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Nuclear Level Density in the X(1122, 330) System

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ABSTRACT

Single particle levels for the X(1122, 330) system as a function of energy are modeled using various functional forms. A power law form $N(E) = a E^b$ for the total number of energy levels provides a better fit than the constant temperature and equidistant models. The number of energy levels/MeV is not well-fit by the commonly used constant temperature model.

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Introduction

Level density data provide important information regarding the systematics of nuclear structure. These calculations have been performed over a range of mass values, and most are performed for nuclei with $A \geq 20$ [1]. These calculations have been extended to superheavy nuclei including the Z=204 A=610 system X(610, 204), X(636, 204), X(692, 214), X(730, 226), X(766, 242), X(888, 274), X(926, 282), and X(1062, 312) [2-15].

This paper extends those calculations to the X(1122, 330) system [16]. This system, and previous calculations in the ^{4}He system, represents endpoints for nuclear level density calculations [17]. As such, they provide additional information regarding the level density systematics in nuclear systems.

Calculational Methodology

Nuclear single particle energies are calculated from the Schrödinger Equation. For a spherically symmetric potential, single particle energies for a nuclear core plus nucleon, can be solved using the methodology of Lukasiak and Sobiczewski and Petrovich et. al. [18,19].

For a nucleon in the field of a nuclear core, the single particle binding energy E_{NLSJ} is derived from the solution of the radial Schrödinger Equation

$$[(h^2/2\mu)\,(d^2/dr^2 - L(L+1)/r^2) - E_{NLSJ} - V_{LSJ}(r)\,]\,U_{NLSJ}(r) = 0 \qquad (1)$$

In Eq. 1, the radial coordinate r defines the relative motion of the nuclear core and the particle, $V_{LSJ}(r)$ is the model interaction [20,21] and $U_{NLSJ}(r)$ is the radial wave function. L, S, and J are the orbital, spin, and total angular momentum quantum numbers, respectively, and N is the radial quantum number. Finally, μ is the reduced mass.

Specific details of the model and its solution are provided in Refs. 2, 4, 7, 10, 12, 14, and 16. The numerical methods of Refs. 22 and 23 are utilized to obtain a converged solution.

Single Particle Level Calculations

Figure 1 summarizes the results of the X(1122, 330) level calculations [16]. The X(1122, 330) system has an alpha (beta) decay half-life of 2.5 h (23 min) [16].

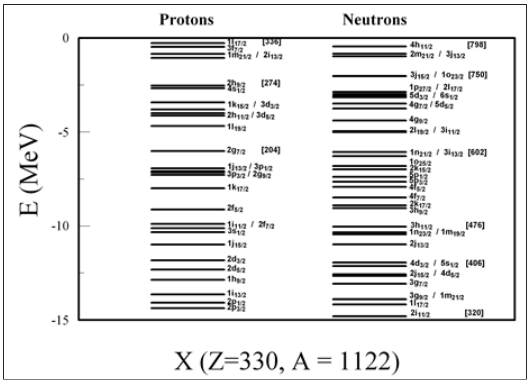

Figure 1: Calculated single particle energy levels for nucleus X(1122, 330). The notation A/B/C is used to indicate adjacent energy levels with level A more tightly bound than level B, which is more tightly bound than level C.

The level systematics summarized in Sections 4.0, 5.0, and 6.0 are based on Figure 1. These results are compared to the constant temperature model [24] in Section 4.0, power law in Section 5.0, and equidistant model [25-28] in Section 6.0. In addition to illustrating nuclear level systematics, level density calculations have also been applied to superheavy nuclei, nuclear astrophysical reaction rates, and machine learning [29-31].

Constant Temperature Model

The constant temperature model [24] is commonly utilized to evaluate nuclear level data and theoretical models, and it has the form

$$\rho(E) = a \exp(E/T) \quad (2)$$

In Eq. 2, ρ(E) is the number of energy levels per MeV. The terms a and T are constants derived from a numerical fit to the X(1122, 330) single particle levels summarized in Figure 1. Simple level density approximations may not reproduce the nuclear density that exhibits a unique single particle level structure. Accordingly, the systematics of level density parameters illuminates the unique differences between nuclei.

The level density ρ(E) functional form is expected to be a simple exponential form based on data from A = 36-66 even-even nuclei [24]. When the level scheme of Figure 1 is fit to Eq. (2), the values a = 1.9132061 and T = 1/0.114237337 = 8.754 are obtained. These values are similar to the fits to the X(610, 204) [3], X(636, 204) [5], X(692, 214)[6], X(730, 226) [8], X(766, 242) [9], X(888, 274) [11], X(929, 282) [13], and X(1062, 312) [15]. Figure 2 illustrates the fit of the constant temperature model to the model results for the X(1122 330) system.

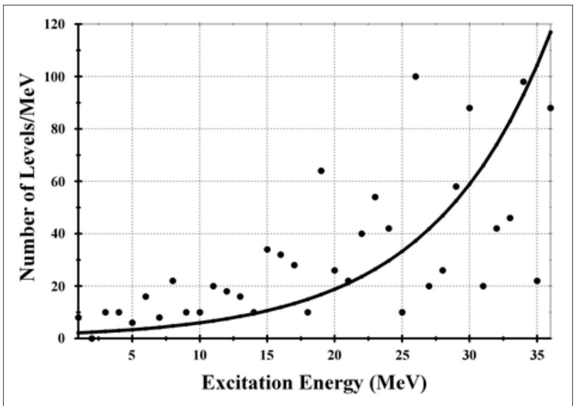

Figure 2: Energy level density for X(1122, 330) using the constant temperature model. The "•" symbol represents the state density for a 1 MeV energy bin. The solid curve is a fit to the constant temperature functional form ρ(E) = a exp(E/T).

In view of the limited number of energy levels, there is considerable variation between the model and constant temperature values of ρ(E). This variation is minimized by considering the total number of levels N(E). N(E) is also assumed to have the constant temperature model form

$$N(E) = c \exp(E/d) \quad (3)$$

The curve shown in Figure 3 results when the X(1122, 330) levels N(E) of Figure 1 are fit to the constant temperature model of Eq. (3). Fitting parameters c = 21.36547563 and d = 1/0.124900521 = 8.006 are utilized in Figure 3. The values of these parameters are similar to the results of Refs. 3. 5, 6, 8. 9, 11, 13, and 15.

The fit to the functional form of Eq. 3 to the X(1122, 330) system underestimates the number of energy levels below about 28 MeV, and overestimates N(E) above about 29 MeV. Table 1 provides a comparison of the X(1122, 330) system d value to lighter systems using the constant temperature model.

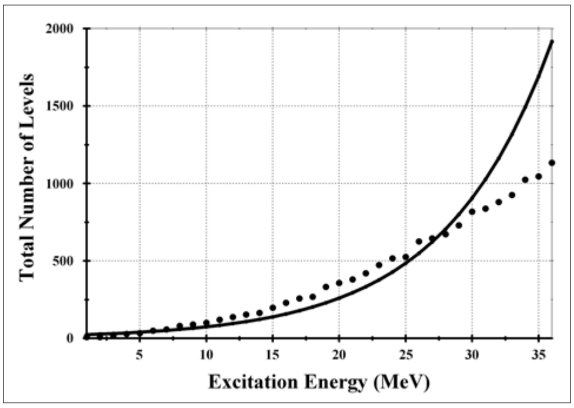

Figure 3: Total number of energy levels N(E) for X(1122, 330) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the constant temperature functional form N(E) = c exp(E/d).

Table 1: Constant Temperature Model Parameters for Nuclear Densities[a-j]

Nucleus	d (MeV)
^{4}He	2.79[a]
^{36}Ar	1.87
^{38}Ar	1.47
^{40}Ca	1.73
^{50}Cr	1.29
^{52}Cr	1.43
^{54}Cr	1.22
^{54}Fe	1.40
^{56}Fe	1.40
^{58}Fe	1.31
^{68}Zn	0.90
X(610, 204)	7.31[b]
X(636, 204)	7.15[c]
X(692, 214)	6.90[d]
X(730, 226)	7.26[e]
X(766, 242)	8.25[f]
X(888, 274)	7.65[g]
X(926, 282)	7.41[h]
X(1062, 312)	7.51[i]
X(1122, 330)	8.01[j]

[a]Ref. 17. [b]Ref. 3. [c]Ref. 5. [d]Ref. 6. [e]Ref. 8. [f]Ref. 9. [g]Ref. 11, [h]Ref, 13, and [i]Ref. 15. [i]This work. All others Ref. 24.

Table 1 summarizes the systematics of the constant temperature model parameter d. This parameter behaves differently in the A < 70 region compared to A > 600 systems. The A > 600 d values are about a factor of four larger than those in the A < 70 region.

Power Law Model

The total number of levels N(E) is also fit to the power law functional form

$$N(E) = a E^{b} \quad (4)$$

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where a = 3.28006914and b = 1.57380959. Figure 4 summarizes the use of these parameters in Eq. 4 to fit the X(1122, 330) energy levels of Fig, 1. The power law (Eq. 4) yields an improved fit compared to the constant temperature model (Eq. 3) for the total number of energy levels. The X(1122, 330) parameters are similar to the values derived from Refs. 3. 5, 6, 8. 9, 11, 13, and 15.

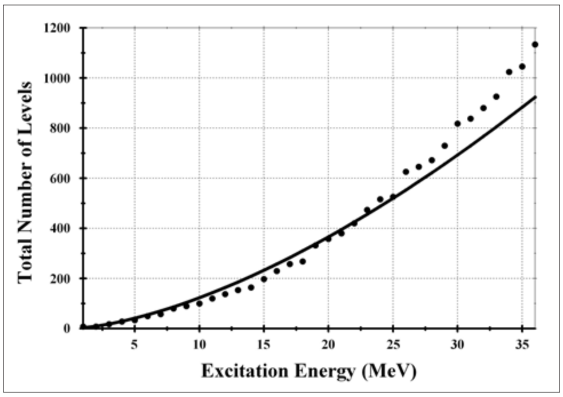

Figure 4: Total number of energy levels N(E) for X(1122, 330) as a function of energy. The "•" symbol represents the total number of energy levels up to energy E. The solid curve is a fit to the power law functional form N(E) = = a E^b.

Equidistant Model
Single particle energy levels are assumed to be equidistant and nondegenerate in the equidistant model [25-28]. The total number of energy levels for a system of neutrons and protons is given by

$$N(E) = (\pi)^{1/2} \exp (2 [a E]^{1/2}) / (12 E^{5/4} a^{1/4}) \quad (5)$$

where a is a level density parameter. For known nuclei, the parameter a has the value of about A/8 [28].

The X(1122, 330) energy levels are fit to the functional form of Eq. (5), and are illustrated in Figure 5. An a = 1.336 value is utilized in Eq. 5. This a value is similar to the X(610, 204) [3], X(636, 204) [5], X(692, 214) [6], X(730, 226) [8], X(766, 242) [9], X(788, 274) [11], X(926, 282) [13], and X(1062, 312) [15] parameters.

The reader should note that Eq. 5 is an asymptotic expression that should become more accurate as the atomic mass tends to infinity. This suggests that Eq. 5 should provide an improved description of the level density as the nucleus mass increases. However, the equidistant model of Eq. 5 does not reproduce the systematics of the level spectrum of Figure 1 as well as the power law model.

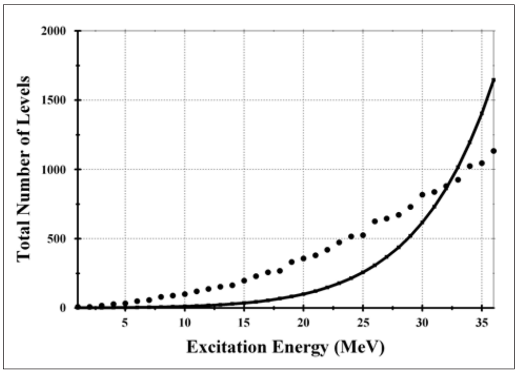

Figure 5: Total number of energy levels N(E) for X(1122, 330) as a function of energy using the equidistant model. The "•" symbol represents the total number of energy levels up to energy E.

Conclusions
The single particle levels for the X(1122, 330) system as a function of energy can be modeled using various functional forms. A power law form N(E) = a E^b for the total number of energy levels provides a better fit than the than the constant temperature and equidistant models. The number of energy levels/MeV is not well-fit by the commonly used constant temperature model.

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