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On the Expanding Universe – Accurately Calculate the Hubble Constant by Lunar Laser Ranging

Xiao En Wang

Weifang Engineering Vocational College (Weifang Educational College), Qingzhou City, Shandong Weifang, China and Zhaoqing KAIST Battery Material Co., Ltd., Gaoyao District of Zhaoqing City, Guangdong Province, China. ORCID: https://orcid.org/0000-0001-8397-0640.

ABSTRACT

Hubble's Law is an empirical formula. We use the product of the speed of light and time instead of distance for differentiation, and obtain the theoretical differential equation $dV=Wdt$. Then, the expansion velocity, radius, total mass, singularity radius, and redshift formula of the universe were calculated, and the gravitational constant was derived and proved to be proportional to the fourth power of cosmic time. The mathematical relationship between lunar laser ranging data and Hubble constant was obtained. This is a newly discovered precise method for measuring the Hubble constant.

***Corresponding author**

Xiao En Wang, Weifang Engineering Vocational College (Weifang Educational College), Qingzhou City, Shandong Weifang, China and Zhaoqing KAIST Battery Material Co., Ltd., Gaoyao District of Zhaoqing City, Guangdong Province, China. ORCID: https://orcid.org/0000-0001-8397-0640.

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Preface

In 1927, based on the equations of general relativity and combined with the observations of American astronomers Hubble et al., Lemaitre proposed the hypothesis that the universe is expanding and estimated the value of the expansion rate [1]. Based on his and previous observational data,in 1929, Hubble proposed that the retrograde velocity V of celestial bodies is proportional to their distance D [2].

$$V = H_0 D \qquad (1)$$

where H_0 is called the Hubble constant. Its SI unit is sec^{-1}, but the commonly used one is km s^{-1} Mpc^{-1}. Among them, 1 Mpc=10^6 parsec=3.26×10^6 light year. Due to measurement reasons, the value of H_0 has been constantly changing. For most of the second half of the 20th century, the value of H_0 was estimated to be between 50 and 90 km s^{-1}Mpc^{-1}. In 2020, the values obtained by Coughlin, M. W. et al.[3] are $H_0 = 73.8^{+6.3}_{-5.8}$ km s^{-1} Mpc^{-1} and $H_0 = 71.2^{+3.2}_{-3.1}$ km s^{-1} Mpc^{-1} by two different kilonova models respectfully.

Based on this two numerical values and theoretical calculations later in this article, H_0=72.7 km s^{-1} Mpc^{-1} is taken here. The reciprocal of the Hubble constant, also known as Hubble time, corresponds to the age of the universe. So the corresponding age of the universe today is: $t_2=1/H_0=1.344 \times 10^{10}$ years. Equation (1) was originally called Hubble's Law, but in 2018, it was renamed Hubble-Lemaitre's Law.

Differential Formula of Hubble-Lemaitin Law

Equation (1) is an empirical formula, and the physical meaning of the variables in the equation is not very clear, which hinders people's understanding and application of it. Theoretical physicist Yaso Kopanets once pointed out that "for many applications, the differential form is more convenient than the integral form" [4]. For rewrite formula (1) into differential form, we write the distance D as the product of the speed of light and time, i.e. $D = ct$, and then differentiate its two sides separately

$$dV=H_0c\ dt$$

Since the product of constants is still constant, let $H_0c = W$, and the above equation becomes

$$dV=Wdt \qquad (2)$$

Obviously, W in equation (2) is a uniform acceleration constant, indicating that our universe is expanding at a uniform acceleration. We call equation (2) the differential equation of the Hubble Lemaitin law, and for comparison purposes, we write it together with equation (1)

$$\begin{cases} V = H_0 D & (1) \\ dV = Wdt & (2) \end{cases}$$

Compared with equation (1), the equation (2) has a clearer physical meaning, and more cosmological information and practical applications. According to the above definition, $W = cH_0$ = 72.7 km s^{-1} (3.26×10^6 years)$^{-1}$, approximately 0.0223 m s^{-1} year^{-1}, or 7.07×10^{-10} m s^{-2}. is to say, today our universe is expanding at a rate of 2.23 centimeters per second per year. There are still doubts today about the expansion of the universe. So we should first clarify relationships between a process and time and space. The expansion is a process that is both time dependent and spatially dependent. Time is one-dimensional, unidirectional, and uniform, and has no starting or ending point. Space is also infinite. But a process always has a starting point, an intermediate evolutionary path, and an endpoint. The universe is a process, a process of uniform accelerating expansion, involving finite time and space. The age of the universe is calculated from the time when the universe began to expand, which we call the starting point or singularity. At the starting point of expansion $t_1 = 0$, the universe is not a mathematical point, but rather the densest universe with a radius denoted as Rs. The information currently available to humans about the universe is limited to the process from the beginning of its expansion to the present. As for what the universe will look like before and after the expansion ends, we cannot obtain this information, nor can we obtain it in the future.

The Expansion Rate of the Universe

We integrate equation (2) over the entire time period of the universe's age and assume that the time when the universe began to expand is $t_1 = 0$. Today's expansion rate of the universe is denoted as V_2. The left-hand integral of equation (2) yields; $\int_{V_1=0}^{V_2} dV = V_2$;

Integrate on the right side, get

$$V_2 = \int_0^{t_2} W dt = \int_0^{t_2} cH_0 dt = \frac{c}{t_2} \cdot t_2 = c \qquad (3)$$

Equation (3) indicates that our universe is currently expanding at the speed of light, and also suggests that the speed of light is proportional to cosmic time, but at every cosmic moment, the speed of light remains constant.

Cosmi Radius

When we perform a quadratic integration on equation (2), we can obtain today's cosmic radius R_2.

The starting radius R_S of the universe is not zero, but as an integral constant, $R_S <<< R_2$.

$$\begin{aligned} R_2 &= R_s + \int_0^{t_2} W\ t dt \approx \int_0^{t_2} W t dt = \frac{1}{2} W t_2^2 \\ &= \frac{1}{2} \times (7.07 \times 10^{-10}\,\mathrm{m \cdot s^{-2}}) \times (1.344 \times 10^{10} \times 3.1557 \times 10^7\,\mathrm{s})^2 \\ &= 6.36 \times 10^{25}\,\mathrm{m} \\ \text{or } R &= 6.72 \times 10^9 \text{ light-yr.} \end{aligned} \qquad (4)$$

We can get cosmic diameter by formula (4)

$2R_2 \approx 2 \times 6.36 \times 10^{25}$ m $= 1.272 \times 10^{26}$ m, or 1.344×10^{10} light-yr. (5)

The above calculations indicate that the radius of the universe increases by 0.5 light years per year, and the diameter increases at a rate of one light year per year. That is to say, the universe has been expanding at the speed of light. It can also be known that all macroscopic lengths (L) in the universe are proportional to the radius of the universe, that is, proportional to the square of time, $L \propto R \propto t^2$.

The Total Mass of the Universe

There are a large number of celestial bodies in the universe orbiting around their central celestial body. The gravitational force exerted by the central celestial body is equal to the centrifugal force generated by the orbital motion of the sub star itself, that is

$$\frac{GMm}{R^2} = \frac{m\upsilon^2}{R} \qquad (6)$$

where G, M, m, R, and υ represent the gravitational constant, the mass of the central celestial body, the mass of the sub celestial body, the semi major axis of the elliptical orbit (which can be approximated as the radius of a circular orbit), and its average velocity of orbital motion, respectively. The formula (6) can be simplified as

$$GM = \upsilon^2 R \qquad (7)$$

If equation (7) is extended and applied to the entire universe, M_T represents the total mass of the universe, R represents the radius of the universe, and υ takes the maximum limit value, that is, the speed of light, c. We got

$$M_T = \frac{Rc^2}{G} = \frac{6.356 \times 10^{25} \times (2.998 \times 10^8)^2}{6.67 \times 10^{-11}} = 8.565 \times 10^{52}\,(\mathrm{kg}) \qquad (8)$$

Radius of Dense Universe at Singularity

When calculating the radius of the universe by equation (4), we take $t = 0$ as the starting point of the expansion of the universe. The cosmic radius R_s at the starting point is used as an integral constant, but it is omitted in the calculation because it is too small. But it is not a mathematical point. At the starting point, the density of the universe is at its maximum and will not be less than the density of any object in the universe today. Next, we estimate the volume size of the universe based on its total mass and density.

According to the Encyclopedia Britannica, the density of the neutron star is equivalent to that in the atomic nucleus, and the core density can reach 10^{15}g/cm^3, which is equivalent to $\rho_n = 10^{18}$ kg/m^3. Assuming cosmic density at singularity $\rho_n = 10^{18}$ kg/m^3, the total cosmic mass is M_T, and we can calculate the volume V_s and radius R_s of the universe at the singular point in turn

$$V_S = \frac{M_T}{\rho} = \frac{8.565 \times 10^{52}\,\mathrm{kg}}{10^{18}\,\mathrm{kg/m^3}} = 8.565 \times 10^{34}\,\mathrm{m^3} \qquad (9)$$

$$R_s = \sqrt[3]{\frac{3V_s}{4\pi}} = \sqrt[3]{\frac{3 \times 8.565 \times 10^{34}}{4\pi}} = \sqrt[3]{20.448 \times 10^{33}} = 2.735 \times 10^{11}\,(\mathrm{m}) \qquad (10)$$

Formula (10) shows that the radius of the universe at the singularity is of the same order of magnitude as the astronomical unit.

Cosmic Redshift should include Spatial Redshift and Temporal Redshift

The redshift value of celestial bodies observed during the Hubble era, $z << 1$, was determined using the redshift distance relationship of $cz = H_0 D = V$. Today, $z = 1$ is no longer the maximum limit for redshift values. As Edward Harrison [5] pointed out, the redshift-distance ($z\,(D)$) law and the velocity-distance $V\,(D)$ law are theoretically equivalent only at the limit of small redshifts. The linear $V\,(D)$ law is widely applicable to cosmological models with uniform expansion and isotropy, where the retrograde velocity can exceed the speed of light when the distance is not limited. However, the linear form of the $z\,(D)$ relationship ($cz = H_0 D$) has no theoretical basis and can only be used for the limit of small redshifts. Space expansion can only be suitable for cosmological models with $z < 1$. And now that celestial bodies with $z \geq 1$ have been observed, the original redshift-distance ($z\,(D)$) law and velocity-distance $V(D)$ law are no longer applicable, and both will exhibit the superluminal paradox. That is to say, for celestial bodies with $z \geq 1$, there are observational results today but no suitable theories. In 2017, I proposed and discussed that cosmic redshift can be divided into spatial redshift and temporal redshift [6]. Not only did it solve the problem of about superluminality, but it also obtained an accurate mathematical relationship between redshift and velocity. Here is a brief introduction.

According to the definition of redshift, We can write the formula (11-0). Because the wavelength λ_0 of the spectral lines emitted by distant celestial bodies is unknown, we can only measure the wavelength λ and identify the identity of its spectral lines when it reaches Earth. We can only use the wavelength λ_e of the spectral line with the same identity in today's laboratory instead of λ_0. Then the redshift formula becomes to formula (11)

$$z = \frac{\lambda - \lambda_0}{\lambda_0} = \frac{\lambda}{\lambda_0} - 1, \text{ or } \frac{\lambda}{\lambda_0} = 1 + z. \qquad (11\text{-}0)$$

$$z = \frac{\lambda}{\lambda e} - 1, \quad \text{or} \quad \frac{\lambda}{\lambda e} = 1 + z. \qquad (11)$$

Assuming that the age of the universe when a distant celestial body emits light is t_1, and the age of the universe we receive the light today is t_2, formula (11) actually assumes that the wavelength λ_0 of the photons emitted by our Earth at cosmic time t_1 is the same as the wavelength λ_e of the same spectral line emitted in the laboratory today, that is, $\lambda_0 = \lambda_e$. Only photons occur redshift when their undergo through long-distance propagation, and it has been assumed that atoms in ancient times emit photons of the same wavelength as atoms today. The redshift assumed in this way is called spatial redshift. If the wavelength of photons emitted by atoms in ancient times is shorter than the wavelength of the same spectral line emitted by atoms in the same location and by the same element today, we call it as time redshift. But we cannot preserve the photons emitted from the Earth in ancient times until now. Therefore, we can only speculate on the existence of time redshift.

The formula (11) only considers spatial redshift and does not include temporal redshift. If there is only spatial redshift and no temporal redshift, then there will be no distant celestial bodies with redshift values greater than 1.

Suppose we have a fiber optic cable long enough to circle the Earth in countless circles, allowing photons to run inside for billions of years. At the ancient time t_1, a photon with a wavelength of λ_0 is input into the optical fiber and allowed to propagate continuously in the fiber. At the modern time t_2, extract it from the laboratory and measure its wavelength. The wavelength of this photon is not affected by the expansion of space in the universe, and we compare it with the wavelength λ_e of photons of the same identity in the laboratory. As mentioned above, if there is no time redshift, $\lambda_e = \lambda_0$, according to the theory of relativity, the redshift value of distant celestial bodies we observe cannot exceed 1. Now that we have observed celestial bodies with redshifts greater than 1, it is definitely not true that they exceed the speed of light. It precisely proves that $\lambda_e \neq \lambda_0$, which also proves the existence of time redshift. The time redshift is represented by the following equation

$$z = \frac{\lambda_e - \lambda_0}{\lambda_0}, \quad \text{or} \quad \frac{\lambda_e}{\lambda_0} = 1 + z, \quad \text{or} \quad \lambda_e = (1+z)\lambda_0 \qquad (12)$$

By substituting equation (12) into equation (11), we obtain

$$\lambda = (1+z)^2 \lambda_0 \qquad (13)$$

Equation (13) represents the cosmic spacetime redshift. Because the expansion rate of the universe is directly proportional to its age, and $V_2 = c$, we have

$$1 + z = \frac{\lambda_e}{\lambda_0} = \frac{t_2}{t_1} = \frac{V_2}{V_1} = \frac{c}{V_1}. \qquad (14)$$

$$t_1 = \frac{t_2}{1+z} \qquad (15)$$

Because V_1 and t_1 are unknown, the measured V in Hubble's law is actually equivalent to $V_2 - V_1$, corresponding to $t_2 - t_1$, here

$$t_2 - t_1 = t_2 - \frac{t_2}{1+z} = \frac{zt_2}{1+z}; \text{ or } \frac{t_2 - t_1}{t_2} = \frac{z}{1+z} \qquad (16)$$

According to equation (14)

$$\frac{t_2}{t_2 - t_1} = \frac{c}{c - V_1} = \frac{c}{V} \qquad (17)$$

By equations (17) and (16), we get

$$V = \frac{(t_2 - t_1)c}{t_2} = \frac{zc}{1+z} \qquad (18)$$

Due to our consideration of both spatial and temporal redshift, the equation (18) avoids the superluminal paradox, and obtained reasonable results, $0 \leq z \leq \infty$ and $V \leq c$.

When $z \ll 1$, $1+z \approx 1$. By equation (18), we obtain the approximate formula used by Hubble, which is the redshift-distance law

$$V = H_0 D \approx zc \qquad (19)$$

Equation (19) is only applicable when $z \ll 1$. Table 1 lists the wavelength and velocity of Hα spectral line at different redshift values.

Table 1: The λ_0, λ_e, λ, V, and V_H (value used by Hubble) for different redshifts

Z	0	0.001	0.01	0.1	0.5	1	4	9	19	49	99	…
t_2/t_1=1+z	1	1.001	1.01	1.1	1.5	2	5	10	20	50	100	…
$\lambda_0=\lambda_e/(1+z)$	121.6	121.5	120.4	110.5	81.1	60.8	24.32	12.16	6.08	2.432	1.216	…
$\lambda=(1+z)^2\lambda_0$	121.6	121.7	122.8	133.8	182.4	243.2	608.0	1216	2432	6080	12160	…
$V=zc/(1+z)$	0	0.001c	0.010c	0.091c	0.333c	0.500c	0.800c	0.900c	0.950c	0.980c	0.990c	
$V_H=zc$	0	0.001c	0.01c	0.1c	0.5c	c	4c	9c	19c	49c	99c	…

The bottom row of Table 1 shows the velocity-redshift relationship used by Hubble. It can be seen that when $z < 0.01$, the velocity calculated by redshift is approximately correct. When $z > 0.1$, there is a significant deviation, and the larger the redshift value, the greater this deviation. When $z > 1$, the underlined parts indicate that the principle of maximum speed of light has been violated and are those impossible values.

The second to last row of Table 1 shows the velocity-redshift relationship presented in this article, with no restrictions on the increase of redshift. Because the redshift (z + 1) derived in this article is equal to the ratio of the current age of the universe to the age of the universe when celestial bodies emitted light. The larger the redshift value, the younger the universe when celestial bodies emit light.

Due to the lack of a correct redshift-velocity relationship in current theories, astronomical distances determined by redshift often have significant deviations, and the Hubble constant measured by redshift has also been subject to significant uncertainty. In the future, using the redshift-velocity relationship of equation (18) will improve these situations.

The Variation of Gravitational Constant with Cosmic Time

In 1994, Dickey et al. pointed out that "Lunar range data also provide a test of a possible change in the gravitational constant (G) with time because of the lunar orbit sensitivity to solar longitude" [7]. This article has proven that as the universe evolves, the speed of light is proportional to the age of the universe and is no longer a constant; The radius of the universe is directly proportional to the square of its age. However, the universe is expanding at a uniform acceleration, and W is a constant that does not change with the age of the universe. We take the age of the universe t as the independent variable, represent the speed of light and the radius of the universe as functions of t, and assume that the total mass of the universe does not change with the age of the universe. Therefore, we can obtain from equation (8)

$$G=\frac{c^2R}{M}=\frac{(Wt)^2\times\frac{1}{2}Wt^2}{M}=\frac{W^3t^4}{2M};\ \text{or}\quad G\propto t^4 \qquad (20)$$

We can draw two important conclusions by equation (20):

(i) The gravitational constant G is directly proportional to the fourth power of the age of the universe;

(ii) When $t = 0$, $G = 0$.

Before the universe began to expand, universal gravity did not exist. Of course, there is no need for centrifugal force, and there is no relative motion. At any moment after the beginning of expansion, the gravitational force acting on an object is equal to the centrifugal force caused by the expansion of the universe.

If R is the radius of a circular or nearly circular orbit and T is its period, then $\upsilon = 2\pi R/T$. Substituting it into equation (7), we obtain Kepler's third law corresponding to a certain cosmic time

$$\frac{GM}{4\pi^2}=\frac{R^3}{T^2} \qquad (21)$$

At a certain cosmic time, equation (21) can be used for any macroscopic binary system in the universe. The “macroscopic” here refers to a world dominated by universal gravity. Excluding the microscopic world of atoms and molecules, as the gravitational force in the microscopic world can be completely ignored.

The deduction of equation (20) is astonishing! $G \propto t^4$!! Is it true that increases with the fourth power of cosmic time? In 2020, J. L. P. R. Fernandes [8] proposed that as the universe expands, the potential energy density ρ_u in space decreases while G increases. The product of the two is a constant,that is,

$$G\cdot\rho_u = \text{constant} \qquad (22)$$

We believe that the Fernandes formula represented by equation (22) is correct, but his subsequent processing may have been incorrect due to the lack of use of the Hubble model. We start by deriving the relationship between G and cosmic time t from equation (22).

The expansion of the universe is a spherical expansion from the center outward, and there is a relationship between density and radius as follows

$$\rho_u \propto \frac{1}{4\pi R^2} \qquad (23)$$

By equations (22) and (23), we obtain

$$G\propto\frac{1}{\rho_u}\propto R^2 \qquad (24)$$

According to equation (4), $R \propto t^2$ can be substituted into equation (24) to obtain equation (20), that is, $G \propto t^4$

G increases with the fourth power of cosmic time, which can be theoretically proven, but can observational evidence be found?

Observational Evidence that G increases with the Fourth Power of Cosmic Time

1) The Hubble-Lemaitre law is one of the observational evidences. The Hubble-Lemaitre law states that the universe is expanding at a uniform acceleration.

According to Newton's second law, uniform acceleration motion must be accompanied by a constant force. The main interaction force between celestial bodies in the universe is universal gravitation

$$F=\frac{GMm}{R^2} \qquad (25)$$

If we consider that mass is a constant and F is a constant force, then G/R^2 must be a constant. As discussed earlier, all macroscopic lengths in the universe are proportional to the square of time, i.e., $R \propto t^2$, so $G \propto t^4$. Only when $G \propto t^4$, F is a constant force. Only constant force can balance and constrain expansion. Although the universe is expanding, it is still in dynamic equilibrium, otherwise the expansion of the universe will lose control and even explode. This is the cosmological evidence that G increases with the fourth power of cosmic time.

2) If G does not change with time, according to equation (4), at the beginning of the expansion of the universe, $R = R_s \approx 0$, or $R \to 0$, then $F \to \infty$, the universal gravitation between all celestial bodies is infinite, and the result is that the expansion of the universe cannot begin.

3) As long as G does not change with time, the universe is an unstable universe, and it must be a shrinking universe, an accelerating shrinking universe, and the final result must be the collapse of the universe.

4)As astronomical observations continue to advance into deep space, that is, to understand the older universe, the velocity-distance relationship represented by equation (1) has not been violated. This indicates that the expansion of the universe has been accelerating uniformly, and there has been no wave like expansion observed in the observed cosmic time so far. The observed uniform acceleration expansion indicates that the gravitational force is a constant force.

5) Einstein did not consider the variation of G over time. In his early days, he believed that the universe was in a steady state, meaning it neither expanded nor contracted. When he established the cosmological equation, in order to balance gravity and prevent the contraction of the universe, he added a cosmological constant term to the equation. After visiting the Hubble Observatory, he abandoned his cosmological constant in an article published in PNAS [9]. However, after abandoning the cosmological constant, his model, although not in a steady state, actually became a contraction model. If Einstein had G/R^2 = constant as his cosmological constant, his model would be completely consistent with the Hubble model. That is

$$\Lambda = \frac{G}{R^2} = \frac{6.67\times10^{-11}\mathrm{N\cdot m^2\cdot kg^{-2}}}{(6.356\times10^{25}\mathrm{m})^2} = 1.651\times10^{-62}\ \mathrm{N\cdot kg^{-2}} \qquad (26)$$

Today, we have obtained $G \propto t^4$, or Λ = constant, which has unraveled the mystery of the cosmological constant for over a century. $G \propto t^4$ can be seen as a stabilizer of the universe. If Einstein's spirit in heaven had heard this news, he would have laughed happily.

6) As previously proven, Kepler's third law is also observational evidence that G increases with the fourth power of cosmic time.

7) The annual increase in lunar distance is the strongest evidence for $G \propto t^4$

In 2020, J. L. P. R. Fernandes wrote, "With the available information, that the Moon is moving away from Earth, at a rate of a constant 3.82 ±0.07 cm per year obtained as measured since 1969, i.e. measurements taken for over 51 years, through the Apollo Laser Ranging Experiments Yield Results and Jupiter has more than 60 natural satellites, but only the top four deserve particular attention: Io, Europe, Ganymede and Calisto [8]. They have nearly circular orbits, and exhibit the same face toward Jupiter. They are also slowly moving away from the planet. Saturn has over forty satellites, except for two, always run with the same face toward the planet, and they are slowly moving away." Below, we use the relationship of $G \propto t^4$ to derive and calculate the annual increase in lunar distance.

We will express the formula (21) for Kepler's third law and write the expressions for it at cosmic times t_1 and t_2, respectively

$$G_1 = \frac{4\pi^2}{M}\frac{R_1^{\ 3}}{T_1^{\ 2}};\quad \text{and}\quad G_2 = \frac{4\pi^2}{M}\frac{R_2^{\ 3}}{T_2^{\ 2}}; \qquad (27)$$

By the formula (20), we obtain

$$\frac{G_2}{G_1} = \left(\frac{t_2}{t_1}\right)^4;\quad G_2 = \left(\frac{t_2}{t_1}\right)^4\cdot G_1 \qquad (28)$$

By formulae (27) and (28), get

$$\frac{R_2^{\ 3}}{T_2^{\ 2}} = \left(\frac{t_2}{t_1}\right)^4\cdot\frac{R_1^3}{T_1^2};\ \text{or}\ \frac{R_2^3}{R_1^3} = \left(\frac{t_2}{t_1}\right)^4\cdot\frac{T_2^2}{T_1^2} \qquad (29)$$

Equation (29) is Kepler's third law containing cosmic time. Because we are calculating the variation of distance with the age of the universe, we take an approximate value of T_2/T_1 =1 for the ratio of periods. Then the equation (29) can be simplified as

$$R_2 = \left(\frac{t_2}{t_1}\right)^{\frac{4}{3}}\cdot R_1 \qquad (30)$$

We assume that last year's age of the universe was an accurate 13.44 billion years, and this year's age of the universe is an accurate (13.44 billion +1) years. That is, $t_1 = 1.344\text{x}10^{10}$ years; $t_2 = (1.344\text{x}10^{10} + 1)$ years;

We need to expand the cosmic age ratio term in equation (30).

$$\left(\frac{t_2}{t_1}\right)^{\frac{4}{3}} = \left(\frac{1.344\times10^{10}+1}{1.344\times10^{10}}\right)^{\frac{4}{3}} = \left(1+\frac{1}{1.344\times10^{10}}\right)^{\frac{4}{3}} \qquad (31)$$

According to the generalized binomial theorem, the equation (31) can be expanded in series form as $(1 + 1/x)^n$,

$$\left(1+\frac{1}{x}\right)^n = 1 + n\cdot\frac{1}{x}+\cdots \qquad (32);$$

We only take the first two ters of the series and ignore the higher - order terms of $1/x$.

Then substitute x= 1.344 x 10^{10} and n= 4/3 into equation (30) to obtain

$$R_2 = \left(\frac{t_2}{t_1}\right)^{\frac{4}{3}}\cdot R_1 = \left(1+\frac{4}{3}\cdot\frac{1}{1.344\times10^{10}}\right)\cdot R_1 = (1+9.9206\times10^{-11})\cdot R_1$$
$$= R_1 + (9.9206\times10^{-11})R_1 \qquad (33)$$

We take the radius of the lunar orbit as 3.85 x10^8 m, and it can be obtained by equation (33)

$$\Delta R = R_2 - R_1 = 9.9206\times10^{-11}\times3.85\times10^8\ \text{m} = 3.819\times10^{-2}\ \text{m} = 3.819\ \text{cm} \qquad (34)$$

Equation (34) indicates that the derivation and calculation results in this paper are completely consistent with the measurement results of increasing 3.82 ± 0.07 centimeters per year during the 51-year Apollo laser ranging experiment [8]. Although this article uses cyclic calculations, we have obtained an accurate mathematical relationship between lunar laser ranging data and the age of the universe.

Because the derivation of equations (28) to (33) uses the age of the universe, equations (33) and (34) not only apply to the moon, but also to the annual increase in orbital radii of all planets and satellites in the solar system; Even applicable to the annual increase in the orbital radius of all orbiting celestial bodies in the entire universe. For example, by replacing R_1 in equation (34) with the orbital radius of the Earth, the annual increase in the distance between the Earth and the Sun can be obtained

$$\Delta R = R_2 - R_1 = 9.9206\times10^{-11}\times1.496\times10^{11}\ \text{m} = 14.84\ \text{m} \qquad (34\text{-}0)$$

Table 2: lists the annual increase in orbital radius of the eight major planets in the solar system

	Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
R_1	(AU)	0.387	0.723	1.000	1.524	5.202	9.567	19.303	30.243
	($\times 10^{11}$m)	0.579	1.082	1.496	2.279	7.782	14.311	28.876	45.242
ΔR	(m)	5.745	10.734	14.840	22.611	77.201	141.970	286.454	448.803

Calculate the Hubble Constant by Lunar Laser Ranging Data

For nearly a century, the value of the Hubble constant has been correcting towards a decreasing trend. The value used by Hubble at his era was 500 km s^{-1}Mpc^{-1}. The Cepheid scale standard he used was later revised, and the revised H_0 value was only about 1/7 of the original value. For most of the second half of the 20th century, the value of H_0 was estimated to be between 50 and 90 km s^{-1}Mpc^{-1}. The precise determination of the Hubble constant remains an important frontier topic to this day.

The determination of the Hubble constant mainly comes from two methods. The first method is Cepheid calibrated type Ia supernovae (SNe Ia), which has been used since Hubble for calibrating Cepheid variables; The second method is the prediction of the Planck cosmic microwave background observation under the cosmological constant (Λ) cold dark matter (Λ CDM) model.

But the Hubble constant measured by the two methods reflects a significant difference of at least 4 σ, known as the "Hubble tension" [10].

For example, for the first method, in 2020, Coughlin, M. W. et al. [3] obtained values of $H_0 = 73.8^{+6.3}_{-5.8}$ km s^{-1} Mpc^{-1} and $H_0 = 71.2^{+3.2}_{-3.1}$ km s^{-1} Mpc^{-1}. For the second method, in 2024 Pasham, D. R. et.al [11] used H_0 = 67.4 km s^{-1} Mpc^{-1} when using the standard ΛCDM cosmological model. The deviation between the two methods original from early cosmology. American science writer Anil Ananthaswamy said, "Galli and her Planck colleagues calculate H_0 to be about 67.8. It has proved impossible to reconcile this with the SH0ES value of 73, says Galli. "[12] Therefore, there is an urgent need for an accurate method to measure H_0 values. When calculating the annual increase in lunar distance above, we derived equations (33) and (34). By the equations, it can be seen that the reciprocal of the age of the universe is directly proportional to the annual increase in the distance from the moon, ΔR. The reciprocal of the age of the universe is the Hubble constant. Here, the ages of the universe for this year and last year are considered equal, that is, $t_1 = t_2 = t$. Therefore, as long as the annual increase in distance from the moon is accurately measured, the Hubble constant can be accurately calculated. According to equation (33)

$$H_0 = \frac{1}{t} = \frac{3}{4} \cdot \frac{\Delta R}{R_1} = \frac{3 \times 3.82 \times 10^{-2}}{4 \times 3.85 \times 10^{8}} = \frac{1}{1.344 \times 10^{10}} (\text{yer}^{-1});$$

$$\text{that is,} \quad H_0 = 72.7 \text{ km s}^{-1} \text{ Mpc}^{-1}. \qquad (35)$$

The derivation of all formulas in this article is rigorous and does not introduce any artificial cosmological parameters. In equation (35), 3/4 is an accurate number introduced by two exponents. If the annual increase in the distance from the moon ΔR = 3.82±0.07 cm is accurate and reliable, then the Hubble constant H_0 = 72.7kms^{-1} Mpc^{-1} and the age of the universe t = 1.344x10^{10} years must also be accurate and reliable.

Conclusion

Einstein was the founder of modern cosmology, and his cosmological equations laid the foundation for quantitative cosmological research. But his cosmological equation is difficult to obtain accurate solutions and contains many unknown factors. In contrast, the Hubble model is a summary of measurement results, which only uses luminous celestial bodies in the universe as some markers for space measurement, without relying on the mass, size, or density of celestial bodies in the sky. So, the Hubble formula is a quantitative empirical formula. This article transforms the Hubble model into a rigorous quantitative theoretical model by differentiating equation (1). Based on this, we have found a new method for measuring the Hubble constant using lunar laser ranging data.

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