV2} into a single coherent V3-level document.

Gauge-Invariant Flow and a Local Flow Operator**Sec: Gauge**

Consider coherent single-mode propagation along an arc-length coordinate s in an AB ring threaded by magnetic flux Φ and described by a vector potential $A_s(s)$. Define the kinetic (gauge-invariant) momentum operator

$$\hat{\pi}_s \equiv \hat{p}_s - eA_s(\hat{s}).$$

\label{eq:pi}

For an orbital wavefunction $\psi(s,t)$, the standard gauge-invariant density and probability current are [5].

$$\rho(s,t) = |\psi(s,t)|^2, \quad j(s,t) = \frac{1}{m} \text{Re} \left[\psi^*(s,t) \hat{\pi}_s \psi(s,t) \right].$$

\label{eq:current}

When $\rho(s,t) \neq 0$, define the local velocity field

$$v(s,t) = \frac{j(s,t)}{\rho(s,t)}.$$

\label{eq:vfield}

$\end{equation}$

The integral curves of $\mathbf{v}(\mathbf{s}, t)$ are streamlines (flow lines) of the probability current field.

To probe flow locally around position $\mathbf{s}_0$ (segment width Δs), introduce a segment projector $\hat{\Pi}(\mathbf{s}_0)$ and define the Hermitian local kinetic-momentum density operator

$$\hat{O}(\mathbf{s}_0) = \frac{1}{2} \left(\hat{\Pi}(\mathbf{s}_0) \hat{\Pi}(\mathbf{s}_0) + \hat{\Pi}(\mathbf{s}_0) \hat{\Pi}(\mathbf{s}_0) \right)$$

$\text{\label{eq:O}}$

This is the minimal gauge-consistent local flow observable used throughout this work.

Weak Values, Postselection, and Conditional Flow Reconstruction Sec: Weakvalues

Let $|\psi(t)\rangle$ be the forward-evolving (preselected) state injected into the AB ring. Postselection is performed by conditioning on the detected output drain $\alpha \in \{D1, D2\}$. In the standard weak-value framework the postselected weak value of an observable $\hat{X}$ is [1,2]

$$X_w^{(\alpha)}(t) = \frac{\langle \phi_{\alpha}(t) | \hat{X} | \psi(t) \rangle}{\langle \phi_{\alpha}(t) | \psi(t) \rangle},$$

$\text{\label{eq:weakvalue_general}}$

where $|\phi_{\alpha}(t)\rangle$ is the backward-evolving postselected state associated with drain α .

For spatially local flow, take $\hat{X} = \hat{O}(\mathbf{s}_0)$ and also define the weak value of the segment projector $\hat{\Pi}(\mathbf{s}_0)$:

$$O_w^{(\alpha)}(\mathbf{s}_0, t) = \frac{\langle \phi_{\alpha}(t) | \hat{O}(\mathbf{s}_0) | \psi(t) \rangle}{\langle \phi_{\alpha}(t) | \psi(t) \rangle},$$

$$\Pi_w^{(\alpha)}(\mathbf{s}_0, t) = \frac{\langle \phi_{\alpha}(t) | \hat{\Pi}(\mathbf{s}_0) | \psi(t) \rangle}{\langle \phi_{\alpha}(t) | \psi(t) \rangle}.$$

$\text{\label{eq:Ow_Piw}}$

I define the reconstructed $\text{\emph{drain-conditioned}}$ velocity field

$$\mathbf{v}_{\alpha}(\mathbf{s}_0, t) = \frac{1}{m} \frac{\langle \text{Re} | O_w^{(\alpha)}(\mathbf{s}_0, t) | \text{Re} \rangle}{\langle \text{Re} | \Pi_w^{(\alpha)}(\mathbf{s}_0, t) | \text{Re} \rangle}.$$

$\text{\label{eq:valpha}}$

Flow lines for the postselected ensemble are then defined by integrating

$$\dot{\mathbf{s}} = \mathbf{v}_{\alpha}(\mathbf{s}, t).$$

$\text{\label{eq:stream}}$

Operationally, Eqs.~(\ref{eq:Ow_Piw})--(\ref{eq:stream}) define what is reconstructed from experimental data: a conditional flow field. This does not constitute a strong which-path record.

Pulsed Spin-Pointer Meter (Time-Gated Von Neumann Coupling) Sec: Pulsed

A standard von Neumann weak measurement couples a system observable to a meter pointer via a short interaction. Here the meter is taken to be the electron spin (pointer), and the system observable is the local flow operator $\hat{O}(\mathbf{s}_0)$.

Time-Gated Interaction Hamiltonian

I propose a time-gated (pulsed) coupling

$$\hat{H}_y(t) = \lambda f(t-t_0) \hat{O}(\mathbf{s}_0) \otimes \hat{\sigma}_y,$$

$$\text{\quad\quad\quad} \int dt, f(t) = 1,$$

$\text{\label{eq:Hint}}$

$\end{equation}$

with small measurement strength λ and pulse center t_0 chosen when the wave packet overlaps the metered segment. The crucial feature is that the interaction window is set externally by $f(t-t_0)$.

Why Pulsed Coupling Matters (Time-Of-Flight Cancellation)

A static interaction region of fixed length L can become an unreliable momentum-flow meter because electrons with different velocities spend different times in the region. In simplified terms, a transit time $T \sim L/v$ can compensate factors of v in the interaction, suppressing momentum dependence when averaged over realistic wave packets. A time-gated pulse avoids this by fixing the interaction time independently of L/v . Appendix~\ref{app:tof} summarizes the cancellation logic in a compact form.

Pointer Readout and Weak-Value Signal

Prepare the pointer spin in $|+\rangle$ and postselect on drain α . In the weak-coupling limit, the conditional pointer shift obeys the standard weak-measurement relation [1,2]

$$\langle \sigma_z \rangle_{\alpha, s_0} \simeq \frac{2\lambda}{\hbar} \langle \text{Re} | O_w^{(\alpha)}(\mathbf{s}_0, t_0) | \text{Re} \rangle,$$

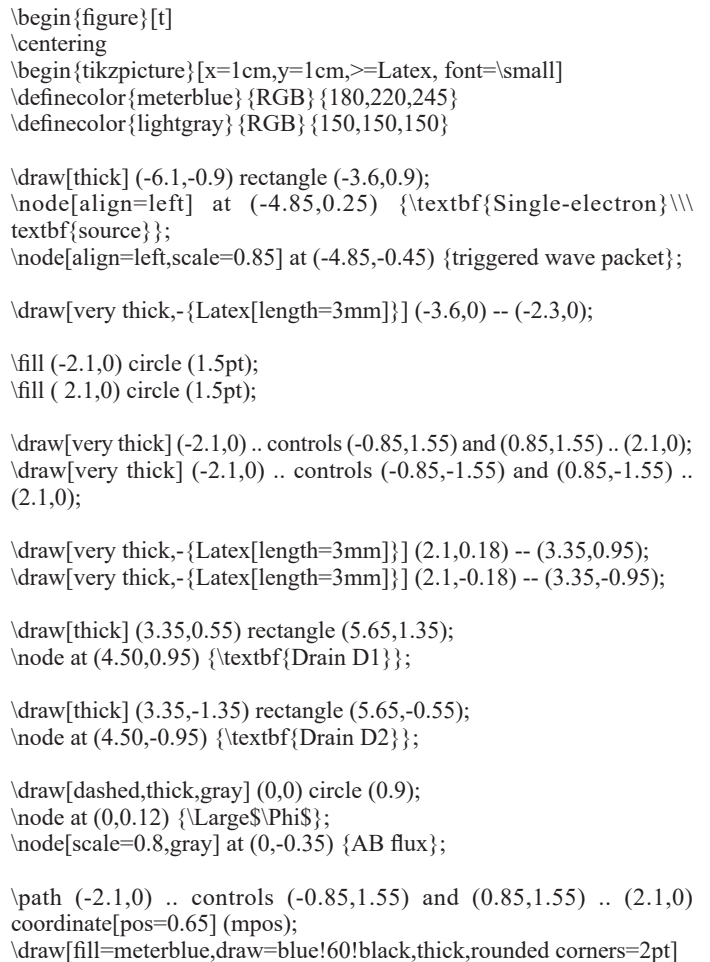
$\text{\label{eq:pointer}}$

providing direct access to $\langle \text{Re} | O_w^{(\alpha)} | \text{Re} \rangle$ and thus to v_{α} via Eq.~(\ref{eq:valpha}). A complementary pointer basis can access $\langle \text{Im} | O_w | \text{Im} \rangle$, useful for systematic checks.

Tomographic Protocol in an AB Ring

Sec: Protocol

Figure~\ref{fig:device} sketches the proposal: a single-electron source injects a wave packet into an AB ring; a local segment implements the pulsed coupling; postselection is performed by conditioning on the output drain; and spin readout yields the weak-value signal.



```

($mpos)+(-0.60,0.18)$) rectangle ($mpos)+(0.60,-0.30)$);
\node[scale=0.78] at ($mpos)+(0.0,-0.06)$) {\textbf{pulsed meter}};
\node[scale=0.75] at ($mpos)+(0.0,0.25)$) {\$s_0$};

\draw[blue!60!black,thick] ($mpos)+(0.95,0.05)$) -- ++(0.22,0) --
++(0,0.22) -- ++(0.22,0) -- ++(0,-0.22) -- ++(0.22,0);

\node[scale=0.72,blue!60!black] at ($mpos)+(1.55,0.05)$) {\$t_0$};

\draw[thick] (1.6,-2.45) rectangle (5.65,-1.55);
\node[anchor=west] at (1.75,-1.85) {\textbf{Postselection + spin
readout}};
\node[anchor=west,scale=0.82] at (1.75,-2.20) {\textbf{condition on D1/D2;
measure $\angle\sigma_z$}};

\draw[->,thick] (4.90,-2.25) -- (5.35,-2.25) node[anchor=west,scale=0.8]
{\$x$};
\draw[->,thick] (4.90,-2.25) -- (4.90,-1.75) node[anchor=south,scale=0.8]
{\$z$};
\node[scale=0.75] at (4.50,-2.33) {\$|x\rangle$};

\draw[dashed,thick,lightgray,-{Latex[length=2.5mm]}] (4.50,0.55) ..
controls (4.10,-0.4) and (3.90,-1.1) .. (3.25,-1.55);
\draw[dashed,thick,lightgray,-{Latex[length=2.5mm]}] (4.50,-0.55) ..
controls (4.20,-1.0) and (4.05,-1.3) .. (3.70,-1.55);
\end{tikzpicture}
\caption{Device-level schematic of the proposed protocol (illustrative).
A time-gated (pulsed) weak coupling measures a local gauge-invariant
flow operator, and drain postselection defines conditioned subensembles.}
\label{fig:device}
\end{figure}

```

Measurement Sequence

A practical sequence (conceptual level) is:

- (i) inject a single electron wave packet with prepared spin;
- (ii) apply the pulse at time t_0 when the packet overlaps the metered segment;
- (iii) detect which drain clicks (postselection);
- (iv) read out the pointer spin (or, equivalently, perform spin-resolved detection);
- (v) repeat over an ensemble to estimate $\angle\sigma_z$ and hence $\text{Re}\langle O_w^{(\alpha)} \rangle$.

Spatial tomography follows by repeating the experiment for multiple s_0 (multiple gates or reconfigurable gating). Time-resolved tomography can be obtained by shifting t_0 .

Internal Weak-Regime Certification and Systematic Errors

Sec: Certification

Any high-level claim requires self-auditing criteria. This proposal includes two non-negotiable checks:

Linear Signal VS Coupling

Equation~(\ref{eq:pointer}) implies $\angle\sigma_z \propto \lambda$ in the weak limit.

Quadratic Disturbance (Visibility Loss)

The coupling entangles spin and orbital degrees of freedom. Coherence loss is second order in λ ; a generic scaling is

```

\begin{equation}
\Delta V \sim \mathcal{O}\left(\frac{\lambda^2}{\hbar^2} \Delta O^2\right),
\label{eq:visloss}
\end{equation}

```

with ΔO the spread of $\hat{O}(s_0)$ over the wave packet. The weak regime is certified by demonstrating $\Delta V \propto \lambda^2$ and convergence of reconstructed v_α as $\lambda \rightarrow 0$.

AB Falsifiability

Flux Provides an Independent Falsifiability Axis

- (i) the reconstruction must be periodic in Φ_0 with $\Phi_0 = h/e$;
 - (ii) drain-conditioned reconstructions generally differ: $v_{D1} \neq v_{D2}$.
- Gauge-inconsistent choices (e.g., ignoring kinetic momentum) will not reproduce correct flux dependence [5].

Error Budget Template

Table~\ref{tab:errorbudget} lists an explicit reporting checklist.

```

\begin{table}[t]
\caption{Error budget template (to report in an experimental realization).}
\label{tab:errorbudget}
\begin{ruledtabular}
\begin{tabular}{p{0.34\textwidth} p{0.58\textwidth}}
Error source & Impact / mitigation \\
\hline
Pulse timing jitter ( $t_0$ , width) & Blurs locality in time; characterize
with independent timing calibration. \\
Finite segment width  $\Delta s$  & Spatial averaging of flow; reduce with
smaller gates; model as convolution. \\
Orbital dephasing (baseline visibility) & Limits maximum recoverable
contrast; measure visibility independently vs flux. \\
Spin dephasing/relaxation & Reduces pointer contrast; bound with control
experiments and short readout windows. \\
Calibration of  $\lambda$  & Affects absolute scaling of  $O_w$ ; calibrate
in a known reference configuration. \\
Drain postselection fidelity & Misclassification biases conditioned
averages; quantify leakage / cross-talk. \\
Statistical error (finite trials) & Standard error  $\sim 1/\sqrt{N}$ ; report
 $N$  and confidence intervals. \\
\end{tabular}
\end{ruledtabular}
\end{table}

```

Illustrative Signal Maps and Flow Lines (Schematic Figures)

Sec: Figs

Figures in this paper are illustrative/schematic; they show what data products look like (pointer maps, streamlines, scaling), not claimed experimental results.

```

\begin{figure}[t]
\centering
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=6.0cm,
xlabel={meter position along ring,  $s_0$  (arb.)},
ylabel={conditional pointer shift  $\angle\sigma_z$  (arb.)},
xmin=0, xmax=10,
ymin=-0.10, ymax=0.20,
grid=both,
grid style={line width=.1pt, draw=gray!25},
major grid style={line width=.2pt, draw=gray!35},
legend style={at={(0.98,0.98)}, anchor=north east, draw=none, fill=white},
ticklabel style={font=\small},
label style={font=\small},
legend cell align=left
]
\addplot[blue!75!black, thick, domain=0:10, samples=300]
{0.05 + 0.05*\sin(deg(0.9*x)) + 0.03*\cos(deg(1.7*x+15))};
\addlegendentry{\Phi=0}
\addplot[red!75!black, thick, domain=0:10, samples=300]
{0.06 + 0.06*\sin(deg(0.9*x+30)) + 0.02*\cos(deg(1.7*x+25))};
\addlegendentry{\Phi=\Phi_0/4}
\addplot[green!60!black, thick, domain=0:10, samples=300]
{0.00 + 0.07*\sin(deg(0.9*x+90)) + 0.03*\cos(deg(1.7*x+40))};
\addlegendentry{\Phi=\Phi_0/2}
\node[anchor=south west, font=\small] at (axis cs:0.2,-0.095) {postselected
on D1 (illustrative)};
\end{axis}
\end{tikzpicture}
\end{figure}

```

```
\end{axis}
\end{tikzpicture}
\caption{Illustrative flux-dependent conditional pointer signal versus meter position. In an experiment, these curves are estimated from postselected pointer statistics.}
\label{fig:pointer}
\end{figure}

\begin{figure}[t]
\centering
\begin{minipage}{0.49\textwidth}
\centering
\begin{tikzpicture}[x=1cm,y=1cm,>=Latex, font=\small]
\draw[very thick] (-2.2,0) -- (-1.4,0);
\fill (-1.4,0) circle (1.5pt);
\fill (1.4,0) circle (1.5pt);
\draw[very thick] (1.4,0) -- (2.2,0);
\draw[very thick] (-1.4,0) .. controls (-0.7,1.2) and (0.7,1.2) .. (1.4,0);
\draw[very thick] (-1.4,0) .. controls (-0.7,-1.2) and (0.7,-1.2) .. (1.4,0);

\foreach \y in {-0.65,-0.35,0.035,0.65} {
\draw[blue!70!black,thick,->] (-2.0,0) .. controls (-1.0,0.25+\y) and (0.4,0.25+\y) .. (1.8,0.05);
}
\node[anchor=west] at (-2.2,1.45) {\textbf{Postselect }$D1$};
\node[anchor=west] at (-2.2,1.18) {\textbf{$\Phi=0$ (schematic)}};
\end{tikzpicture}
\end{minipage}\hfill
\begin{minipage}{0.49\textwidth}
\centering
\begin{tikzpicture}[x=1cm,y=1cm,>=Latex, font=\small]
\draw[very thick] (-2.2,0) -- (-1.4,0);
\fill (-1.4,0) circle (1.5pt);
\fill (1.4,0) circle (1.5pt);
\draw[very thick] (1.4,0) -- (2.2,0);
\draw[very thick] (-1.4,0) .. controls (-0.7,1.2) and (0.7,1.2) .. (1.4,0);
\draw[very thick] (-1.4,0) .. controls (-0.7,-1.2) and (0.7,-1.2) .. (1.4,0);

\foreach \y in {-0.2,0.05,0.25,0.45,0.65} {
\draw[red!75!black,thick,->] (-2.0,0) .. controls (-1.0,0.35+\y) and (0.4,0.35+\y) .. (1.8,0.05);
}
\foreach \y in {-0.65,-0.5} {
\draw[red!75!black,thick,->] (-2.0,0) .. controls (-1.0,0.15+\y) and (0.4,0.15+\y) .. (1.8,0.05);
}

\node[anchor=west] at (-2.2,1.45) {\textbf{Postselect }$D1$};
\node[anchor=west] at (-2.2,1.18) {\textbf{$\Phi=\Phi_0/2$ (schematic)}};
\end{tikzpicture}
\end{minipage}
\caption{Schematic drain-conditioned flow lines obtained by integrating $\dot{s}=v_\alpha(s,t)$}
\label{fig:flowlines}
\end{figure}
```

```
\begin{figure}[t]
\centering
\begin{minipage}{0.49\textwidth}
\centering
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=5.2cm,
xlabel={\lambda$ (arb.)},
ylabel={\angle\sigma_z$ (arb.)},
xmin=0, xmax=1.0,
ymin=0, ymax=1.1,
grid=both,
grid style={line width=.1pt, draw=gray!25},
major grid style={line width=.2pt,draw=gray!35},
```

```
ticklabel style={font=\small},
label style={font=\small},
title={(a) Signal $\propto \lambda$},
title style={font=\small}
]
\addplot[only marks, mark=*, mark size=1.8pt, blue!75!black]
coordinates {(0.0,0.02) (0.2,0.22) (0.4,0.42) (0.6,0.61) (0.8,0.82) (1.0,1.02)};
\addplot[blue!75!black, thick, domain=0:1.0] {1.0*x+0.02};
\end{axis}
\end{tikzpicture}
\end{minipage}\hfill
\begin{minipage}{0.49\textwidth}
\centering
\begin{tikzpicture}
\begin{axis}[
width=\textwidth,
height=5.2cm,
xlabel={\lambda$ (arb.)},
ylabel={\mathcal{V}$ (arb.)},
xmin=0, xmax=1.0,
ymin=0.5, ymax=1.05,
grid=both,
grid style={line width=.1pt, draw=gray!25},
major grid style={line width=.2pt,draw=gray!35},
ticklabel style={font=\small},
label style={font=\small},
title={(b) Loss $\propto \lambda^2$},
title style={font=\small}
]
\addplot[only marks, mark=*, mark size=1.8pt, red!75!black]
coordinates {(0.0,1.00) (0.2,0.98) (0.4,0.92) (0.6,0.82) (0.8,0.68) (1.0,0.52)};
\addplot[red!75!black, thick, domain=0:1.0] {1 - 0.48*x^2};
\end{axis}
\end{tikzpicture}
\end{minipage}
\caption{Internal weak-regime certification strategy (illustrative): linear pointer response and quadratic visibility loss.}
\label{fig:scaling}
\end{figure}
```

Minimum Viable Experiment (MVE)

Sec: Mve

A full spatial scan over s_0 is not required to establish the core claim: that a gauge-consistent weak-value access to local flow is experimentally available.

Minimum Viable Experiment (MVE).

A minimal demonstration requires a single metering segment at a fixed location s_0 (no spatial scan). A triggered single-electron wave packet is injected into an AB ring with two output drains. A time-gated weak interaction is applied when the packet overlaps the segment, and the pointer readout is recorded conditioned on detection at drain $D1$ or $D2$. Success criteria are:

- a conditional pointer shift scaling linearly with coupling strength λ ;
 - an independently measured interference visibility whose reduction scales quadratically with λ ;
 - a flux-dependent conditional signal periodic with $\Phi_0 = h/e$ and differing between drain postselections.
- Meeting (i)–(iii) establishes an experimentally certified weak-value access to local flow. Spatial tomography follows by repeating the same measurement at multiple s_0 (multiple gates or reconfigurable gating).

Roadmap to Electron Quantum Backflow

Sec: Backflow

After calibrating the meter and reconstruction pipeline in the AB ring, the same pulsed weak-value readout can be deployed in a straight single-mode channel to target quantum backflow negative local probability current despite preparation with momentum-space support restricted to

positive kinetic momenta [6]

Operationally, a credible backflow program requires:

- (i) a preparation protocol intended to enforce a positive-momentum support constraint (within experimental resolution);
- (ii) an independent diagnostic/verification of that constraint;
- (iii) reconstruction of the local current via the reconstructed velocity field and density, and identification of spacetime regions where $\langle j \rangle < 0$ under the verified constraint.

Backflow is a decisive target because it is a sign test under a preparation constraint rather than a qualitative path narrative.

Interpretation and Scope

This work is operational: it defines how a gauge-invariant local flow observable can be accessed via postselected weak values and how to certify the weak regime. Interpretations (e.g., Bohmian mechanics) may identify reconstructed flow lines with ontic trajectories under additional assumptions but the empirical protocol itself does not depend on adopting a particular ontology [4,7].

Conclusion

I have presented a detailed protocol for pulsed weak-value tomography of gauge-invariant electron flow in an AB interferometer. The core ingredients are a kinetic-momentum density target observable, a time-gated von Neumann coupling to a spin pointer, and output-drain postselection, yielding a drain-conditioned velocity field whose integral curves define flow lines. The pulsed interaction avoids a key failure mode of static regions (time-of-flight cancellation) and enables internal weak-regime certification through coupling-strength scaling and visibility loss. I have further specified a minimum viable experiment and an extension roadmap to electron quantum backflow.

The code used to generate the figures in this manuscript is available at: [url{https://github.com/barackEinstein/ndenga-flow-tomography}](https://github.com/barackEinstein/ndenga-flow-tomography)

Appendix

Weak-Measurement Pointer Shift (Derivation)

App: Pointer

For the pulsed interaction

$$\hat{H}_{\text{int}}(t) = \lambda f(t-t_0) \hat{O} \otimes \hat{\sigma}_y, \quad \int dt f(t) = 1,$$

the unitary is

$$\hat{U} = \exp\left(-i \frac{\lambda}{\hbar} \hat{O} \otimes \hat{\sigma}_y\right) \simeq 1 - i \frac{\lambda}{\hbar} \hat{O} \otimes \hat{\sigma}_y + \mathcal{O}(\lambda^2).$$

With preselection $|\psi\rangle$, postselection $|\phi_\alpha\rangle$, and pointer state $|x\rangle$, the postselected pointer state is proportional to

$$\langle \phi_\alpha | \hat{U} | \psi \rangle |x\rangle \simeq \langle \phi_\alpha | \psi \rangle \left(1 - i \frac{\lambda}{\hbar} \langle \sigma_y \rangle_{\psi} \right) |x\rangle,$$

with weak value $\langle \sigma_y \rangle_{\psi}^{\phi_\alpha} = \langle \phi_\alpha | \hat{\sigma}_y | \psi \rangle / \langle \phi_\alpha | \psi \rangle$.

Expanding to first order yields $\langle \sigma_z \rangle_{\psi}^{\phi_\alpha} \simeq (2\lambda/\hbar) \text{Re}\langle \sigma_y \rangle_{\psi}^{\phi_\alpha}$, consistent with Eq. (1) [1,2,8,9].

Time-Of-Flight Cancellation (Static Region Intuition)

App: ToF

In a static region of length L , a particle with velocity v spends time $T \sim L/v$. If an effective pointer rotation accumulates as $\int_0^T dt g(t)$ (text{momentum-like}) then the dependence on v (or momentum) can be partially compensated by $T \propto 1/v$ under

realistic averaging. A time-gated pulse fixes the interaction time externally, preventing this cancellation mechanism and preserving sensitivity to the targeted observable in the weak limit.

References

1. Aharonov Y, Albert DZ, Vaidman L (1988) How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. Physical Review Letters <https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.60.1351>.
2. Dressel J, Malik M, Miatto FM, Jordan AN, Boyd RW (2014) Colloquium: Understanding quantum weak values: Basics and applications. Reviews of Modern Physics 86: 307.
3. Kocsis S, Braverman B, Ravets S, Stevens MJ, Mirin RP (2011) Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer. Science 332: 1170-1173.
4. Wiseman HM (2007) Grounding Bohmian mechanics in weak values and Bayesianism. New Journal of Physics 9: 165.
5. Aharonov Y, Bohm D (1959) Significance of electromagnetic potentials in the quantum theory. Physical Review 115: 485.
6. Bracken AJ, Mello GF (1994) Probability backflow and a new dimensionless quantum number. Journal of Physics A: Mathematical and General <https://iopscience.iop.org/article/10.1088/0305-4470/27/6/040>.
7. Bohm D (1952) A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables I. Physical Review 85: 166.
8. Barack N (2026) Pulsed Weak-Value Tomography of Gauge-Invariant Electron Flow in an Aharonov--Bohm Interferometer, with a Route to Quantum Backflow (Version V2). Zenodo <https://zenodo.org/records/18718203>.
9. Barack N (2026) Weak-Value Tomography of Electron Flow in an Aharonov--Bohm Interferometer (Version V1), Zenodo <https://zenodo.org/records/18621793>.

Copyright: ©2026 Ndenga Lumbu Barack. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.