

Revealing the Essence of Differential and Integral Calculus based on Scientific Philosophy

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ABSTRACT

As it is known in the way where the basic equations of mathematical analysis and mathematical physics were taken as a basis it was possible to come to the main results of the Cantor set theory. However, contradictions and paradoxes were inherent to the results obtained in this field. In the way where the basic equations of theoretical physics, as well as empirical and probabilistic physics were taken as a basis, it was possible to arrive at the basic results of quantum theory. The results obtained in this area proved to be very useful in describing the experimental data. This was a sign of their truth. Therefore, there is reason to come to the following conclusion. That taking as a basis the ideas and results obtained in the field of quantum theory, there is a possibility to come to the disclosure of the essence of differential and integral calculus. As well as the results obtained in the field of new set theory. The main purpose of this work is an attempt to solve this problem.

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§1. The development of the foundations of theoretical physics led to the results inherent to quantum theory. The same results were obtained also in the field of a new variant of set theory.

As it is written about it [1-3] managed to come to the realization of the following truth. That there are fundamental ideas of scientific philosophy of which it was possible to systematize with the help of scheme 1

					Sociology
				Psychology	
			Biology		
		Physics			
	Kinematics				
Algebra					
Arithmetic					

Scheme 1

Then it was possible to come to the realization of the following. That there are results of which it is possible to take into account with the help of: scheme-2 and 3 (theoretical physics); scheme-4 and 5 (probable physics); and also with the help of scheme-6 and 7 (unification of the basis of physics). The following should be noted. That these results were obtained from the solution of Hamilton's equation

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} \quad (1)$$

for : α) of many orderly moving particles obeying the force,
 β) of many chaotically moving particles.

Thus the equation was obtained

$$\begin{aligned} \text{a) } \frac{\partial S}{\partial t} + H\left(q_i, \frac{\partial S}{\partial q}, t\right) &= 0, \\ \text{б) } H\left(q_i, \frac{\partial S}{\partial q}\right) &= E, \\ \text{в) } \Delta \psi + \frac{8\pi^2 m}{\hbar^2} (E - V)\psi &= 0, \end{aligned} \quad (2)$$

$$\begin{aligned} \text{a) } \frac{\partial p}{\partial t} - [H\rho] &= 0, \\ \text{б) } [H\rho] &= 0, \\ \text{в) } \rho_l &= \exp \frac{F - \varepsilon_l}{kT}, \\ \text{г) } \rho_{l,n} &= \exp \frac{\Phi + \mu_l - \varepsilon_l}{kT}. \end{aligned} \quad (3)$$

Here (2) is the Hamilton-Jacobi- Schrödinger equation and (3) is the basic equation of Gibbs statistical mechanics. At obtaining these equations from (1) were made suggestions that on this way had to take advantage of the possibility of $3N+1$ and $6N+1$ dimensional spaces. The natures of these same numbers can still be understood as the number of degrees of freedom. Therefore, it makes sense to understand their natures as the basic equations of classical statistical mechanics. Then on the basis of these equations the results were obtained

$$\begin{aligned} E_l &= \alpha + k\beta_l, \\ \Psi_i &= \sum_{ir} C_{ir} x_r, \end{aligned} \quad (4)$$

$$\begin{aligned} n_A^0 &= \frac{n^0}{1 + \exp \frac{\Phi - f}{kT}}, \\ n_\Phi^0 &= \frac{n^0}{1 - \exp \frac{\Phi - f}{kT}}. \end{aligned} \quad (5)$$

On the basis of (4) it is possible to take into account numbers and natures subordinate to the particle's connection. Whereas on the basis of (5) it is possible to take into account numbers and natures of chaotically moving particles. Therefore, their natures can be understood as the basic results of quantum statistical mechanics. They make sense in an ordinary three-dimensional space.

On the other hand, there is also the following equations as

$$\boxed{\begin{aligned} \nabla^2 \bar{E} - \frac{1}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2} &= 0, \\ \nabla^2 \bar{H} - \frac{1}{c^2} \frac{\partial^2 \bar{H}}{\partial t^2} &= 0. \end{aligned}} \quad (6) \quad \begin{aligned} dU &= TdS - PdV \\ dH &= TdS + VdP \\ dF &= -SdT - PdV \\ dG &= -SdT + VdP \end{aligned} \quad (7)$$

$$\begin{aligned} dU &= TdS - PdV + \sum \mu dn_i \\ dH &= TdS + VdP + \sum \mu dn_i \\ dF &= -SdT - PdV + \sum \mu dn_i \\ dG &= -SdT + VdP + \sum \mu dn_i \end{aligned} \quad (8)$$

derived from the analysis of experimental data. Therefore are the equations obtained with the accuracy of empirical physics. Here (6) are the basic equations of Maxwell's electrodynamics. Whereas (7) and (8) are the basic equations of technical and chemical thermodynamics. In these equations U is the internal energy of the system; H is enthalpy; F is free energy; G is thermodynamic potential; S is entropy; P is pressure; V is volume; T is absolute temperature; μ is chemical potential.

There are still results to be had

$$\boxed{\begin{aligned} \text{a) } E &= -\frac{me^4}{2\hbar^2} \cdot \frac{1}{n^2}, \\ \text{б) } 2\pi r &= n\lambda; \end{aligned}} \quad (9) \quad \boxed{\begin{aligned} \text{a) } K &= \frac{n_{AB}}{n_A \cdot n_B}, \\ \text{б) } \theta &= \frac{bn_A}{1 + bn_A}. \end{aligned}} \quad (10)$$

They were obtained by solving the problem for many orderly and chaotically moving particles based on the results of probability theory. In these expressions K is the equilibrium constant, b is the adsorption constant.

As it is written in [1-3] on the basis of the results (2), (3) and (4), (5) obtained with accuracy for theoretical physics it was possible to obtain a justification for the results (6), (7), (8) as well as for the results (9), (10 a, c). Exactly at reception of such proofs it was possible to reveal the true essence of quantum theory. Turn out (9) and also (10 a, c) are results inherent to quantum theory. And mainly because of the following reasons. Equations (2) and (3) of (1) are obtained for cases when the number of degrees of freedom are $3N+1$ and $6N+1$. The results (4) and (5) obtained from (2) and (3) make sense in three-dimensional space. Therefore these results (4) and (5) have sense inherent for quantum physics. However, obtained with precision when the nature of the constant and remains to be discovered. Thus along the way succeeded in proving the following. That such relations as (10 a, c) with which in the field of physical chemists used as early as the 19th century are results inherent to quantum physics. That is for a long time 1900, when Planck introduced the concept of quantum of energy.

§2. On how accepting the possibilities of results obtained in theoretical physics, it was possible to arrive at an interpretation of the nature of the basic results obtained in mathematical physics.

As we know from the basic equation of mathematical physics

$$\frac{\delta^2 u}{\delta t^2} - v^2 \Delta u = 0 \quad (11) \quad \frac{\delta u}{\delta t} - v^2 \Delta u = 0 \quad (12)$$

Equation (11) was obtained by taking as a basis the possibility of the algebraic method. For this equation is obtained as a consequence of the solution of Newton's equations. However, equation (12) was obtained by assuming the possibility of a non-algebraic method. For this equation (12) is not obtained as a consequence of solving Newton's equations. They are obtained as a consequence of analyzing the empirical equations obtained for the heat and matter flux balance. Therefore further solutions which have been received from the possibility of these equations also have been received within the framework of possibility of not algebraic method. Then already in the works on the basis of which the nature of such results were analyzed the following difficulties were found. That these solutions describe curves at no point of which it is impossible to draw a tangent. In a word, all these facts led to the fact that the basis of analysis began to develop in a way where there are contradictions. In this connection I would like to say the following. M. Kline in chapter 7 of the book [4] wrote

"At the heart of all mathematical analysis are the concepts of the continuous function and the derivative...Purely intuitively, a continuous function is a curve that can be drawn with a single stroke of the pen without taking it off the paper. It would seem obvious that a continuous function must have a tangent at every point. However, some mathematicians in the 19th century managed to rise above intuitive notions and set out to prove everything possible by purely logical means." (13)

On the other hand in chapter 6 of book [4] Kline still wrote "Mathematical analysis, the core of which is differential and integral calculus - the most delicate area of all mathematics - was built on the completely non-existent logical foundations of arithmetic and algebra and on the not quite clear foundations of Euclidean geometry". (14)

In my opinion, if we analyze the thoughts contained in lines (13) and (14) together, then we can come to the following conclusion. That in principle there is a possibility to eliminate all those contradictions, which appeared at development of the basis of mathematical analysis. In order to do this, the following should be done. For this purpose there is a necessity to refuse all the services of Aristotle's logic at working out the bases of the scientific theory of cognition. On the other hand it means the following. For the main results fulfilling the roles of basic theory of thought (15)

There is a need to take
(a) algebraic equations, (b) arithmetic equations themselves. (16)

Then solve problems on these bases
(a) geometry, (c) kinematics, (d) physics. (17)

Thereby getting the results inherent to
(a) algebraic geometry, (b) arithmetic geometry, (18)
(a) algebraic kinematics, (b) arithmetic kinematics, (19)
(a) algebraic physics, (b) arithmetic physics. (20)

Of course, after obtaining such results, it becomes necessary to interpret the philosophical natures (16) as well as (18), (19), (20). This means the following. There arises the necessity to reveal the roles of interrelation between subject and object in understanding the nature of these results. And in such a way that contradictions and paradoxes do not appear later when obtaining results in the field of function theory and set theory. It turns out that this kind

of goal is actually possible to achieve. Moreover, if in the course of calculation we manage to use correctly the possibility of
a) the method of separation of variables, b) the method of abolition of variables. (21)

The necessity of such steps is due to the following. All these main results are obtained in the path where, from the very beginning, (16) was taken as a basis for (15). Then we began to solve problems (17).

Now I want to tell about the following. What I mean when I speak about philosophical natures of (16) and results obtained in (18), (19), (20).

A) It turns out that in the case when one takes advantage of the possibility (16 a) one means the following. In this case it is possible to carry out calculations over abstract quantities while taking into account their nature. When the possibility (16 b) is used, calculations over a finite number of sets are carried out taking into account their number and nature. Thereby, with this understanding of the nature of (16 a) and (16 b), the relationship between subject and object is successfully revealed. Thus it is possible to realize that these results can be used as a basis for the theory of thinking.

B) On the other hand, in the case when using the possibility (18 a) it is possible to carry out calculations over geometric quantities. Whereas when using the possibility (18 b) it is possible to carry out calculations over geometrical points of subordinate links whose number tends to infinity. Thus, in this case also the interrelation of subject and object is successfully revealed. This means that the basis of theoretical geometry is successfully developed. And for this purpose at this stage it is necessary to make optimal use of the possibility of multidimensional space.

C) In the same case when using the possibility (19 a) it is possible to carry out calculations over kinematic quantities. Whereas when using the possibility (19 b) it is possible to carry out calculations over kinematic points of subordinate links whose number tends to infinity. It means that in this case the interrelation of subject and object can be successfully revealed. And also the basis of theoretical kinematics is successfully developed. And at this stage it is also necessary to use the possibility of multidimensional space.

D) In the case when the possibility (20 a) is used, calculations over physical quantities are carried out. Whereas when using the possibility (20 b), calculations are carried out over finite numbers of physical particles subordinated to the connection or moving chaotically. Thus in these cases it is possible to complete successfully the development of the foundations of theoretical physics. And also successfully take into account the interrelation of subject and objects. One still has to take advantage of the possibility of multidimensional space.

At this stage one can realize the following. Analyzing the thoughts contained in A, B, C, D the following conclusions can be drawn. It is possible to take these thoughts as the basic one when interpreting the nature of the main results taken into account when constructing scheme-2 and 3. It means for the results on the basis of which it is possible to reveal the essence of the obtained in the field
(a)Theoretical Geometry, (b)Theoretical Kinematics, (c) Theoretical Physics (22)

I would also like to emphasize the following. For example, analyzing the results obtained in the field of theoretical physics one can make the following conclusions. It turns out that the natures (4) and (5) can be taken as the main results of the new

variant of the function theory, and also as the main results of the new variant of the set theory. And at obtaining of which it was possible to take into account the roles of the number of particles and their nature.

On the other hand the following facts are well known. In the way where the basic equations obtained in the field of mathematical physics (11) and (12) were taken as a basis, it was also possible to come to the results inherent for the theory of function and for the theory of infinite set. However in this case contradictions and paradoxes were inherent to such results. The main reason for this is of course the following. The main results in these areas were obtained in the way where the possibility of such results as

$$\sum_{n=1}^{\infty} a_n \sin nx + b_n \cos nx, \quad (23)$$

In connection with the use of the possibility of expression (23) within the framework of mathematical physics, we can note the following. As it is known before the object of study of mathematicians were Fourier series. Nevertheless Riemann began to study arbitrary trigonometric series (23) as a means of representing functions [5].

Of course, this was a step taken on a new path. As it was pointed out in the book it was on this path that the main results of Cantor's infinite set theory were further obtained [6].

§3. On the consequences of accepting the possibility of algebra and arithmetic in place of the results of Aristotle's logic.

In his book entitled " The Joy of Knowing" Richard Feynman wrote the following
"The next great era of awakening human intelligence may create a methodology for understanding the qualitative content of equations" [7]. (24)

In my opinion, the adoption of the fundamental ideas of scientific philosophy makes it possible to come to the realization of the following. That it is reasonable to take the possibilities of algebra and arithmetic equations as a basis for the theory of thinking. Thus to realize the necessity to refuse the necessity to use the possibility of Aristotle's logic. There is every reason to suppose that further on this path it will be possible to achieve the next goal. That is, that goal which is referred to in the thoughts contained in lines (24). In order to show that this is indeed a possibility the following may be pointed out. That by taking the basic ideas of scientific philosophy, i.e. the ideas that are taken into account with the help of scheme-1, it becomes possible to solve the problems that were formulated by Descartes [8]

The first is never to take as true anything that I do not recognize as such with obviousness, ...and to include in my judgments only those things that appear to my mind so clearly and so distinctly that they give me no reason to question them.

The second is to divide each of the difficulties I'm exploring into as many parts as is possible and necessary to better overcome them.

Third, to adhere to a certain order of thinking, beginning with the simplest and most easily cognizable subjects and ascending gradually to the cognition of the most complex,

Fourth, always make lists so complete and reviews so general that there is no omission.

Analyzing the results taken into account by means of scheme-1, 2, 3, 4, 5, 6, 7 one can notice the following [1-3]. That there is a certain correspondence with the essence of ideas which are contained in these rules of Descartes, and also with those results considered by means of these schemes. For in obtaining these results it is possible to come to the realization of the following truth. That it is necessary to accept the possibilities of algebra and arithmetic for the results of correctly fulfilling the role of the basis of the theory of thinking. Then on this basis solving problems of geometry-kinematics-physics to come to obtaining results inherent to (24). And also to obtain the results considered with the help of scheme-6 and 7. It means to put in order all intellectual achievements of mankind on a scientific basis. Thereby it is actually possible to successfully realize the program, about which is written in lines (24). For when obtaining the results taken into account with the help of these schemes it actually manages to reveal the qualitative content of the equation. That is, not only numbers, but also the nature of objects can be correctly taken into account.

Problem #1: Back in the days of antiquity, the Pythagoreans had a desire to reduce all mathematics to the doctrine of integers. It seemed to them that in order to properly develop the foundations of the scientific theory of cognition it is reasonable to take the possibilities of arithmetic of integers as a basis (17) from the very beginning. The final account of natural numbers. It means to take as a basis the possibilities of thought which they formulated as "the axiom of the indivisibility of the unit". (25)

Further on this path difficulties began to appear. When trying to solve the problem of geometry there was a need to introduce the concept of irrational numbers. To overcome such difficulties, Aristotle formed the idea that became known as "axiom on the necessity of separating arithmetic and geometry". (26)

However on the basis of the analysis of ideas and results (4) and (5) obtained at development of the basis of quantum theory as a new variant of set theory it is possible to draw the following conclusions. That these results potentially contain proofs that the basic ideas contained in (25) and (26) are true. For it is not difficult to guess that in order to study any concrete objects the possibility of natural numbers must be taken as a basis. The following may be further noted. At one time Cavalieri and Descartes came very near to realizing that such thoughts were really maturing. They began to realize that for this purpose it is necessary to give a special status to the possibility of algebra and arithmetic. For example, as it is written in Cavellieri in the thoughts contained in a letter to Galileo (1622) [9]. Descartes in the same case when he was introduced the basic results of Cartesian coordinate systems [10]. However unfortunately Descartes further stated that those new results which he began to develop still contained the following possibilities. That in the future it will be possible to obtain proofs of the fallacy of the thought contained in (25) and (26). Thus he, like Cavellieri, also advocated the unification of the separate theories of numbers and continuous quantities into one general theory. As it is known, many mathematicians began to think in this way. For example, Newton, Leibniz, Euler, Lagrange, Dalember, Carnot, Cauchy, Balzano, Weierstrass, Dedekind, Cantor ... Therefore, at the very end of this path were obtained the results of the theory of infinite abstract sets [11]. That is, a theory which has no cognitive possibility at all.

Thus, on the basis of analyzing the highest set forth thoughts we can come to the following conclusion. It turns out that those thoughts that were first expressed by the Pythagoreans (25) and Aristotle (26) are actually true. What so managed to prove in the following way. In the way where for the basis of the results fulfilling the roles (15) the possibilities (16) were accepted, then the problems (18) were solved at the very end it was possible to come to the results (4) and (5). That is the results of the nature of which it is possible to understand as inherent for quantum theory. And also inherent for a new variant of the set theory. That is the theory at development of which there is a possibility to use possibilities of natural numbers only.

Problem #2: As it is known in his time L Carnot [12] in his book "Reflections on the Metaphysics of the Calculus of Infinitesimals" formulated the essence of his goal as follows

"I seek to know what is the true spirit of the calculus of infinitesimals; the reflections ... are distributed in three chapters: in the first I state the general principles of the analysis of infinitesimals; in the second I investigate how this analysis has been brought to an algorithm through the discovery of the differential and integral calculus; in the third I compare this analysis with other methods that can replace it, such as the method of exhaustiveness, the method of indivisibles, the method of indefinites, and so on." (27)

As is well known L Carnot was not able to achieve his goal. Mainly because of the following reasons. In order to discover the true spirit of the calculus of the small, he took the possibility of results obtained by Leibniz as a basis. However, the possibility of Leibniz's algorithm was not quite enough to solve such problems. Mainly due to the fact that Leibniz in obtaining his results for the results of correctly fulfilling the role (15) considered possible to take Aristotle's logics. Therefore Leibniz though used the possibility of the method of conducting tangents, however he began to use its possibility only with the accuracy inherent in geometrical concepts. That is, he made his conclusions on the basis of analyzing the properties of the triangle introduced by Pascal. However, as Leibniz himself began to realize in due time, the possibilities of this approach were not quite sufficient for the full completion of the development of the foundations of analysis. In this he admitted in a letter written by him to L'Hopital [13]. Here he wrote: "our calculus of differences and sums leaves us". Thus he began to realize even then that some ideas were still missing for the complete development of the foundations of analysis. Unfortunately, however, at that time he did not manage to realize the essence of those ideas that were missing. He could not realize that there is a necessity to obtain differential equations for the 1st geometrical and for the 1st kinematic points. For this purpose, the necessity to take as a basis the possibilities of the method of conducting tangents. However, with such precision, when the possibilities of the basic concepts of physical theory are taken as a basis. Here I want to say the following. When it is said about the possibility of the concept of physical theory it means the following. It is when for geometrical and kinematic curves the property of elasticity is inherent. In order to successfully take this fact into account at this stage it would be optimal to use the possibility of the concept of multidimensional space. For with such an approach the results of the inherent properties for 22 a, b, c could actually be obtained correctly.

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