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Electromagnetic Waves Traveling on a Line of Force and Gravitational Force Waves Traveling along a Line of Force

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ABSTRACT

This paper proposes, based on Maxwell's equations and the fundamental fields of the electric flux density vector **D** [As/m²] and magnetic flux density vector **B** [Vs/m²] in SI units, that the vector product of **D** and **B** can be defined as a quadrupled density $q(h)$, which is equivalent to the electromagnetic impulse density $\rho(I)$. These quantities are mathematically equivalent in unit analysis:

$$q(h) = \rho(I) = \mathbf{D} \times \mathbf{B}\ [Js/m^4] = [Ns/m^3] = [kg \cdot m/s]$$

Under the condition that the unidimensional terms satisfy an exact differential equation, and assuming that the derivative of $q(h)$equals that of $\rho(I)$, the unidimensional Maxwell's equations (based on permittivity εand permeability μin vacuum) can theoretically establish relationships among energy density $\rho(E)$, impulse density $\rho(I)$, and mass density $\rho(m)$.

These are expressed as:

$$\rho(E) = \frac{1}{2}\left(\frac{B^2}{\mu} + \frac{D^2}{\varepsilon}\right), \rho(I) = b\varepsilon\mu\rho(E), \rho(m) = \frac{1}{2}(\varepsilon B^2 + \mu D^2) = \varepsilon\mu\rho(E) = \frac{\rho(E)}{b^2}$$

where b is the speed of wave beam with mass in itself, different from the conventional speed of the light c that is regarded a photon as a particle with no mass, equals the inverse of electromagnetic constant product of permittivity ε and permeability μ, εb=1/μb, $\varepsilon\mu b^2$=1.

These relations correspond to Einstein's mass–energy equation. Furthermore, integrating $\rho(I)$over an infinitesimal volume dVand spatial element dzleads to an electromagnetic indeterminacy, expressed as the Planck constant wave:

$$h = \int_0^{\Delta z}\left(\int_0^V \rho(I)\, dV\right) dz = M\Delta z$$

where $M=\int_0^V \rho(I)\, dV$ represents electromagnetic momentum. This expression corresponds to Heisenberg's uncertainty principle.

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Introduction

This study aims to provide simpler solutions to unresolved problems in physics, favoring more intuitive approaches than existing theories.

The paper challenges several established concepts, including:

- Electromagnetic waves being purely transverse,
- Lorentz transformations based solely on the speed of light,
- Einstein's postulates.

The key contributions of this study are as follows:

1. The coexistence of traditional transverse waves with a newly proposed progressive longitudinal wave in electromagnetic propagation, a system of the coexistence can be easily explainable A description of light bending within a Cartesian coordinate framework.
2. A new wave transformation based on the reciprocal of the speed of light squared ($1/b^2$) and $\varepsilon\mu$.
3. A new definition of a quadrupled density wave vector q(h) z, defined as the vector product of the electric field density Dx and magnetic field density By, equivalent to impulse density ρ(I)z, where x, y, z are the unit vectors in the x-axis, the y-axis, and the z axis for the wave to travel, respectively.
4. An electromagnetic indeterminacy derived from integrating

q(*h*), corresponding to Heisenberg's uncertainty principle [1-6].

A Simple Mathematical Line
In this paper, the properties of a unidimensional line in mathematics are defined and postulated as follows:
- Straightness
- Continuity
- Integrability
- Differentiability
- A line with no extent
- The line is associated with a function g(z,t), which satisfies an exact differential equation, with time variable tand spatial variable zalong the z-axis in a Cartesian coordinate system [7].

Properties of the Exact Differential Equation
According to the mathematical text, the differential function *g*(*z*,*t*) in a rectangular (Cartesian) coordinate system is expressed as [7]:

$$dg = \frac{\partial g}{\partial z}dz + \frac{\partial g}{\partial t}dt = 0 \tag{1}$$

$$\frac{\partial g}{\partial z} = -\left(\frac{dt}{dz}\right)\frac{\partial g}{\partial t} \tag{2}$$

$$\frac{\partial g}{\partial t} = -\left(\frac{dz}{dt}\right)\frac{\partial g}{\partial z} \tag{3}$$

$$\frac{\partial^2 g}{\partial z\,\partial t} = \frac{\partial^2 g}{\partial t\,\partial z} \tag{4}$$

$$\frac{\partial^2 g}{\partial z^2} = \left(\frac{dt}{dz}\right)^2\frac{\partial^2 g}{\partial t^2} \tag{5}$$

$$\frac{\partial^2 g}{\partial t^2} = \left(\frac{dz}{dt}\right)^2\frac{\partial^2 g}{\partial z^2} \tag{6}$$

$$\left(\frac{dz}{dt}\right)^2 = \frac{\frac{\partial^2 g}{\partial t^2}}{\frac{\partial^2 g}{\partial z^2}} \tag{7}$$

$$\left(\frac{dt}{dz}\right)^2 = \frac{\frac{\partial^2 g}{\partial z^2}}{\frac{\partial^2 g}{\partial t^2}} \tag{8}$$

The wave equations (5) and (6) indicate that no physical medium is required for wave propagation, in special, applying their equations to gravitational wave to travel along a line of the line, detailed hereafter. However, a guiding structure is necessary for the wave to travel along a defined line, such as the mathematical line described above.

Electromagnetic Waves Based on Fundamental Terms
According to standard electromagnetic theory, different unit systems lead to different interpretations of field products:
- **Case 1:** Electric field (V/m) and magnetic field (A/m)→ Product gives W/m^2, representing electromagnetic power density.
- **Case 2:** Electric field (V/m) and magnetic flux (Vs/m²)→ Product gives V^2 s/m^2, which lacks clear physical interpretation.
- **Case 3 (Proposed):** Electric flux density **D** [As/m²] and magnetic flux density **B** [Vs/m²]

→ Vector product:

$$\mathbf{D} \times \mathbf{B}[Js/m^4] = [Ns/m^3]$$

This represents electromagnetic energy density and mechanical impulse density.

Based on this, **D** and **B** are selected as the fundamental variables in this study.

According to Maxwell's theory, **D** is perpendicular to **B**, hence:

$$\mathbf{D}\cdot\mathbf{B}=0 \tag{9}$$

$$\mathbf{D}\times\mathbf{B}\neq 0 \tag{10}$$

The vector product produces a directional propagation along the z-axis with unit vector **z**, representing wave motion at speed *b*:

$$\mathbf{bz}\times\mathbf{D}\equiv\varepsilon\mathbf{B} \tag{11}$$

$$\mathbf{B}\times\mathbf{bz}\equiv\mu\mathbf{D} \tag{12}$$

Taking scalar products:

$$\mathbf{B}\cdot(\mathbf{bz}\times\mathbf{D})=\varepsilon B^2 \tag{13}$$

$$\mathbf{D}\cdot(\mathbf{B}\times\mathbf{bz})=\mu D^2 \tag{14}$$

Using the triple scalar product identity:

$$\mathbf{B}\cdot(\mathbf{bz}\times\mathbf{D})=\mathbf{D}\cdot(\mathbf{B}\times\mathbf{bz})$$

we obtain:

$$\varepsilon B^2=\mu D^2$$

$$\frac{B^2}{\mu} = \frac{D^2}{\varepsilon}\left[\frac{J}{m^3}\right] \tag{15}$$

This relationship represents electromagnetic energy density.

Furthermore, the vector product **D×B**indicates that electromagnetic waves consist of both transverse and longitudinal components, exhibiting both electromagnetic and mechanical properties. These characteristics can be theoretically derived from unidimensional Maxwell's equations, as discussed in the following sections [8-11].

Electromagnetic Waves Expressed Using D and B Fields
According to Maxwell's formulation (without explicit vector notation), electromagnetic waves (EMWs) consist of both electric flux density D and magnetic flux density B, under the condition that D is perpendicular to B, and they oscillate in phase but at right angles in vacuum [12].

For waves with frequency fand wave-number vector k, propagating along the z-axis with unit vector z, the fields can be expressed as:

$$\mathbf{D} = \boldsymbol{x}\frac{A(D)}{\sqrt{2}}\exp(j2\pi ft)\exp(j2\pi kz) \quad (16)$$

$$\mathbf{B} = \boldsymbol{y}\frac{A(B)}{\sqrt{2}}\exp(j2\pi ft)\exp(j2\pi kz) \quad (17)$$

where √2represents the root mean square, and $A(D)$and $A(B)$are the amplitudes of the respective fields.

Since **D** is perpendicular to **B**, their scalar product is zero:

$$\mathbf{D}\cdot\mathbf{B}=0$$

This corresponds to a transverse standing wave component. However, the vector product **D**×**B**is non-zero and provides a directional propagation along the z-axis.

Thus, the vector product generates a progressive longitudinal wave component along the z-axis. Based on this, it is proposed that electromagnetic waves consist of both transverse and longitudinal components.

Electromagnetic Energy Density and Mass Density

Using the relationship between **D** and **B** from Equation (15), the empirical Poynting energy density can be expressed as [13]:

$$\rho(E) = \frac{1}{2}\left(\frac{D^2}{\varepsilon} + \frac{B^2}{\mu}\right) = \frac{D^2}{\varepsilon} = \frac{B^2}{\mu}\left[\frac{J}{m^3}\right] \quad (18)$$

Here, $\varepsilon$and μrepresent the permittivity and permeability in vacuum, respectively.

Multiplying $\rho(E)$by the electromagnetic constant $\varepsilon\mu$, we obtain the electromagnetic mass density:

$$\rho(m) = \frac{1}{2}(\mu D^2 + \varepsilon B^2) = \mu D^2 = \varepsilon B^2\left[\frac{kg}{m^3}\right] \quad (19)$$

This expression represents electromagnetic mass and suggests that both **D** and **B** fields can be described as wave quantities associated with mass density.

Substituting $\rho(m)$into the wave equation (20), we obtain:

$$\frac{\partial^2\rho(m)}{\partial z^2} = \left(\frac{dt}{dz}\right)^2\frac{\partial^2\rho(m)}{\partial t^2} \quad (20)$$

Using the relations:

$$\left(\frac{dt}{dz}\right)^2 = \left(\frac{k}{f}\right)^2 = \varepsilon\mu \quad (21)$$

$$b = \pm\frac{dz}{dt} = \pm\frac{f}{k} = \pm\frac{1}{\sqrt{\varepsilon\mu}} \quad (22)$$

Here, b represents the propagation speed of the longitudinal wave. The notation b is used instead of the conventional speed of light cto avoid assumptions related to photon particle interpretation.

Thus, the objective is to examine whether Equations (18) and (19) can be derived directly from Maxwell's equations.

Derivation of Electromagnetic Energy and Mass from Unidimensional Maxwell's Equations

A Simple Force Line for Wave Propagation

Experimental observations suggest that:

- A straight line can represent a gravitational longitudinal force between masses.
- A straight line can also represent an electromagnetic longitudinal force between charged entities.

Two important properties are noted:

1. Particles with mass cannot reach the speed of light.
2. Waves propagate at the speed of light in vacuum.

Based on this, the properties of a line of force are defined as:

- Homogeneity
- Isotropy
- Perfect elasticity (no energy loss)
- Perfect flexibility (no energy loss)
- Perfect transmission (no energy loss)
- Mathematical integrability
- Mathematical differentiability
- Mathematical continuity
- Finite extent
- No dispersion without external interaction

Under these assumptions, both electromagnetic and gravitational waves propagate along such lines of force and satisfy exact differential equations in terms of time tand space z (equation 7 & 8).

Gravitation

Newton's Law of Universal Gravitation

According to Newton's law of universal gravitation, every particle attracts another with a force proportional to the product of their masses and inversely proportional to the square of the distance between them [14]:

$$F = G\frac{m_1 m_2}{d^2} = m_2\alpha[N]$$

where:

- $d$is the distance between the centers of mass,
- $m_1$and $m_2$are the masses,
- $\alpha$is the acceleration.

To avoid singularity at d=0, the force is expressed using mass density $\rho(m_1)$over volume V:

$$F = \frac{V}{d^2}G\rho(m_1)m_2$$

$$\rho(F) = \frac{F}{V} = \frac{G\rho(m_1)m_2}{d^2}[N/m^3]$$

As d→0:

$$\lim_{d\to 0}\frac{V}{d^2} = 0$$

Thus, the force density formulation avoids singularity issues.

Under conditions of no external force, the force density satisfies:

$$d\rho(F) = \frac{\partial\rho(F)}{\partial t}dt + \frac{\partial\rho(F)}{\partial z}dz = 0$$

$$\frac{\partial\rho(F)}{\partial t} = 0,\text{or}\frac{\partial\rho(F)}{\partial z} = 0$$

Hence, the force density remains constant in time and space.

Newton's Laws of Motion

According to Newton's laws of motion, these laws describe the relationship between the motion of an object and the forces acting upon it [15]. Since motion can be represented in a unidimensional form, it is postulated here that motion satisfies an exact differential equation.

For a particle of mass m[kg] moving with velocity v[m/s] along a line, the momentum is defined as:

$$M \equiv mv[kg \cdot m/s]$$

The impulse is defined as:

$$I \equiv F\Delta t[Ns]$$

First Law

According to Newton's first law, a body at rest or moving at constant velocity in a straight line will remain in that state unless acted upon by an external force [16].

Assuming no change in mass distribution, the momentum remains constant in time and space. This can be expressed as:

$$d\rho(M)=d(\rho(m)v)=0[kg \cdot m/s] \tag{23}$$

Second Law

According to Newton's second law, the rate of change of momentum is equal to the applied force [16].

Thus:

$$d\rho(M)=d(\rho(m)v)\neq 0$$

The force density is defined as:

$$\rho(F) = \frac{d\rho(I)}{dt} = \frac{d(\rho(m)v)}{dt} = \rho(m)\frac{dv}{dt}$$

The impulse density becomes:

$$\rho(I) \equiv \rho(F)\,dt = \rho(m)\,dv$$

Integrating:

$$\rho(I) = \int \rho(F)\,dt = \int \rho(m)\,dv = \rho(M_a) - \rho(M_b)$$

where subscripts aand brepresent the states after and before interaction, respectively.

Third Law

According to Newton's third law, forces between two bodies are equal in magnitude and opposite in direction [16].

Assuming unidimensional force densities $\rho(F_s)$(second law) and $\rho(F_t)$(third law), and applying the exact differential condition:

$$d\rho(I) = \frac{\partial\rho(I)}{\partial z}dz + \frac{\partial\rho(I)}{\partial t}dt = 0 \tag{24}$$

$$\rho(F_s) = \frac{\partial\rho(I)}{\partial t} = -\frac{\partial\rho(I)}{\partial z}\frac{dz}{dt} \tag{25}$$

Using the relation:

$$\rho(E) = \rho(I)\frac{dz}{dt}$$

we obtain:

$$\frac{\partial\rho(E)}{\partial z}dz + \frac{\partial\rho(E)}{\partial t}dt = \rho(F_s)dz + \frac{\partial\rho(E)}{\partial t}dt = 0 \tag{26}$$

Also:

$$d\rho(E) = \frac{\partial\rho(E)}{\partial z}dz + \frac{\partial\rho(E)}{\partial t}dt = \rho(F_t)dz + \frac{\partial\rho(E)}{\partial t}dt = 0 \tag{27}$$

Thus:

$$\rho(F_t)dz+\rho(F_s)(-dz)=0 \tag{28}$$

This represents the balance of forces in accordance with Newton's second and third laws.

Substructure of a Corpuscular Element

Experimental observations show that electromagnetic waves interact with external electric and magnetic fields [17-19]. Additionally:

- Electron–positron annihilation produces electromagnetic waves [20].
- High-energy electromagnetic waves can generate electron–positron pairs [21].

Based on these, the following postulates are proposed for an electromagnetic corpuscular substructure:

- An infinitesimal dipole element
- Indivisible element
- Contains positive and negative fractional charges
- Oscillates continuously
- Positive charge produces diverging electric fields; negative charge produces converging fields
- Electric field generates current density:

Notation: The current density can have an indispensable circuit due to a dipole with diverging and converging flux density D in vacuum.

$$J_A = \frac{\partial D}{\partial t}[A/m^2]$$

- Magnetic field induced by current:

$$J_V = \frac{\partial B}{\partial t}[V/m^2]$$

- Closed current paths form due to dipole structure, when making the paths consecutive, so the paths can produce an electric circuit with a half path to go head from a source of

the wave and the other path to return the source.
- Rosaries of continuous and consecutive corpuscular with a dipole produce electromagnetic waves in vacuum

The fields are expressed as:

$$\boldsymbol{D} = \boldsymbol{x}\frac{A(D)}{\sqrt{2}}e^{\pm j2\pi ft}e^{\mp j2\pi(k_x x+k_y y)} \tag{29}$$

$$\boldsymbol{B} = \boldsymbol{y}\frac{A(B)}{\sqrt{2}}e^{\pm j2\pi ft}e^{\mp j2\pi(k_x x+k_y y)} \tag{30}$$

Since **D**·**B**=0, the scalar product represents a standing wave. The vector product leads to propagation along the z-axis:

$$k=k_x\times k_y=k_z$$

Thus, the wave equation becomes:

$$\frac{\partial^2 g}{\partial z^2} = \left(\frac{k}{f}\right)^2\frac{\partial^2 g}{\partial t^2}$$

A new electromagnetic impulse density is defined as:

$$\rho(I) = \frac{A(I)}{2}e^{\pm j4\pi ft}e^{\mp j4\pi(k_z)}$$

Substituting into the wave equation gives:

$$\frac{\partial^2 \rho(I)}{\partial z^2} = \left(\frac{dt}{dz}\right)^2\frac{\partial^2 \rho(I)}{\partial t^2}$$

$$\frac{dt}{dz} = \pm\frac{k}{f}, b\boldsymbol{z} = \pm\frac{f}{k}\mathbf{z}$$

where b represents the longitudinal wave speed to travel along the z-axis with the unit vector **z**.

Electromagnetic Waves and Impulse Density Equation

This study derives electromagnetic wave mass from unidimensional Maxwell's equations (31), assuming a line of electromagnetic force characterized by permittivity εand permeability μ.
The constancy of wave propagation is expressed as:

$$d\left(\frac{dt}{dz}\right) = 0, d\left(\frac{dz}{dt}\right) = 0 \tag{31}$$

The electromagnetic impulse density is defined as:

$$q(h)_z = \rho(I)_z = \mathbf{D}\times\mathbf{B}[Js/m^4] = DB\boldsymbol{z}\left[\frac{Ns}{m^3}\right] \tag{32}$$

For wave propagation along the z-axis:

$$\frac{\partial}{\partial x} = 0, \frac{\partial}{\partial y} = 0, D_z = 0, B_z = 0 \tag{33}$$

From Maxwell's equations:

$$-\frac{\partial B_y}{\partial z} = \mu\frac{\partial D_x}{\partial t}, \frac{\partial B_x}{\partial z} = \mu\frac{\partial D_y}{\partial t}$$

$$-\frac{\partial D_y}{\partial z} = \varepsilon\frac{\partial B_x}{\partial t}, \frac{\partial D_x}{\partial z} = \varepsilon\frac{\partial B_y}{\partial t}$$

Thus, the derivative of the vector product becomes:

$$d\rho(I)\mathbf{z}=d(\boldsymbol{D}\times\boldsymbol{B})=b\ \mathbf{z}d\rho(m) \tag{34}$$

Electromagnetic Waves and Impulse Density Equation (Continued)

From Equations (35)- (36), the total differential of impulse density is:

$$d\rho(I) = \frac{\partial\rho(I)}{\partial t}dt + \frac{\partial\rho(I)}{\partial z}dz = b\left(\frac{\partial\rho(m)}{\partial t}dt + \frac{\partial\rho(m)}{\partial z}dz\right) \tag{35}$$

The differential mass density is defined as:

$$d\rho(m) \equiv \frac{1}{2}(\mu\, dD^2+\varepsilon\, dB^2)\left[\frac{kg}{m^3}\right] \tag{36}$$

where:

$$dD^2 = dD_x^2 + dD_y^2, dB^2 = dB_x^2 + dB_y^2$$

$$dD_x = \frac{\partial D_x}{\partial t}dt + \frac{\partial D_x}{\partial z}dz, dD_y = \frac{\partial D_y}{\partial t}dt + \frac{\partial D_y}{\partial z}dz$$

$$dB_x = \frac{\partial B_x}{\partial t}dt + \frac{\partial B_x}{\partial z}dz, dB_y = \frac{\partial B_y}{\partial t}dt + \frac{\partial B_y}{\partial z}dz$$

Using these relations together with the energy density (18), we obtain:

$$\rho(I) = b\rho(m) = b\varepsilon\mu\rho(E)\left[\frac{Ns}{m^3}\right] \tag{37}$$

$$\rho(m) = \varepsilon\mu\rho(E) \tag{38}$$

Thus, the longitudinal wave speed squared becomes:

$$b^2 = \frac{\rho(m)}{\rho(E)} = \frac{1}{\varepsilon\mu}\left[\frac{m^2}{s^2}\right] \tag{39}$$

Integrating the impulse density over volume:

$$m(\text{EMW}) = E(\text{EMW})\ \varepsilon\mu \tag{40}$$

$$E(\text{EMW}) = m(\text{EMW})\ b^2 \tag{41}$$

where m("EMW")and E("EMW")represent electromagnetic wave mass and energy, respectively [22].

$$\boldsymbol{D} = \boldsymbol{x}\frac{A(D)}{\sqrt{2}}e^{\pm j2\pi ft}e^{\mp j2\pi(k_x x+k_y y)} \tag{42}$$

$$\boldsymbol{B} = \boldsymbol{y}\frac{A(B)}{\sqrt{2}}e^{\pm j2\pi ft}e^{\mp j2\pi(k_x x+k_y y)} \tag{43}$$

Since:

D·B=0

the scalar product corresponds to a standing transverse wave in the x"-" yplane.

The vector product:

$$\mathbf{D}\times\mathbf{B} \propto e^{j2\pi kzz}$$

produces a progressive longitudinal wave along the z-axis, where:

$$k=k_x\times k_y=k_z \tag{44}$$

Thus, the unidimensional wave equation becomes:

$$\frac{\partial^2 g}{\partial z^2} = \left(\frac{k}{f}\right)^2 \frac{\partial^2 g}{\partial t^2}$$

A corresponding impulse wave is defined as:

$$\rho(I) = \frac{A(I)}{2} e^{\pm j4\pi f t} e^{\mp j4\pi k_z z} \tag{45}$$

Substitution yields:

$$\frac{\partial^2 \rho(I)}{\partial z^2} = \left(\frac{dt}{dz}\right)^2 \frac{\partial^2 \rho(I)}{\partial t^2} \tag{46}$$

$$\frac{dt}{dz} = \pm\frac{k}{f}, b = \pm\frac{f}{k} \tag{47}$$

Electromagnetic Wave Forces and Newton's Laws

First Law

Electromagnetic waves do not have a stationary state. Their propagation is characterized by:

$$db=0 \tag{48}$$

$$\varepsilon\mu\neq 0, f\neq 0, k\neq 0$$

This implies continuous propagation without rest.

Second Law

The energy density satisfies:

$$d\rho(E) = \frac{\partial \rho(E)}{\partial z}dz + \frac{\partial \rho(E)}{\partial t}dt = 0 \tag{49}$$

Defining force density:

$$\rho(F_s) \equiv \frac{\partial \rho(E)}{\partial t}$$

Similarly:

$$\rho(F_t) \equiv \frac{\partial \rho(I)}{\partial t}$$

Third Law

Using the relation:

$$\rho(E) = b\rho(I)$$

we obtain:

$$\rho(F_t) = \frac{1}{b}\frac{\partial \rho(E)}{\partial t} \tag{50}$$

$$\rho(F_t)dz = \rho(F_s)dt \tag{51}$$

Integrating:

$$\rho(E) = \int \rho(F_t)\, dz = \int \rho(F_s)\, dt \tag{52}$$

This expresses equivalence between forces analogous to Newton's second and third laws [1].

Electromagnetic Indeterminacy Equations

Integrating impulse density:

$$h = \int \rho(I)\, dV\, dz = M\Delta z \tag{53}$$

where:
- Mis electromagnetic momentum: M=∫ρ(I)" " dV
- his Planck's constant

Using $dz= b\ dt$:

$$h = Mb\Delta t = E\Delta t \tag{54}$$

This represents an uncertainty relation between energy and time.

Using equation (47) and an arbitrary of the equation,

$k\left[\frac{1}{m}\right]\Delta z[m] = k\Delta z[1], or, f\left[\frac{1}{s}\right]\Delta t[s]\ f\Delta t[1]$, applying equation (53), (54) we can get equations of EM energy E and momentum M as follows.

$$M\mathbf{z} = \frac{h}{\Delta z} = hk_z\mathbf{z} \tag{55}$$

$$E = \frac{h}{\Delta t} = hf \tag{56}$$

The above equation can easily be explainable to the thermal radiation in the electromagnetic energy E, or, to the photoelectric effect for the momentum M to be forced an electron emitted from a surface of a metal [24].

Difference between Electromagnetic and Gravitational Waves

Both electromagnetic and gravitational waves propagate along lines of force; however, they differ fundamentally in origin, interaction, and propagation characteristics [25].

Table 1: Difference of Electromagnetic Wave and Gravitational Wave

No.	Items	Unidimensional Longitudinal Wave (ULW)	
		Electromagnetic Wave (EMULW)	Gravitational Wave (GULW)
1	Straightness	The wave travels along on a line	
2	Guide	A line of force between entities with charge	A line of force between particles with mass
3	Medium to travel in vacuum space	A line of the inherent medium with permittivity ε and permeability μ	Line of the inherent medium with perfect elasticity with bulk modulus K
4	Properties of the longitudinal speed of the wave	The speed of light : b > v, $b^2 = \frac{1}{\varepsilon\mu} = \frac{\rho(E:EM)}{\rho(m:EM)}$, where ρ(E: EM) ρ(m: EM) are electromagnetic energy density and mass density, respectively. The derivative of b is zero.	The speed of light unattainable: v $v^2 = \frac{\rho(E:g)}{\rho(m:g)}$, where ρ(E: g) and ρ(m: g) are an elastic line of energy density and mass density, respectively. The derivative of v is non-zero.
5	Estimation of speed of gravitation based on ratio of the dark matter to dark energy	$\frac{\rho(E:dark)}{\rho(m:dark)} = \frac{\rho(E:EM)}{b^2\rho(m:EM)} + \frac{\rho(E:g)}{v^2\rho(m:g)} = 1 + \frac{\rho(E:g)}{v^2\rho(m:g)} = \frac{68\%}{27\%} \approx 2.52$ [3.8.1] $\therefore\ \rho(E:g) = 1.52v^2\rho(m:g) = b^2\rho(m:g)(\frac{1.52v^2}{b^2}) \approx b^2\rho(m:g)\left(\frac{v}{0.8b}\right)^2$ $\therefore$ *gravitational speed*: $v \approx 0.8b$ where $\rho(E{:}dark) = \rho(E{:}EM) + \rho(E{:}g)$, $\rho(E{:}\ EM{:}dark) = b2\rho(m{:}EM)$, $\rho(E{:}\ g{:}dark) = v2\rho(m{:}g)$	
6	Coexistence of the waves	Coexistence of the transverse and longitudinal wave	Unknown
7	Path for the wave to travel	A circuit with progressive and retrogressive paths for the wave to travel in itself	One way path in itself under no external force
8	Help the other wave	Help the sound wave use the line	Help the sound wave use the line
9	Exclusive property	Optical reaction of waves radiated from a source	Unknown
10	Bunching	A line making all lines bunch up	A line making all lines bunch up
11	Non-determinacy	Electromagnetic Indeterminacy	Hesterberg uncertainty

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